Illiquidity in Sovereign Debt Markets*

Juan Passadore † **Yu Xu** ‡

December 5, 2020

Abstract

We study sovereign debt and default policies when credit and liquidity risk are jointly determined. To account for both types of risks we focus on an economy with incomplete markets, limited commitment, and search frictions in the secondary market for sovereign bonds. We quantify the effect of liquidity on sovereign spreads and welfare by performing quantitative exercises when our model is calibrated to match key features of the Argentinean default in 2001. From a positive point of view, we find (a) that a substantial portion of sovereign spreads is due to a liquidity premium, and (b) the liquidity premium helps to resolve the "credit spread puzzle," by generating high mean spreads while maintaining a low default frequency. From a normative point of view, we find that reductions in secondary market frictions improve welfare.

Keywords: Credit Risk, Liquidity Risk, Sovereign Debt, Open Economies.

JEL classification: D83, E32, E43, E44, F34, G12, O11.

*We would like to thank Robert Townsend, Ivan Werning, George-Marios Angeletos, Alp Simsek, Gadi Barlevy, Nicolas Caramp, Satyajit Chatterjee (discussant), Arnaud Costinot, Claudio Michelacci, Athanasios Orphanides, Juan Carlos Hatchondo, Ignacio Presno, Dejanir Silva, Robert Ulbricht, Francesco Lippi, Saki Bigio, Laura Sunder-Plassmann (discussant) and seminar participants at MIT, European University Institute, International Monetary Fund, Central Bank of Chile, University of Naples, Toulouse School of Economics, Tinbergen Institute, Universidad Di Tella, and conference participants at SED Meetings in Poland, RIDGE, New Faces in Macro Madrid, LACEA Medellin, and the Central Bank of Uruguay, for helpful comments. We thank Marzio Bassanin, Lucas Belmudes, Adriana Grasso, Cesar Urquizo and Santiago Varela for excellent research assistance and the Macro-Financial Modeling Group from the Becker and Friedman Institute for financial support. All errors are our own. First version: November 2014. This version: December 2020.

†Einaudi Institute for Economics and Finance. Email: juan.passadore@eief.it

‡Lerner College of Business and Economics, University of Delaware. Email: yuxu@udel.edu.

1 Introduction

Sovereign countries borrow to smooth shocks. One friction that prevents the smoothing of expenses over time and states of nature is governments' inability to commit to future debt and default policies. This lack of commitment implies that the government will repay its debts only if it is convenient to do so and will dilute debt holders whenever it sees fit. To compensate investors for bearing these risks the sovereign pays a *credit risk premium* that reduces the available resources for domestic consumption and can substantially increase borrowing costs during bad times. The sovereign debt literature has helped us to understand how a lack of commitment shapes the outcomes of sovereign countries from a positive point of view and what policies are desirable from a normative point of view.[1]

The recent European debt crisis, however, has underscored that decentralized markets impose additional frictions that prevent smoothing by sovereign countries.[2] Their bonds are traded in over-the-counter markets, where trading is infrequent. Thus, if an investor holds a large position in a sovereign bond, it might take time to find a counterparty willing to trade at a fair price. For this reason, investors need to be compensated not only for the risk of default or dilution but also for illiquidity, which introduces, in addition, a *liquidity risk premium*. This liquidity premium further reduces available resources and constrains sovereign policies. So far, the literature has been silent about this feature of sovereign borrowing. Our objective in this paper is to fill this gap by answering the following questions. How do credit and liquidity premiums interact? What portions of total spreads can be explained by credit and by liquidity? What are the welfare gains of reducing frictions in the secondary market?

Our paper contributes to the literature on sovereign borrowing in two ways. First, we propose a tractable model of sovereign borrowing in which credit and liquidity premia jointly determine borrowing and default decisions. Second, in a quantitative exploration focusing on one of the most studied cases of sovereign default, Argentina's default in 2001, we show that the liquidity premium is a substantial component of total spreads,

[1]See for example Aguiar and Gopinath (2006), Arellano (2008), and see Aguiar and Amador (2014) for a recent review . Recent contributions to the quantitative literature of sovereign debt, among many others, include Arellano and Bai (2014), Chatterjee and Eyigungor (2015), Hatchondo et al. (2016), Pouzo and Presno (2016), Aguiar et al. (2017), Arellano and Bai (2017), Bianchi et al. (2018), Dovis (2018), Roch and Uhlig (2018), Ottonello and Perez (2018), Perez et al. (2019), Sanchez et al. (2018), Arellano et al. (2019) and Bocola and Dovis (2019).

[2]For example, Pelizzon et al. (2016) documents substantial liquidity frictions for Italian bonds during the recent crises. In addition to the recent evidence from sovereign debt markets, a large literature that studies corporate bonds has documented sizable trading frictions and liquidity premia in the US corporate debt market. See for example Edwards et al. (2007) and Longstaff et al. (2005).

and helps to rationalize the "credit spread puzzle" by simultaneously generating high credit spreads and a low default frequency. Furthermore, we show that, through the lens of our model, the welfare gains from eliminating liquidity frictions are, quantitatively, of the same order of magnitude as the gains from eliminating income fluctuations.

We begin our paper by constructing a model of sovereign debt where debt and default policies take into account credit and liquidity risk. We focus on a small open economy that borrows from international investors to smooth income shocks following the quantitative literature on sovereign debt that builds on Eaton and Gersovitz (1981). A benevolent government issues non-contingent long-term debt and chooses debt and default policies to maximize household utility. The government cannot commit to future debt and default policies and might default in some states of nature. The distinctive feature of our model, in comparison to the previous literature, is the introduction of search frictions in the secondary market for sovereign bonds, following the literature on over-the-counter markets, such as Duffie et al. (2005). In our model, investors buy bonds in the primary market and can receive idiosyncratic liquidity shocks. If a shock occurs, they will bear a cost for holding the bond and therefore become natural sellers. Due to search frictions in the secondary market, it will take time for them to find a counterparty with whom to transact. As a result, bid-ask spreads arise endogenously through the bargaining between investors and dealers in the over-the-counter bond market.

One of the main features of the model we propose is that default and liquidity risk will be jointly determined. On the one hand, the presence of search frictions in the secondary market introduces a liquidity risk premium that affects prices in the primary market, thereby affecting debt and default policies, which in turn affect the credit risk premium. On the other hand, as the credit risk premium increases, the probability of default also increases, and because investors foresee worse liquidity conditions in the future should a default occur, liquidity conditions will also deteriorate, which in turn increase the liquidity risk premium. Therefore, in our model, the default and liquidity risk premia are jointly determined. This joint determination is important because it will enable us to decompose total spreads into liquidity and credit components and to study the effects on welfare of reducing liquidity frictions in the secondary market.

After building a model of sovereign borrowing in which both credit and liquidity premia constrain the choices of the government, we perform quantitative exercises to assess (1) how much of total spreads are due liquidity frictions, and (2) what the welfare gains from eliminating these frictions would be. To do so, we calibrate the model to match key features of Argentina's default in 2001. In particular, we match debt levels, the mean and volatility of spreads, bid-ask spreads, and bond trading turnover, to counterparts in the data.

Our first quantitative finding is that the liquidity premium is a substantial component of total spreads. For the 1993:I to 2001:IV period, our calibrated model attributes to the liquidity premium (on average) roughly one quarter of the total Argentine sovereign spread. Furthermore, the liquidity component of total sovereign spreads generated by our model is time-varying and is low during good times (when debt levels are low and output is high) and high during bad times. This prediction is consistent with the empirical findings in Bai et al. (2012) and Pelizzon et al. (2016).

A corollary of our first quantitative finding is that accounting for the liquid component of total spreads helps to resolve the "credit spread puzzle. Standard sovereign default models fully attribute the sovereign spread to default risk. As a result, such models require a counter-factually high default rate to match observe levels of sovereign spreads. In contrast, our model attributes a significant portion of the total sovereign spread to liquidity risk and is therefore able to match the mean level of Argentine spreads while simultaneously matching a low annual default frequency of 2.4 percent.

Our second quantitative finding is that the welfare gains from eliminating liquidity frictions are substantial. In particular, using our calibrated model, we find that the welfare gains induced from eliminating secondary market frictions are 0.3 percent in consumption equivalent terms. To put this number in perspective, given the volatility of consumption for Argentina in the period of study, a representative agent in an economy as in Lucas (2003) would pay 0.40 percent in consumption equivalent terms to eliminate income fluctuations.

The source of variation of the liquidity premium in our model, over time and states of nature, is due to what we term as the *maturity extension channel*. The intuition for this channel is that a sovereign default results in the deferral of debt repayment, which results in an increase in the expected maturity of outstanding debt in the run up to a default. The latter extension in expected maturity increases the sensitivity of bond prices to liquidity shocks. As a result, investors require a higher compensation for bearing liquidity risk (i.e., a higher liquidity premium) as the default probability increases.

We believe that the distinction between credit and liquidity risk is important for the design of debt policies for at least two reasons. First, in the long run, the policies to mitigate lack of commitment differ from those to mitigate frictions in the secondary market. For example, Hatchondo et al. (2016) and Chatterjee and Eyigungor (2015) show that fiscal rules and covenants on debt improve welfare in models when the government lacks commitment. However, policies that would decrease the liquidity premium in the long run include the development of a centralized exchange for sovereign bond trading or increasing transparency in the secondary market, as reported in Edwards et al. (2007). Second, the policies implemented during a short-term crisis might also be different. For example, a government could use resources to repay debt or to bail-out financial institutions that hold government debt and are in distress. An alternative policy, focusing on the secondary market, would be to provide liquidity to intermediaries.

Literature Review. The main contribution of our paper is to incorporate secondary bond market liquidity frictions into quantitative models of sovereign default (see Aguiar and Gopinath, 2006 and Arellano, 2008 for early examples) that build on Eaton and Gersovitz (1981). Our model incorporates long-term debt (see, e.g., Hatchondo and Martinez, 2009, Arellano and Ramanarayanan, 2012 and Chatterjee and Eyigungor, 2012) and debt recovery after default (see, e.g., Yue, 2010).

One of the key takeaways of our analysis is that accounting for the liquidity premium allows our model to simultaneously match a high level of credit spread and a low default frequency, thereby providing a potential resolution of the "credit spread puzzle." There are alternative explanations of the this puzzle in the literature. Borri and Verdelhan (2009) empirically document the existence a sovereign risk premium and propose an explanation of this premium by introducing time varying risk aversion, following the work of Campbell and Cochrane (1999). Other explanations of credit spreads that give a role to risk averse international investors are Arellano (2008), Tourre (2017), and Morelli et al. (2019). Pouzo and Presno (2016) provide a resolution of the credit spread puzzle in the same context of our paper, the 2001 Argentinean default, by introducing model uncertainty and ambiguity aversion on the investors' side. Our paper proposes an explanation of the credit spread puzzle which focuses on an aspect of sovereign debt markets that has been overlooked by the previous literature and therefore complements the previous findings.

To model secondary market frictions, we build on the random over-the-counter search framework in Duffie et al. (2005). In this setting, bondholders naturally charge a liquidity premium for holding illiquid sovereign bonds due to the possibility of costly delays when bondholders attempt to offload their positions. In subsequent work to our paper, Chaumont (2020) also incorporates liquidity frictions into a model of sovereign borrowing as in Eaton and Gersovitz (1981), adopting a framework with competitive search (see, for example, Moen, 1997) in which investors direct their search to different sub-markets. In contrast, our paper adopts a framework with random search, which has a long tradition in studying over the counter markets (see, for example, Duffie et al., 2005 and He and Milbradt, 2014). In addition, the source of the liquidity premium in our setting is the *maturity extension channel*. In contrast, this channel is absent in the model of Chaumont (2020) as that model does not feature debt recovery. Instead, the source of variation in the liquidity premium Chaumont (2020) is the equilibrium choice of trading fees.

A recent corporate credit risk literature studies the implications of secondary market search frictions for corporate default risk. He and Milbradt (2014) and Chen et al. (2017) decompose corporate credit spreads into liquidity and credit components. We adapt their decomposition to our sovereign default setting. Our decomposition further accounts for the role of dynamic debt issuance, a feature that is common to sovereign debt models. We show that optimal dynamic debt management by governments can partially mitigate the adverse liquidity-default risk feedback loop. This highlights an additional margin over which active sovereign debt management policies can be of value.

Finally, there is a growing empirical literature which documents a sizeable liquidity component in sovereign spreads.[3] Pelizzon et al. (2016) document a strong relationship between sovereign risk and secondary bond market liquidity for Italian bonds. Bai et al. (2012) documents the presence of a liquidity component in sovereign spreads for a wider set of Eurozone countries. Liquidity risk is documented even for sovereigns with no meaningful default risk. For example, Fleming (2002) finds evidence of liquidity effects in U.S. treasury markets. Our paper complements these empirical studies by quantifying the size of the liquidity premium over the business cycle and to assess the welfare losses due to liquidity frictions.

2 Model

In this section, we present a model of sovereign default with trading frictions in the secondary market. We first describe the setting: section 2.1 describes the macroeconomic environment, section 2.2 describes the secondary bond market, and 2.3 describes the the timing of the model. We then proceed to characterize the equilibrium: section 2.4 characterizes the decisions of the government given prices, sections 2.5 and 2.6 define bond prices and valuations, and section 2.7 defines the equilibrium. Finally, section 2.8 discusses our model's ingredients.

[3]The importance of liquidity risk has also been documented for corporate bond spreads. See, for example, Longstaff et al. (2005), Chen et al. (2007), Edwards et al. (2007), Bao et al. (2011), and Friewald et al. (2012).

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. The small open economy receives a stochastic stream of income denoted by y_t. Income follows a first-order Markov process $\mathbb{P}(y_{t+1} = y' \mid y_t = y)$. The government is benevolent and wants to maximize the utility of the household. To do this, it trades bonds in the international bond market to smooth the household's consumption. The household evaluates consumption streams, c_t, according to:

$$(1-\beta)\,\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

with time-preference $\beta \in (0,1)$ and utility function $u(\cdot)$, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The sovereign issues long-term debt when it is not in default. As in Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012), each unit of debt matures with probability m each period. Non-maturing bonds pay coupon z. This memoryless formulation of the debt maturity structure means that the face value of outstanding debt is the only relevant state variable for the obligations of the government. The government can issue bonds at a price q_t in the primary bond market. In equilibrium, the price of debt depends on current income, y_t, and next period's bond position, b_{t+1} (our convention is that $b_{t+1} > 0$ denotes debt). The budget constraint for the economy is given by:

$$c_t = y_t - [m + (1-m)\,z]\, b_t + q_t\,[b_{t+1} - (1-m)b_t]\,, \tag{2.1}$$

where mb_t is the repayment of principal for maturing debt, $(1-m)zb_t$ is the total coupon payment for non-maturing debt, and $q_t\,[b_{t+1} - (1-m)b_t]$ represents the proceeds from newly issued debt.

There is limited enforcement of debt; thus, the government can default at its convenience. There are two consequences of default. First, the government loses access to the international credit market and goes into autarky. Second, output is lower during default and is given by $y_t - \phi(y_t)$. That is, there is also a direct output cost of default, $\phi(y_t)$, which is a standard assumption in the literature.

The government can regain access to the international credit market with probability θ each period. A fraction of defaulted debt is written off when the government regains access to credit markets. In particular, the new face value of outstanding debt is $\mathcal{R}(b_t) = \min\left\{\overline{b}, b_t\right\}$, where b_t is the face value of defaulted debt and $\bar{b}$ is a maximum recovery value.[4]

[4]Our specification of a maximal value for debt recovery is motivated by the renegotiation models of

That is, the fraction of recovered debt in face value terms is $\mathcal{R}(b_t)/b_t = \min\left\{\overline{b}/b_t, 1\right\}$, and this fraction converges to zero as the amount of defaulted debt goes to infinity. Bond holders of (previously) defaulted bonds receive replacement bonds, with each unit, in face value terms, of defaulted bonds being replaced by new bonds of face value $\mathcal{R}(b_t)/b_t$. The coupon rate and maturity probability of the replacement bonds remain unchanged at z and m, respectively.

2.2 Primary and Secondary Bond Markets

There are two bond markets: the primary market and the secondary market. The government issues debt in the primary bond market. International investors can initially purchase newly issued debt in the primary market. Subsequent trading of bonds occurs in the secondary market. Trading in the secondary bond market is subject to search frictions. We adopt the random search framework as in Duffie et al. (2005) and He and Milbradt (2014) to model these frictions.

Each international investor is assumed to be small and can hold at most a single unit of government debt. These investors are risk-neutral and can be either constrained or unconstrained (Section 4 extends our model to incorporate risk averse investors). Unconstrained investors price bonds by discounting future payoffs at the risk-free rate, r. Unconstrained investors can become constrained if they receive a liquidity shock. Liquidity shocks are idiosyncratic in nature and have a per period probability ζ of occurrence. Constrained investors also discount payoffs at the risk free rate, but are additionally subject to per period holding costs $h_c > 0$.[5] As a result, unconstrained investors have a high bond valuation, q^H, while constrained investors have a low bond valuation, q^L (the exact expressions for q^H and q^L are given given in sections 2.5 and 2.6). Therefore, unconstrained investors are the natural buyers of bonds in both the primary and the secondary markets, while constrained investors are the natural sellers of bonds in secondary markets.

Constrained investors try to offload their bond positions in the secondary market. Secondary market trading is intermediated where the per period contact probability between

Yue (2010) and Bai and Zhang (2012) which endogenize recovery as the equilibrium outcome of a Nash bargaining game between the government and its creditors. These models produce recovery values of the form $\mathcal{R}(\overline{b}(y_t), b_t) = \min\left\{\overline{b}(y_t), b_t\right\}$ with state-dependent maximum recovery values, $\overline{b}(y_t)$. In our simple specification, the maximum recovery value does not vary across states.

[5] We interpret liquidity events as investor-specific events that prompt an immediate need to sell (e.g. to meet an expenditure). Holding costs represent the utility loss associated with delayed transactions. Valuations of debt are non-negative due to free disposal of the asset. When solving the model numerically, we additionally assume that bonds can be freely disposed. This is necessary to ensure that the presence of holding costs do not imply negative bond prices for some off-the-equilibrium path values of the state space. Note that bond prices are always positive along the equilibrium path.

constrained investors and intermediaries is λ.[6] Constrained investors and intermediaries bargain upon making contact. The total surplus is:

$$S = A - q^L,$$

where q^L is the low valuation of the constrained investor, and A denotes the ask price at which the intermediary can then offload the bond. Following He and Milbradt (2014), we assume that there is a large mass of competitive unconstrained investors waiting on the sidelines who intermediaries could contact with immediate effect. This simplifying assumption means that we do not have to keep track of the dealer's inventory as intermediaries could instantaneously offload bonds to high-valuation investors. Since the large mass of high-valuation investors on the sidelines act competitively, intermediaries can offload their positions at high valuations, and thus:

$$A = q^H.$$

The total surplus, $S = q^H - q^L$, is then divided according to a Nash bargaining rule with the bargaining power of the constrained low-valuation investors being $\alpha \in [0,1]$. This implies that the price at which constrained investors sell to intermediaries upon contact is:

$$q^S = q^L + \alpha(q^H - q^L).$$

The proportional bid-ask spread is the difference between intermediaries' selling and buying price, $q^H - q^S$, normalized by the mid-price, $\frac{1}{2}\left(q^H + q^S\right)$, and is equal to

$$ba = \frac{q^H - q^S}{\frac{1}{2}\left(q^H + q^S\right)} = \frac{(1-\alpha)(q^H - q^L)}{\frac{1}{2}\left(q^H + q^L\right) + \frac{\alpha}{2}\left(q^H - q^L\right)}. \tag{2.2}$$

2.3 Timing

The timing for the government is as follows and is summarized in Figure 2.1. First, consider the case in which the government has credit access (i.e., is not in default) and begins

[6] For simplicity, we assume that the matching function between constrained investors and intermediaries implies a constant matching probability, which is standard in the OTC search literature (see, e.g., Duffie et al. (2005), Lagos and Rocheteau (2007), He and Milbradt (2014)). Appendix D shows that the fraction of bonds held by constrained investors is approximately constant over time in our calibrated model. This implies that the number of potential sellers (approximately) scale one-for-one with the level of outstanding debt. A constant matching probability would result in standard matching frameworks under the assumption that the number of potential buyers also scale one-for-one with the level of outstanding debt.

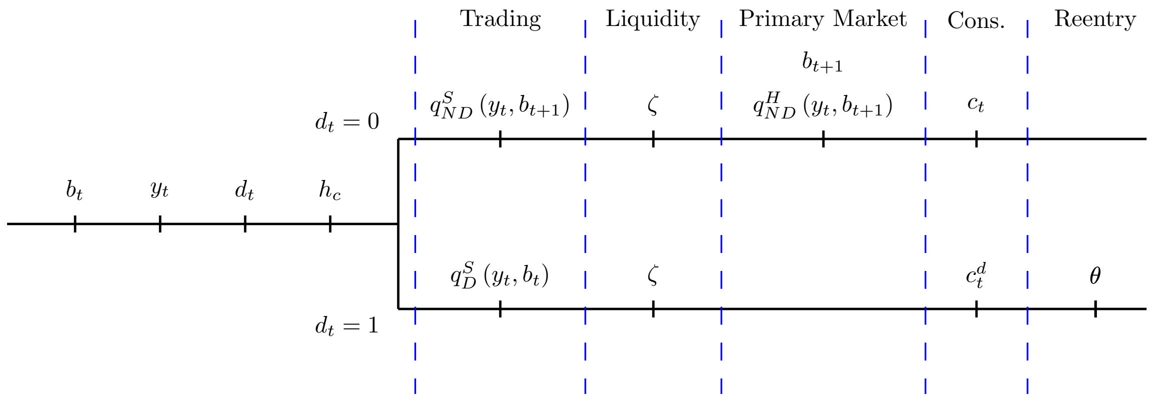

Figure 2.1: **Timing.** This figure summarizes the timing before and after default in period t. The government enters the period with bonds b_t. Then, income, y_t, is realized, and the government chooses whether or not to default, d_t. Constrained investors are subject to holding costs, h_c. The upper branch depicts the sequence of events in the absence of default: secondary market trading of outstanding bonds occurs, and unconstrained investors receive liquidity shocks. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. Then, the liquidity shock is realized. Then, the government issues a face value of debt $b_{t+1} - (1-m)b_t$, facing a price $q^H_{ND}(y_t, b_{t+1})$. Finally, consumption is realized. The lower branch depicts what happens in the case in which the government defaults. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Note that the primary market is closed while the government is in autarky. Then, the government will re-access the debt market in the next period with probability θ. Finally, consumption is equal to $c^d(y_t) = y_t - \phi(y_t)$.

period t with an amount b_t of outstanding debt. Income, y_t, is then realized. The government then decides whether or not to default $d_t \in \{0, 1\}$. If the government chooses not to default ($d_t = 0$), principal payments for maturing debt, mb_t, and coupon payments for non-maturing debt, $(1-m)\, zb_t$, are made. The government can then issue new debt in the primary market. An issuance with face value $b_{t+1} - (1-m)b_t$ leads to outstanding debt with face value b_{t+1} at the beginning of the next period. As previously mentioned, unconstrained investors are the natural buyers of new bond issues, and thus, bonds are always issued at the high valuation, q^H_t. Finally, consumption takes place and is given by $c_t = y_t - [m + (1-m)\, z]\, b_t + q^H_t\, [b_{t+1} - (1-m)b_t]$.

Next, consider the case in which the government is already in default or chooses to default in the current period ($d_t = 1$). In this case, b_t is the amount of debt that is in default. The government is in autarky and cannot borrow. Consumption is simply equal to income adjusted for the costs of default: $c_t = y_t - \phi\,(y_t)$. Nature determines whether the government regains credit access between the end of period t and the beginning of the next period, $t+1$. The probability of regaining credit access is θ, and in that event, the government re-accesses the debt market with an outstanding debt of $\mathcal{R}(b_t) = \min\left\{\overline{b}, b_{t+1}\right\}$

at the beginning of the next period. Otherwise, the government remains in autarky.

The timing for investors is as follows. Investors pay holding costs h_c for the period if they begin period t constrained. Secondary market trading for outstanding bonds occurs once per period, immediately before the government issues new bonds in the primary market. As mentioned in section 2.2, only constrained bond holders attempt to sell in the secondary market. With probability λ, a constrained investor meets an intermediary and offloads his bond position. The transaction price is $q^S_{ND}(y_t, b_{t+1})$ when the government is not in default, and $q^S_D(y_t, b_t)$ when the government is in default. Constrained investors who fail to contact intermediaries remain constrained going into the next period $t+1$.

An investor who is unconstrained at the beginning of period t is not subject to holding costs for the period. However, such an investor can become constrained for the beginning of the next period $t+1$ if he receives a liquidity shock during period t. Liquidity shocks occur with probability ζ and take place after the conclusion of trading in the secondary market. This means that a newly constrained investor in period t is unable to immediately offload his position in the same period. In addition, liquidity shocks occur prior to new bond issuances in the primary market. This implies that the unconstrained investors who purchased newly issued bonds during period t will still be unconstrained at the beginning of period $t+1$.

2.4 The Government's Decision Problem

The government takes the bond price schedule as given and chooses debt and default policies to maximize household welfare. This infinite-horizon decision problem can be cast as a recursive dynamic programming problem. We focus on a Markov equilibrium with income, y, as the exogenous state variable and debt, b, as the endogenous state variable. The value for a government with an option to default, V^{ND}, is the larger of the value of defaulting, V^D, and the value of repayment, V^C,

$$V^{ND}(y,b) = \max_{d\in\{0,1\}} dV^D(y,b) + (1-d)\,V^C(y,b)\,. \tag{2.3}$$

The solution to this problem yields the government's default policy:

$$d = \boldsymbol{D}(y,b) = 1_{\left\{V^D(y,b) > V^C(y,b)\right\}}. \tag{2.4}$$

That is, the government defaults whenever the value of defaulting is higher than the value of repayment.

The value of defaulting is:

$$V^D(y,b) = (1-\beta)\,u(y-\phi(y)) + \beta\mathbb{E}_{y'|y}\left[\theta V^{ND}\left(y',\mathcal{R}(b)\right) + (1-\theta)V^D\left(y',b\right)\right], \quad (2.5)$$

where the flow utility is determined by household consumption in default, $y-\phi(y)$, while the continuation value takes into account the possibility of regaining credit market access with debt level $\mathcal{R}(b)$.

The value of repaying is:

$$V^C(y,b) = \max_{b'}\left\{(1-\beta)u(c) + \beta\mathbb{E}_{y'|y}\left[V^{ND}\left(y',b'\right)\right]\right\} \quad (2.6)$$

and is subject to two constraints. The first constraint is the budget constraint,

$$c = y - [m + (1-m)z]\,b + q^H_{ND}\left(y,b'\right)\left[b' - (1-m)b\right], \quad (2.7)$$

in which the government issues bonds by running an auction with commitment, as is standard in Eaton and Gersovitz (1981) settings. The government obtains an auction price that is equal to the valuation of (unconstrained) high valuation investors, $q^H_{ND}(y,b')$.[7]

In addition, the government faces an upper bound on the ex ante one-period-ahead expected default probability:

$$\delta\left(y,b'\right) \equiv \mathbb{E}_{y'|y}\left[d\left(y',b'\right)\right] \leq \bar{\delta} \quad (2.8)$$

whenever there is positive debt issuance, $b' - (1-m)b > 0$. As explained in Chatterjee and Eyigungor (2015), in long-term debt models with positive recovery, the government has incentives to dilute existing bond holders by issuing large amounts of debt just prior to default. Since the liability of the government upon regaining credit access is at most $\bar{b}$, the government will then have incentives to issue an infinite amount of debt just prior to default in order to fully dilute existing bond holders. Constraint (2.8) is a simple modeling device to rule out such counterfactual behavior.[8]

[7] The budget constraint (2.7) treats bond prices in the event of a debt buyback (i.e. $b' < (1-m)\,b$) symmetrically. In principle, the government could potentially benefit from interacting with constrained investors in order to repurchase outstanding debt at the low valuation price q^L_{ND}. In our quantitative analysis, however, we find that debt buybacks do not occur on the equilibrium path (see Appendix A). This is because debt buybacks imply costly transfers to outstanding creditors. See, for example, Bulow and Rogoff (1991) and Aguiar et al. (2019) for proofs of "no buyback results" in sovereign debt settings. See, also, Admati et al. (2018) for similar results in a corporate finance setting.

[8] In the quantitive section we set the upper bound to 0.75, which amounts to maximal one month ahead default probability of 75 percent for newly issued bonds. In subsection 3.9 we show that even in the case in which this constraint caps the default probability at 99 percent per month, the quantitive properties

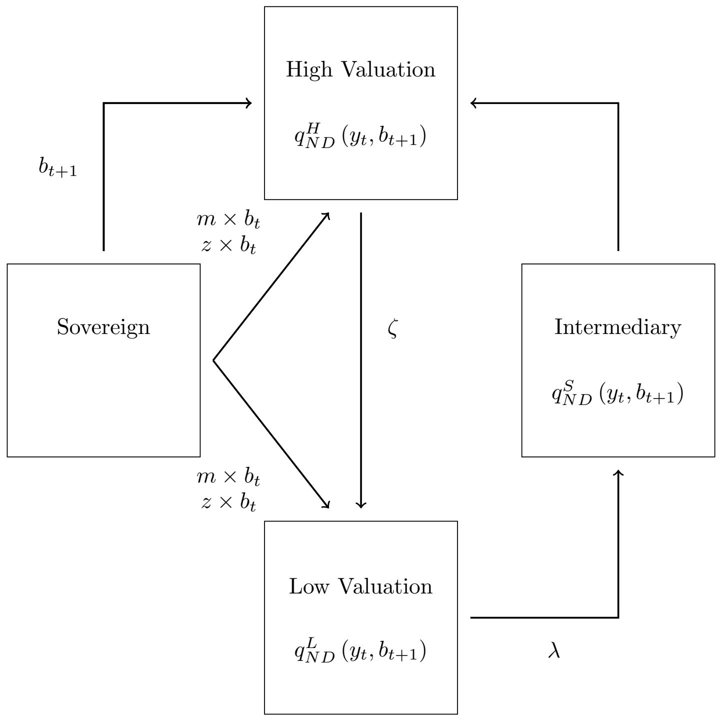

Figure 2.2: **Bond market, not in default.** This figure details the bond market if the sovereign is not in default and does not default in period t. The sovereign begins by issuing debt b_{t+1}. The high-valuation investors buy this debt in the primary market. After that, with probability λ, the low-valuation investors will meet an intermediary. They will sell their bonds at the price $q^S_{ND}(y_t, b_{t+1})$. After selling their bonds, they exit the market. The low-valuation investors who do not meet an intermediary will attempt to sell their bonds in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond in the next period in the secondary market. Both the high- and low-valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

The solution to the repayment problem (2.6) yields the debt policy of the government:

$$b' = \boldsymbol{B}(y, b). \tag{2.9}$$

2.5 Debt Valuations Before Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is not in default. The debt market before de-

of the model are almost unchanged. In addition, as noted in Chatterjee and Eyigungor (2015), sovereign bonds issued in financial centers (e.g. New York) have to be underwritten by some investment bank, and. reputational concerns may prevent them from issuing bonds with very high probabilities of immediate default. Flandreau et al. (2009, Figure 3b), reports that sovereign spreads at issuance where at most 1000 basis points for the period 1993 to 2007. For a 10 year zero coupon bond this spread amount to a 1 month (risk neutral) default probability of less than 1 percent, which is a fraction of the upper bound that we use.

fault is summarized in Figure 2.2. Let y be current income, and suppose that b' is the post-issuance face value of outstanding debt. The value of one unit of debt for an unconstrained investor with a high valuation is:

$$q^H_{ND}\left(y,b'\right) = \mathbb{E}_{y'|y}\left\{\left[\left(1-d\left(y',b'\right)\right]\frac{m+(1-m)\left[z+\zeta q^L_{ND}\left(y',b''\right)+(1-\zeta)q^H_{ND}\left(y',b''\right)\right]}{1+r}\right.\right.$$

$$\left.+d\left(y',b'\right)\frac{\zeta q^L_D\left(y',b'\right)+(1-\zeta)q^H_D\left(y',b'\right)}{1+r}\right\}, \tag{2.10}$$

which reflects the state-contingent payoffs of the bond. An investor receives principal m and coupon $(1-m)\,z$ in the absence of default during the next period ($d\left(y',b'\right)=0$). In this case, the continuation value of the $1-m$ non-maturing fraction of the bond depends on next period's optimal debt policy, $b''=\boldsymbol{B}\left(y',b'\right)$, and the realization of the idiosyncratic liquidity shock. A liquidity shock arrives with probability ζ, in which case the investor obtains a low continuation value, $q^L_{ND}\left(y',b''\right)$. Otherwise, the investor remains unconstrained and assigns a high continuation value, $q^H_{ND}\left(y',b''\right)$, to the bond. An investor does not receive any cashflow in the event of a default ($d\left(y',b'\right)=0$). In this case, the government defaults on b' units of debt, and the per unit price of defaulted debt is $q^H_D\left(y',b'\right)$ and $q^L_D\left(y',b'\right)$ for investors who do not receive and receive liquidity shocks, respectively. The value of defaulted bonds will be described in the next section 2.6.

The price of debt for a constrained investor with a low valuation is:

$$q^L_{ND}\left(y,b'\right) = \mathbb{E}_{y'|y}\left\{\left[\left(1-d\left(y',b'\right)\right]\frac{-h_c+m+(1-m)\left[z+(1-\lambda)q^L_{ND}\left(y',b''\right)+\lambda q^S_{ND}\left(y',b''\right)\right]}{1+r}\right.\right.$$

$$\left.+d\left(y',b'\right)\frac{-h_c+(1-\lambda)q^L_D\left(y',b'\right)+\lambda q^S_D\left(y',b'\right)}{1+r}\right\}. \tag{2.11}$$

The valuation of a constrained investor is similar to that of an unconstrained investor, but reflects the following differences. First, a constrained investor is assessed holding costs h_c, which lowers the effective value of a bond. Second, the continuation value for a constrained investor depends on trading outcomes in the secondary market. As described in section 2.2, a constrained investor sells with probability λ. The selling price (in the next period) is:

$$q^S_{ND}\left(y',b''\right) = \alpha q^H_{ND}\left(y',b''\right)+(1-\alpha)q^L_{ND}\left(y',b''\right)$$

in the absence of default and $q^S_D(y',b')$ in the event of default.

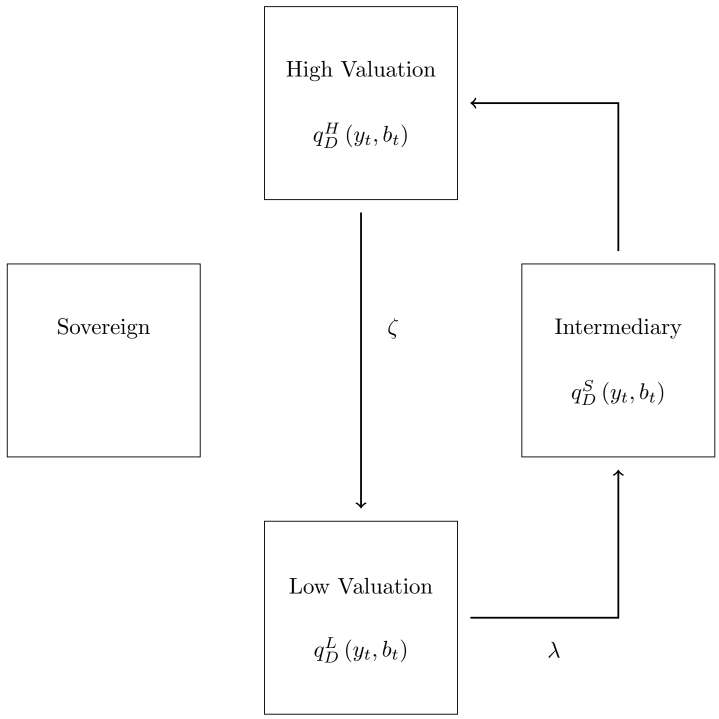

Figure 2.3: **Bond market, in default.** This figure details the bond market if the government is in default or defaults in period t. There is no debt issue or debt service. The sovereign has an outstanding balance of debt b_t. The low-valuation investors will meet an intermediary with probability λ. They will sell their bonds at the price $q_D^S(y_t, b_t)$. After they sell the bonds, they exit the market. The low-valuation-investors who do not meet an intermediary will attempt to sell in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell in the next period in the secondary market. Finally, with probability θ, the government resolves the default and re-accesses the credit market in the next period with a total outstanding debt of $\mathcal{R}(b_t)$.

2.6 Debt Valuations After Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is in default. The debt market after default is summarized in Figure 2.3. Let the current income be y, and let b be the amount of debt in default. In this case, the value of one unit of debt for unconstrained high-valuation investors is:

$$q_D^H(y, b) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q_D^L(y', b) + (1-\zeta) q_D^H(y', b)\right] + \theta\frac{\mathcal{R}(b)}{b} q_{ND}^H(y, \mathcal{R}(b)). \quad (2.12)$$

The government regains credit access in the next period with probability θ, in which case $\mathcal{R}(b)/b$ is the fraction recovered for each unit of defaulted debt. The value of recovered bonds, $q_{ND}^H(y, \mathcal{R}(b))$, is given by (2.10) and takes into account the fact that future debt choices are now determined by the new value of outstanding debt, $\mathcal{R}(b)$. Otherwise, investors receive no payments if default is not resolved and the continuation value reflects the probability ζ of receiving a liquidity shock.

Similarly, the per unit value of debt for constrained low-valuation investors is:

$$q_D^L(y,b) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c + \lambda q_D^S\left(y',b\right) + (1-\lambda)q_D^L\left(y',b\right)\right] + \theta\frac{\mathcal{R}(b)}{b}q_{ND}^L\left(y,\mathcal{R}\left(b\right)\right). \tag{2.13}$$

This valuation is analogous to that of unconstrained investors (2.12), but accounts for holding costs and trading frictions in the secondary market. Constrained investors sell during the next period with probability λ, with the selling price equal to

$$q_D^S(y,b) = \alpha q_D^H(y,b) + (1-\alpha)q_D^L(y,b),$$

for a defaulted bond.

2.7 Equilibrium

We focus on a Markov equilibrium with state variables y and b. An equilibrium consists of a set of policy functions for consumption $\boldsymbol{C}(y,b)$, default $\boldsymbol{D}(y,b)$, and debt $\boldsymbol{B}(y,b)$, as well as bond valuations $q_{ND}^H(y,b')$, $q_{ND}^L(y,b')$, $q_D^H(y,b)$ and $q_D^L(y,b)$ such that: (1) the policies solve the government's problem (2.6) taking bond valuations as given, and (2) the bond valuations satisfy equations (2.10) (2.11), (2.12) and (2.13).

2.8 Discussion

We discuss our modeling assumptions in this section. Our goal is to provide a parsimonious framework to study debt and default policy in a setting where default and liquidity risks are jointly determined.

There is substantial evidence of trading frictions in the secondary market for sovereign bonds (see, e.g., Pelizzon et al. 2013). To model these frictions, we adopt the framework of random search following a large literature on OTC markets that builds on the seminal contribution of Duffie et al. (2005). These secondary market frictions endogenously generate a positive wedge between the bond valuations of unconstrained and constrained investors (i.e., $q^H > q^L$), which results in positive bid-ask spreads and liquidity premiums.

Two additional features of our model, namely, long-term debt and debt recovery, interact with secondary market frictions to generate quantitatively realistic behavior for the liquidity premium. We describe the roles of these two features below.

First, having long-term debt is important for generating quantitatively significant levels for the liquidity premium. This is because investors would be unconcerned about secondary market trading frictions when debt maturity becomes sufficiently short—there is no need to trade if investors expect repayment of principal in a short span of time. Furthermore, long-term debt is now a standard feature of sovereign debt models (see, e.g., Hatchondo and Martinez 2009, Arellano and Ramanarayanan 2012, and Chatterjee and Eyigungor 2012), and is also needed to generate realistic debt levels and sovereign spreads.

Second, our model features a positive debt recovery after a default (i.e., $\mathcal{R}(b) > 0$). Besides being a well-documented feature of the data (see, e.g., Yue 2010, Bai and Zhang 2012, and Cruces and Trebesch 2013), debt recovery allows our model to capture the positive relation between default probabilities and liquidity premiums observed in the data (see, e.g., Pelizzon et al. 2013). The reason is as follows. A sovereign default event effectively results in a deferral of payment for the portion of debt that will eventually be recovered (the payments for recovered debt resume after the government regains credit access). Thus, changes in default probabilities change the expected timing of the payments to investors. This expected payment deferral, which we term as the *maturity extension channel*, increases the bond valuation wedge between constrained and unconstrained investors in the run up to a default—constrained investors expect to pay holding costs for longer before receiving any payments from the bond. This leads to the positive relation between the default probability (equivalently, the probability of a payment deferral) and the liquidity premium. In contrast, the maturity extension channel is missing when there is no debt recovery—all debt is simply written off immediately upon default.

In Appendix B we further discuss our modeling choices using a simple jump-to-default model in which debt policies are fixed and there is an exogenous default probability.

3 Results

We numerically solve a discretized version of the model. As discussed in Chatterjee and Eyigungor (2012) grid-based methods have poor convergence properties when there is long-term debt. To overcome this problem, we follow their prescription and use randomization methods to ensure convergence. Appendix C discusses the details of our numerical algorithm.

3.1 Calibration

We calibrate the model developed in section 2 to account for the main features of Argentina's default in 2001. We focus on Argentina over the period from 1993:I to 2001:IV for three reasons. First, using this period facilitates comparison to prior studies in the sovereign debt literature (see, for example, Hatchondo and Martinez, 2009, Arellano, 2008 and Chatterjee and Eyigungor, 2012), and in this way, we can be transparent about the contribution of our paper. Second, this sample satisfies our model's main assumptions: (1) our model is real and Argentina had a fixed exchange rate vis-a-vis the dollar during this period, and (2) Argentina's bonds were traded in an illiquid secondary market during this period. We calibrate and simulate the model at a monthly frequency because liquidity is inherently a short-run phenomenon. However, we report results at a quarterly frequency to facilitate comparison to previous studies.[9]

Functional Forms and Stochastic Processes. As is standard in the literature, we specify household utility to be CRRA, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We set the endowment process to be:

$$y_t = e^{z_t} + \epsilon_t, \tag{3.1}$$

$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}, \tag{3.2}$$

where z_t is a discretized AR(1) process with persistence ρ_z, volatility σ_z, and normally distributed innovations $\varepsilon_{z,t} \sim N(0,1)$. We also add a small amount of noise $\epsilon_t \sim trunc\, N(0, \sigma_\epsilon^2)$ that is continuously distributed. As shown in Chatterjee and Eyigungor (2012), this is necessary to achieve numerical convergence. We set the output loss during default to:

$$\phi(y) = \max\left\{0, d_y y + d_{yy} y^2\right\}.$$

This loss function is proposed by Chatterjee and Eyigungor (2012) and nests several cases in the literature. When $d_y < 0$ and $d_{yy} > 0$, the cost is zero for the range $0 \leq y \leq -\frac{d_y}{d_{yy}}$ and rises more than proportionally with output for $y > -\frac{d_y}{d_{yy}}$. Alternatively, when $d_y > 0$ and $d_{yy} = 0$ the cost is a linear function of output.[10] As explained in Chatterjee and Eyigungor (2012), the convexity of output costs is necessary to match the volatility of

[9] For example, the quarterly debt to output ratio in our paper is the stock of debt at the end of the quarter divided by the sum of monthly output within the quarter. This implies, for example, that the average debt to quarterly output is one-third of average debt to monthly output.

[10] The case studied in Arellano (2008) features consumption in default that is given by the mean output if the output is over the mean and equal to output if the output is less than the mean. This implies a cost function $\phi^A(y) = \max\{y - \mathbb{E}(y), 0\}$, which closely resembles the case of $d_y > 0$ and $d_{yy} = 0$.

sovereign spreads.

A priori set parameters. Panel A of Table 1 summarizes the values for apriori set parameters. We choose risk aversion to $\gamma = 2$, which is standard in the sovereign debt literature. The parameters for output are estimated from (linearly) detrended and seasonally adjusted data for Argentina for the quarterly sample from 1980:I to 2001:IV available from Neumeyer and Perri (2005). After estimating an AR (1) model for output at a quarterly frequency, we obtain monthly values $\rho_z = 0.983$, and $\sigma_z = 0.0151$, and we fix $\sigma_\epsilon = 0.004$.[11] We set the risk free rate to $r = 0.0033$ per month so that the quarterly risk free rate is 1 percent, which is standard. We set $m = 1/60$ to match an average debt maturity of 5 years based on values reported in Chatterjee and Eyigungor (2012) and Broner et al. (2013). We set the coupon rate to $z = 0.01$, so that the annualized coupon rate is 12 percent, which is close to the 11 percent value-weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012). We fix the reentry probability at $\theta = 0.0128$, following Chatterjee and Eyigungor (2012). This implies an average exclusion period of 6.5 years.[12] We fix $\lambda = 1 - e^{-2} = 0.8647$ so that the meeting intensity between a constrained investor and an intermediary is twice a month or, equivalently, once every two weeks, as in Chen et al. (2017). We also fix $\alpha = 0.5$ so that the bargaining power between constrained investors and market makers is symmetric. As discussed towards the end of Section 2.4, limiting the default probability of newly issued bonds to be less than one is necessary to obtain finite moments for debt and spreads. Following Chatterjee and Eyigungor (2015), we set this limit to $\overline{\delta} = 0.75$. Section 3.9 shows that we obtain very similar results if we set $\overline{\delta} = 0.99$ instead.

Calibrated parameters. Panel B of Table 1 summarizes the values for the remaining six parameters, which are calibrated to hit six targeted moments. We choose time preference $\beta = 0.9846$ to target an average debt to quarterly output ratio of 100 percent based on the mean debt to quarterly output ratio for Argentina for the period between 1993:I and 2001:IV. We set the maximal debt recovery to be $\overline{b} = 0.8238$ to match a mean recovery of $\mathbb{E}\left[\frac{\min\{\overline{b}, b_{def}\}}{b_{def}}\right] = 0.3$ in the model. This is based on a realized recovery rate of 30

[11] The conversion is as follows. We first obtain a quarterly output autocorrelation and volatility of 0.95 and 0.027, respectively, in the data. We then convert these numbers to their monthly counterparts using $0.95^{1/3} = 0.983$ and $\sqrt{0.004^2 + 0.0151^2} = 0.027/\sqrt{3}$, respectively. The latter calculation for the monthly output volatility takes into account the volatility of 0.004 for the noise component.

[12] Beim and Calomiris (2001) report that for the 1982 default episode, Argentina remained in a default state until 1993. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in the default state from 2001 until 2005, when it settled with most of its bondholders.

Parameter	Description	Value
	A. Apriori set parameters	
γ	Sovereign's risk aversion	2
ρ_z	Persistence of output	0.983
σ_y	Volatility of output	0.0156
r	Discount rate of international investors	0.0033
m	Rate at which debt matures	0.0167
z	Coupon rate	0.01
θ	Probability of reentry	0.0128
$\overline{\delta}$	Max. Default Probability	0.75
λ	Probability of meeting a market maker	0.8647
α	Bargaining power of the low-valuation investor	0.5
	B. Calibrated parameters	
β	Sovereign's discount rate	0.9846
$\overline{b}$	Maximum recovery for sovereign bonds	0.8238
d_y	Output costs for default	-0.2738
d_{yy}	Output costs for default	0.3478
h_c	Holding costs for constrained investors	0.0062
ζ	Probability of getting a liquidity shock	0.1121

Table 1: **Baseline parameters.** We simulate the model at a monthly frequency using the parameters in this table. We then aggregate the monthly outputs to a quarterly frequency.

percent for the 2001 Argentine default. In addition, we choose the default cost parameters $d_y = -0.2738$ and $d_{yy} = 0.3478$ to match the first two moments for the quarterly behavior of (annualized) sovereign spreads.[13,14] Following Chatterjee and Eyigungor (2012), this involves targeting mean spreads of 0.0815 and a quarterly volatility of 0.0443.

The two remaining parameters are related to secondary market frictions and are calibrated as follows. First, we choose a holding cost of $h_c = 0.0062$ to target a mean proportional bid-ask spread (2.2) of 56 basis points outside of periods of default.[15] This target is based on Argentine bid-ask spreads for our sample period (see Appendix F for details regarding the computation of bid-ask spreads in the data). Next, we set the probability of receiving a liquidity shock each period to be $\zeta = 0.1121$ in order to match bond turnover (i.e., the fraction of outstanding bonds being traded each period). Appendix D shows that turnover is approximately $\lambda\zeta / \left[m + (1-m)(\lambda + \zeta)\right]$ on average in the model;

[13] The annualized sovereign spread is given by $cs(y, b') = (1 + r^H(y, b'))^{12} - (1+r)^{12}$ where the yield to maturity is given by $r^H(y, b') = \left[m + (1-m)z\right] / q^H(y, b') - m$.

[14] The calibrated costs of default imply a 13% drop in quarterly output, on average, when the government is in default.

[15] Our targeted value for bid-ask spreads is comparable to that documented for European sovereign bonds and non-investment grade US corporate bonds. For example, Pelizzon et al. (2013) report a median bid-ask spread of 43 basis points for their sample of European sovereign bonds, and Chen et al. (2017) report bid-ask spreads of 50 basis points during normal times for their sample of US non-investment grade corporate bonds.

Moment	(1) Target	(2) Baseline	(3) CE (2012)
Mean Debt to GDP	1.0	1.0	0.7
Expected Recovery	0.30	0.30	0
Mean Sovereign Spread	0.0815	0.0816	0.0815
Vol. Sovereign Spread	0.0443	0.0443	0.0443
Mean Bid-Ask Spread, ND	0.0056	0.0056	-
Mean Bid-Ask Spread, D	-	0.0292	-
Mean Turnover (annual)	1.19	1.19	-
Default frequency (annual)	-	0.024	0.068

Table 2: **Model moments.** This table compares the targeted moments (column (1)) against their counterparts from the baseline model (column (2)). Column (3) reports results from Chatterjee and Eyigungor (2012) for comparison.

the corresponding data target for turnover is 119% per anum, or 8.4% per month. Our data target for turnover is based on Argentinean debt trading volume data, which we obtain from the Annual Debt Trading Volume Survey conducted by the Emerging Markets Trade Association (EMTA) for the years 1998-2004.[16] The calibrated value for ζ implies that, on average, an unconstrained investor becomes constrained after 9 months. Section 3.9 conducts a sensitivity analysis for the key parameters of the model and illustrates the relationship between parameter values and target moments.

Calibration Results. Column (2) of Table 2 displays our baseline model's outputs for the six targeted moments; column (3) shows the corresponding moments from Chatterjee and Eyigungor (2012) for comparison.[17] Our calibrated model closely matches the mean sovereign spread in the data (815 basis points in the data vs. 816 basis points in our model). In addition, our calibrated model exactly matches the remaining targeted moments. The mean debt to (quarterly) GDP ratio of 100 percent and an average recovery rate of 30 percent.[18] The volatility of sovereign spread 0.0443, the average bid-ask spread outside of default is 56 basis points, and the mean turnover rate is 119% per annum.

We also report two non-targeted moments. First, the annual default frequency is 2.4%

[16]The start of our sample, 1998, corresponds to the first year for which we obtained access to the EMTA debt trading volume survey; we end the sample in 2004, just prior to Argentina's debt restructuring in 2005. We compute annual turnover for Argentina in a given year by dividing annual trading volume for sovereign Eurobonds (reported by EMTA) by the stock of outstanding public and publicly guaranteed external debt for that year (obtained from the World Bank's WDI database). We then obtain monthly turnover by dividing annual turnover by twelve.

[17]To make our simulation results comparable, we follow Chatterjee and Eyigungor (2012) and discard the first twenty quarters after reentry following each default.

[18]Chatterjee and Eyigungor (2012) target a debt to quarterly output ratio of 70 percent because their model does not feature recovery. We target the full 100 percent because our model features recovery.

in our model, which is in line with the default rates used by the literature (see Section 3.2). Second, the model-implied bid ask spread is 292 basis points on average for defaulted bonds. This larger bid-ask spread for bonds in default arises endogenously as a result of the maturity extension channel discussed in Section 2.7, and is consistent with the maximum post-default bid-ask spread of 301 basis points observed in the data (although the mean post-default bid-ask spread in the data is lower at 170 basis points, see Appendix F for details).

Coming Next. In section 3.2 we discuss the implications of liquidity in solving the (sovereign) credit spread puzzle. In section 3.3, we report the pricing functions in the primary market and the bid-ask spreads. In sections 3.4 and 3.5, we provide a structural decomposition of total spreads into liquidity and default components. In section 3.6 we quantify the welfare implications of liquidity frictions. In section 3.7 we use our calibrated model to impute the liquidity component of Argentinean spreads in the lead up to Argentina's 2001 default. Finally, section 3.8 reports business cycle statistics of the model.

3.2 Credit Spread Puzzle

Table 2 shows that our baseline model matches the mean and volatility sovereign spreads while having an annual default frequency of 2.4%. Our model-implied default frequency is in line with the consensus default frequency used by the literature. For example, Arellano (2008), Hatchondo et al. (2010), Lizarazo (2013) and Pouzo and Presno (2016) all target an annual default frequency of 3 percent; Yue (2010) and Mendoza and Yue (2012) target an annual default frequency of 2.7 and 2.78 percent, respectively. Our model is therefore able to address the "credit spread" puzzle by simultaneously matching both credit spreads and default rates. The underlying reason for our model's ability to address the credit spread puzzle is simple: a portion of the credit spread is not due to default risk, but rather liquidity risk. In contrast, models that match credit spreads without consideration for the role of liquidity frictions would result in a counterfactually high model-implied default rate (e.g. column (4) of Table 2 reports results from Chatterjee and Eyigungor (2012), which does not consider illiquidity for sovereign bonds).[19] In the coming sections, we further investigate the quantitative importance of liquidity for the credit spread puzzle. We show that, on average, liquidity is responsible for roughly a quarter of

[19]Other approaches to the credit spread puzzle involve introducing risk averse investors (see, e.g., Arellano 2008, Borri and Verdelhan 2009, Tourre 2017, and Morelli et al. 2019), and model uncertainty (Pouzo and Presno, 2016). We view our liquidity-based explanation as complementary to the channels proposed in these prior works.

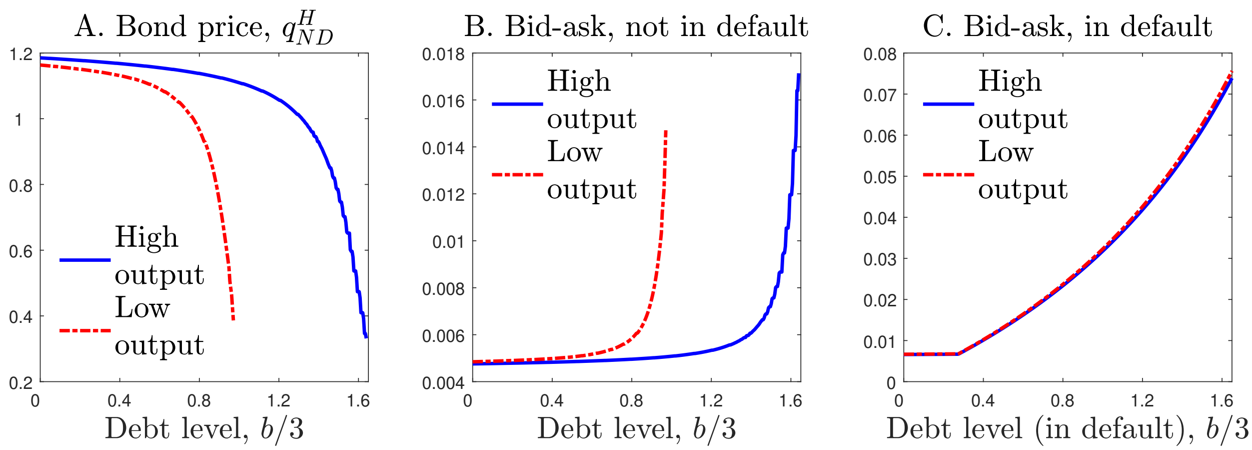

Figure 3.1: **Bond prices and bid-ask spreads.** Panels A and B plot bond prices at issue (2.10) and proportional bid-ask spreads (2.2) during credit access, respectively. These plots are shown as a function of the current debt level b, over the range of debt for the which default does not occur, with next period's choice of debt b' being given by the optimal debt policy (2.9). Panel C plots proportional bid-ask spreads during autarky as a function of the amount of debt in default. Across all panels, log output is one unconditional standard deviation above (below) its mean in the solid (dash-dotted) line. Horizontal axes plot expresses debt levels in units of average quarterly output.

credit spreads, with this fraction changing depending on business cycle conditions.

3.3 Bond Prices and Bid-Ask Spreads: Maturity Extension Channel

Figure 3.1 plots the model-implied bond prices and bid-ask spreads as a function of the level of debt at the beginning of the period. To aid comparison with other sovereign debt models (which are typically calibrated at a quarterly frequency), we express debt levels in units of average quarterly output (i.e., $b/3$) in this and subsequent figures. All Panels in Figure 3.1 are for high and low values of output y that correspond to (log) output values equal to plus and minus one standard deviation of its unconditional distribution, respectively.

Panel A plots bond prices in the primary market during the credit access regime, $q^H_{ND}(y, b')$, as a function of the current debt level ($b/3$), where the debt choice $b' = \boldsymbol{B}(b, y)$ is chosen according to the debt policy (2.9). The presence of bond recovery following a default implies that bond prices are always strictly positive. Standard comparative statics apply: bond prices are increasing in output and decreasing in debt levels, as is usually the case in quantitative models of sovereign debt.

Panel B plots proportional bid-ask spreads (2.2) during periods in which the government is not in default. The bid-ask spread is stable around 50 basis points for low levels of debt (e.g., $b/3 \leq 0.6$) regardless of income levels. Bid-ask spreads increase as the likelihood of default increases. This occurs when output is low or the debt level is high. For example, the bid-ask spread reaches 147 basis points when the debt level is $b/3 = 0.97$ and output is low.

The logic for this increase in bid ask spreads is the endogenous maturity extension channel. To see this, note first that bonds pre-default and after default have a different schedule of payments. In particular, the modified duration of bonds pre-default is shorter. The reason is that pre-defaulted bonds pay a coupon z and m of principal each period, whereas defaulted bonds pay no coupon or principal, and will become non defaulted bonds in the future. Second, as a result of these different payments schedules, as default probabilities change, the expected maturity of payments of the bond changes, which we term as the endogenous *maturity extension channel*. Third, as consequence of the endogenous extension in the expected maturity of promised payments, increases in default probabilities increase the expected maturity of promised payments, which increases the sensitivity of prices. Appendix B.4 formally illustrates the maturity extension channel in the context of our jump to default model.

Panel C shows that the proportional bid-ask spread for defaulted bonds is also increasing in the amount of debt in default. The reasoning is related to the maturity extension channel—a higher amount of debt in default results in a lower recovery rate. In present value terms, having a lower recovery is akin to fixing the recovery rate, but extending the time at which recovery takes place. This effective extension in maturity results in higher bid-ask spreads. Panel C also shows that there is little difference in the proportional bid-ask spreads of defaulted bonds across different output states. From equations (2.12) and (2.13), we see that the value of defaulted bonds depends on the current output state only through its influence on the value of recovered debt upon the government reentering international credit markets. In our calibration, periods of autarky last 6.5 years on average. As a result of mean reversion in output over such a horizon, the influence of current output on the eventual recovered debt value is weak. Therefore, the proportional bid-ask spread for defaulted bonds mainly depends on the amount of debt in default.

In combination, Panels B and C of Figure 3.1 demonstrate a liquidity-default feedback loop by which incentives to default are higher during bad times due to liquidity frictions. This margin was first highlighted by He and Milbradt (2014) in the context of corporate bonds. To see this feedback loop, consider the net proceeds from debt rollover, the difference in the value of newly issued bonds and the repayment of the principal of maturing debt, $q^H_{ND,t}[b_{t+1} - (1-m)b_t] - mb_t$. A larger debt issuance is necessary to break even when rolling over debt if the probability of default is high and the price of newly issued bonds, $q^H_{ND,t}$, is low. This leads to higher levels of indebtedness and further increases the government's incentives to default in future periods. This rollover risk channel is already understood (see, .e.g, Chatterjee and Eyigungor, 2012). The presence of liquidity frictions further amplifies this rollover risk channel. Higher debt levels further increase bid-ask spreads in the secondary market (Panel B), which results in an even lower issuance price in the primary bond market. In turn, this further exacerbates the government's default incentives and results in a liquidity-default feedback loop.[20]

Next, in sections 3.4 and 3.5, we further investigate this feedback by decomposing the sovereign spread into default and liquidity components, as well as their interactions.

3.4 Sovereign Spread Decomposition

In this section, we present a decomposition of total spreads into a liquidity and a credit component. Our main result is that liquidity premia can be a substantial component of total spreads.

As a first step towards the decomposition, we start by defining *total spreads*. Consider a government with debt and default policies given by $\widetilde{\boldsymbol{B}}(y,b)$ and $\widetilde{\boldsymbol{D}}(y,b)$, respectively. Let $q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ be the corresponding value of this government's debt to an unconstrained investor who receives liquidity shocks with probability ζ.[21] That is, $q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} = q^H_{ND}\left(y,\widetilde{\boldsymbol{B}}(y,b)\right)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ where $q^H_{ND}(y,b')\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ is the solution to equation (2.10) under debt policy $\widetilde{\boldsymbol{B}}$ and default policy $\widetilde{\boldsymbol{D}}$. Since our calibration is monthly, the corresponding annualized sovereign spread is defined as:

$$cs(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} \equiv \left(1 + r^H(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}\right)^{12} - (1+r)^{12}, \tag{3.3}$$

where r is the risk free rate, and the bond's yield to maturity is given by $r^H(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} = [m+(1-m)z] \Big/ q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} - m$.

Let $\boldsymbol{B}$ and $\boldsymbol{D}$ be the debt and default policy from the baseline calibration (i.e., the parameters listed in Table 1), respectively. The total spread implied by our baseline model is then obtained by evaluating equation (3.3) at the baseline policies (i.e., by setting $\widetilde{\boldsymbol{B}} = \boldsymbol{B}$ and $\widetilde{\boldsymbol{D}} = \boldsymbol{D}$). Next, we decompose the total spread (3.3) into a credit and a liquidity component.[22]

[20] In Appendix B we discuss this feedback mechanism in a simple model with fixed debt and exogenous default probabilities.

[21] In making this definition, we are assuming that the debt choice b' is set according to the prescribed debt policy. As a result, we write bond prices and credit spreads in terms of output and the current debt level b.

[22] The decomposition is analogous to the decomposition provided in He and Milbradt (2014) in the context of corporate bond spreads. As it will be made clear in the next subsection, one important difference is

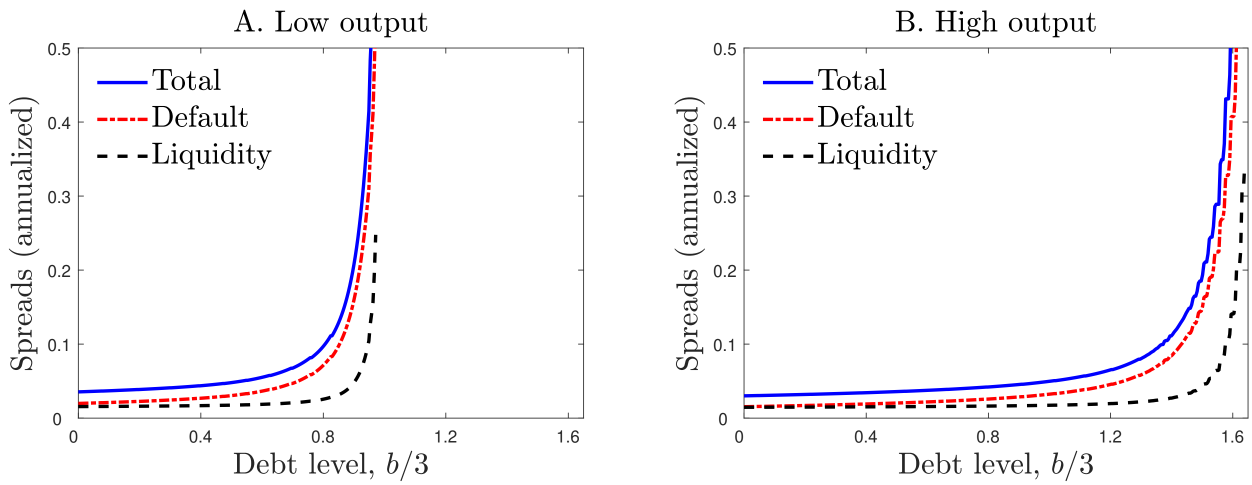

Figure 3.2: **Illustration of the sovereign spread decomposition.** This figure decomposes total sovereign spreads, defined in equation (3.4), into default and liquidity components given by equations (3.4) and (3.5), respectively. Panels A and B plot the decomposition for low and high output values of output corresponding to (log) output values one unconditional standard deviation below and above its mean, respectively.

The *default component* of the sovereign spread is defined as:

$$cs_{DEF}\left(y,b\right) \equiv cs\left(y,b\right)\Big|_{(\boldsymbol{B},\boldsymbol{D},0)}. \tag{3.4}$$

The pricing of the default component uses the baseline debt $\boldsymbol{B}$ and default $\boldsymbol{D}$ policies, but the investor pricing the bond has a zero probability of receiving a liquidity shock.[23] More precisely, the policy pair $(\boldsymbol{B},\boldsymbol{D})$ is the optimal policy of a government that is subject to price schedule $q^H_{ND}(y,b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)}$ in the primary market. However, the bond price associated with $cs_{DEF}(y,b)$ is given by $q^H_{ND}(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)}$.

The *liquidity component* is then defined as the residual:

$$cs_{LIQ}(y,b) \equiv cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} \tag{3.5}$$

and accounts for the portion of the total sovereign spread that is not explained by the default component. These two definitions amount to decomposing the total sovereign spreads as:

$$cs\left(y,b\right) = cs_{DEF}\left(y,b\right) + cs_{LIQ}\left(y,b\right). \tag{3.6}$$

that our decomposition takes into account the endogenous response of debt policy, while the decomposition in He and Milbradt (2014) is for a fixed debt policy.

[23]The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account when choosing debt and default policies), individual investors are heterogeneous so that there may be some investors without liquidity concerns who discount at the risk-free rate.

What portion of total spreads is explained by each component? Figure 3.2 illustrates the decompostion (3.6) for our baseline model. Panels A and B plot the total spreads, credit risk premium, and liquidity premium when log output is one standard deviation below and above its mean, respectively. We highlight two features of this decomposition. First, Panels A and B show that the liquidity component is increasing as debt levels increase or output decreases. The increase in the liquidity component reflects a higher default probability which increases current bid ask spreads (Panel B Figure 3.1). Second, the liquidity component represents a sizable fraction of total spreads. In our simulations, the liquidity and the credit component have a mean of 229 and 586 basis points, respectively. Thus, the liquidity component accounts for 28% of total spread on average.[24] The fraction of total spreads attributable to liquidity is time-varying and is larger when default risk is low (i.e., when output is high and/or debt levels are low). For example, in Panel B, the fraction of the total sovereign spreads attributable to liquidity increases to one third for low levels of debt ($b/3 \leq 0.6$).

How does default risk affect the liquidity premium? To see this, consider a "jump-to-default" model similar to the one developed in section 2 except that debt is fixed, and the default probability is exogenous and given by p_d (we present the jump-to-default model in detail in Appendix B). In this case, the valuation of the unconstrained investor (2.10) can be written as

$$q_{ND}^{H} = \frac{1}{1+r+\ell_{ND}}\left[(1-p_d)\left(m+(1-m)\left(z+q_{ND}^{H}\right)\right)+p_d q_D^H\right]$$

where

$$\ell_{ND} = (1-m)\,\zeta\left[\frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}} + p_d\left(\frac{q_D^H-q_D^L}{q_D^H}\frac{q_D^H}{q_{ND}^H} - \frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}}\right)\right] \tag{3.7}$$

is the liquidity premium that is needed to equate the market price q_{ND}^H to the valuation of an investor who is not subject to liquidity concerns. Quantitatively, the second term of equation (3.7) is second order with respect to the first term,[25] thus we can approximate the liquidity premium as:

$$\ell_{ND} \simeq (1-m)\,\zeta\frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}}.$$

Thus, there is positive co-movement of liquidity and default risk as long as the bid ask

[24]Table 3 reports the decomposition (3.6) when we simulate our baseline model. We further discuss time-variation in the liquidity and default components Section 3.5.

[25]This approximation is exact in continuous time. In particular, in subsection B.6 of the Appendix we show that the second term of (3.7) is exactly equal to zero in the continuous time limit.

spread *endogenously* respond to higher default probabilities. As highlighted in section 3.3 and in Figure 3.1, this endogenous response of bid ask spreads is a result of the endogenous maturity extension channel.[26]

3.5 Zooming into the Interaction of Default and Liquidity Risk

In this section, we further expand upon decomposition (3.6) for the total spread by additionally considering interaction effects between the liquidity component and the default component. The main objective is to illustrate how the credit and liquidity risk premium interact.

A finer decomposition of the default component. We begin by further decomposing the default component (3.4). Let $\boldsymbol{B}_0$ and $\boldsymbol{D}_0$ respectively denote the debt and default policy of a government operating in an economy that is not subject to liquidity frictions (i.e., $\zeta = 0$). We can then decompose the default component (3.4) into

$$cs_{DEF}(y, b) = cs_{pureDEF}(y, b) + cs_{LIQ \to DEF}(y, b), \tag{3.8}$$

where the first "pure default" component, defined as

$$cs_{pureDEF}(y, b) \equiv cs(y, b) \mid_{(\boldsymbol{B}_0, \boldsymbol{D}_0, \zeta=0)}, \tag{3.9}$$

is the spread that the sovereign would pay if there were no liquidity frictions, and the second component:

$$cs_{LIQ \to DEF}(y, b) \equiv cs(y, b) \mid_{(\boldsymbol{B}, \boldsymbol{D}, \zeta=0)} - cs(y, b) \mid_{(\boldsymbol{B}_0, \boldsymbol{D}_0, \zeta=0)}, \tag{3.10}$$

is the liquidity-driven component of default spreads. This is the portion of the default component attributable to liquidity-induced changes in equilibrium debt and default policies, from $(\boldsymbol{B}_0, \boldsymbol{D}_0)$ to $(\boldsymbol{B}, \boldsymbol{D})$, when we transition from an economy absent liquidity

[26] Aside from the endogenous maturity extension channel, there are other reasons why the bid-ask spreads can increase during bad times. For example, Chaumont (2020) develops a model in which search in the secondary market is directed towards different sub-markets (as in Moen, 1997) characterized by a trading probability and a fee, and there is no recovery after default. As a result of zero recovery there is no endogenous maturity extension channel. The source of the liquidity premium in Chaumont (2020) is the equilibrium choice by investors of fees and trading probabilities. During bad times because the asset's price falls, the holding cost (which is set exogenously and fixed over states of nature) is higher relative to the price of the asset. As a result, investors want to offload the asset faster, in the process paying higher equilibrium fees. This equilibrium choice of fees by investors is an alternative motive to generate a liquidity premium.

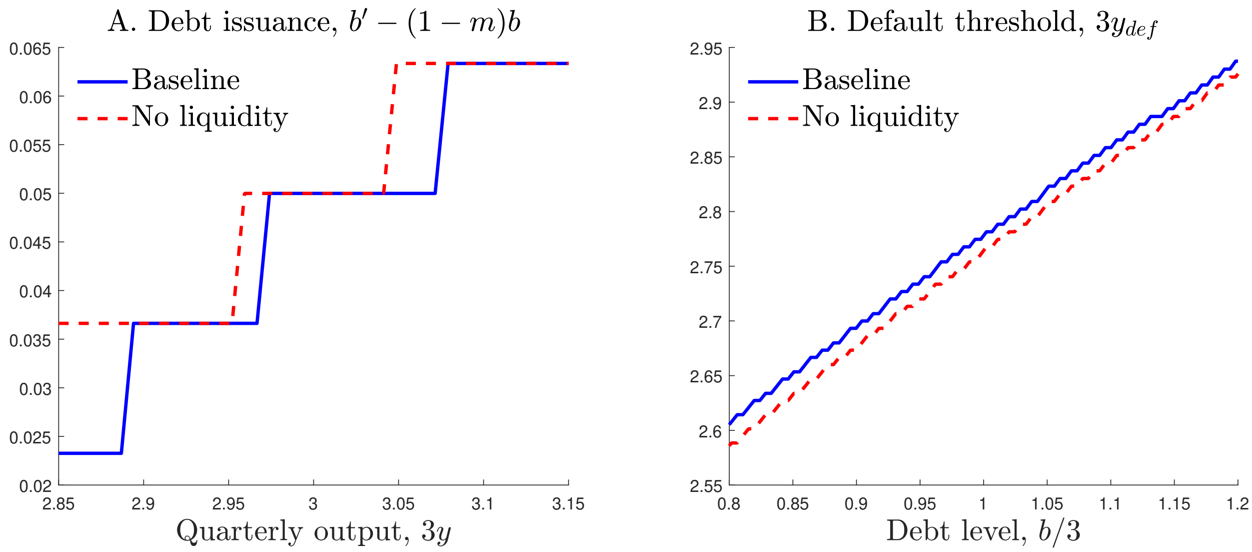

Figure 3.3: **Liquidity and policies.** Panel A plots the debt issuance rate as a function of output with current debt level b fixed at its mean level. Panel B plots the default threshold for a given debt level. The solid lines plot the policies for the baseline model in which liquidity frictions are present. The dashed lines plot the policies in a setting in which liquidity frictions are absent.

frictions to one with liquidity frictions.

The sign of the liquidity-driven default component (3.10) is influenced by two offsetting forces. On the one hand, fixing default policies, an increase in liquidity frictions increases the cost of borrowing which can, in turn, decrease default incentives through lowering the amount of debt that the government chooses to borrow. This effect is illustrated by Panel A of Figure 3.3 which compares the debt issuance policy in our baseline model to that of an economy in which liquidity shocks are absent (but is otherwise identical). We see that debt issuance is more conservative in our baseline model in which liquidity frictions are present. On the other hand, fixing debt policies, an increase in liquidity frictions increases the cost of rolling over debt which, in turn, increases default incentives. This effect is illustrated by Panel B of Figure 3.3 which shows that the default threshold is higher in the baseline economy in which liquidity frictions are present.[27]

We quantify these two effects by further decomposing liquidity-driven default $cs_{LIQ\to DEF}(y,b)$ into two terms:

$$cs_{LIQ\to DEF}(y,b) \equiv cs_{LIQ\to DEF,Debt}(y,b) + cs_{LIQ\to DEF,Def}(y,b), \tag{3.11}$$

[27] For a given level of debt b, the default threshold $y_{def}(b) = \max\{y : D(\tilde{y},b) = 1 \text{ for all } \tilde{y} \le y\}$ is the maximal level of output for which default occurs.

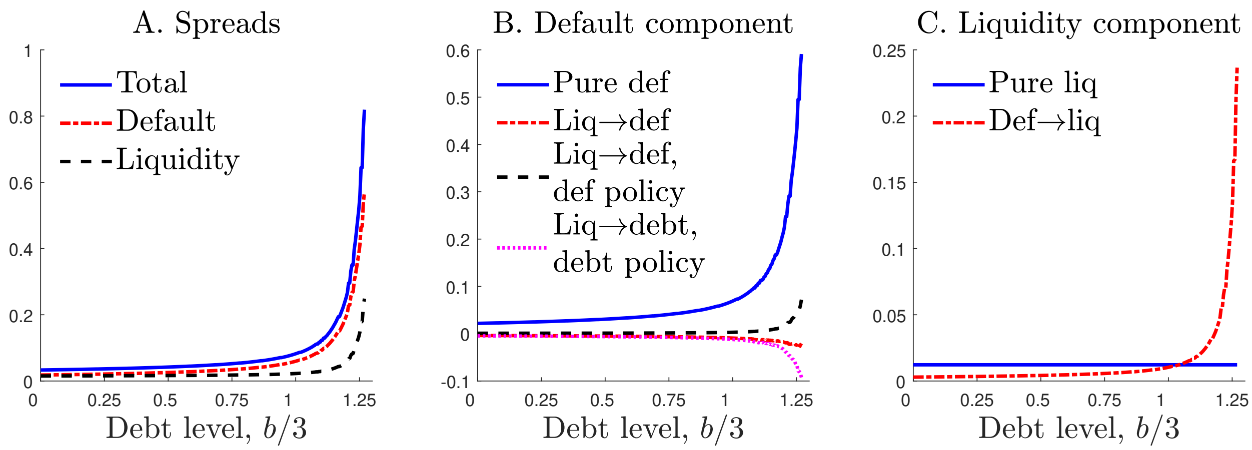

Figure 3.4: **Further decomposing the sovereign spread.** This figure plots the different components of spreads when output is at its median value ($y = 1$). Panel A plots decomposes total spreads into default and liquidity components according to decomposition (3.6). Panel B further decomposes the default component according to decompositions (3.8) and (3.11). Panel C further decomposes the liquidity component according to decomposition (3.14).

where the first term, defined as

$$cs_{LIQ\to DEF,Debt}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)} - cs(y,b)\mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)}, \tag{3.12}$$

measures the degree to which liquidity induces changes in the default component of spreads through its effect on debt policy (holding default policy fixed); the second term, defined as

$$cs_{LIQ\to DEF,Def}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} - cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)}, \tag{3.13}$$

measures the degree to which liquidity induces changes in the default component of spreads through its effect on default policy (holding debt policy fixed).

Panel B of Figure 3.4 illustrates the decompositions (3.8) and (3.11) for our baseline model when output is equal to its median value (i.e., $y = 1$). We see that the $cs_{LIQ\to DEF,Def}$ component is positive while the $cs_{LIQ\to DEF,Debt}$ component is negative, which is in line with our previous discussion. The net effect, captured by the $cs_{LIQ\to DEF}$ term, is (slightly) negative so that, to first order, pure default term $cs_{pureDEF}$ is the dominant term of the overall default component cs_{DEF}. This illustrates the effectiveness of optimal debt management in reducing the impact of liquidity risk and why it can be the case that liquidity frictions could actually decrease spreads through its effect on optimal debt policy. Later on, in Table 3 we further illustrate the quantitative behavior of the decompositions above.

A finer decomposition the liquidity component. We can further decompose the liquidity component (3.5) into two terms:

$$cs_{LIQ}(y,b) = cs_{DEF \to LIQ}(y,b) + cs_{pureLIQ}(y,b). \tag{3.14}$$

The first term, defined as

$$cs_{pureLIQ}(y,b) \equiv cs(y,b) \mid_{(\boldsymbol{D}=0,\zeta)}, \tag{3.15}$$

is a "pure liquidity" term corresponding to the spread for a government which never defaults, but whose bonds are held by investors subject to liquidity shocks (as in Duffie et al. 2005). The absence of default implies that this pure liquidity component is independent of debt policy (i.e., any arbitrary debt policy can be used in definition (3.15)). The second term, defined as the residual

$$cs_{DEF \to LIQ}(y,b) \equiv cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},0)} - cs(y,b) \mid_{(\boldsymbol{B}_0,\boldsymbol{D}=0,\zeta)} \tag{3.16}$$

is a "default-induced liquidity" component that measures the portion of the liquidity component that is due to default risk. Panel C of Figure 3.4 illustrates these two components. The default-induced liquidity component is increasing as debt level increases and default becomes more likely. This is a result of the default-liquidity feedback mechanism of our model.

Quantifying the sovereign spread decomposition. We now investigate the quantitative behavior of the sovereign spread decomposition. Table 3 reports the results for the decomposition when we simulate our baseline model. Column (1) reports the unconditional means, while columns (2) and (3) report the means conditional on output being high and low, respectively. In these simulations, we define the low (high) output regime to correspond to times in which output is below (above) the eleventh percentile of its unconditional distribution. We choose this threshold so that the low output regime corresponds to the worst output year over a nine-year span—the length of our sample period, 1993-2001. As such, we can think of the low output regime as the one year period leading up to a sovereign default.

From Table 3, we see that, on average, the default and liquidity components make up 28% and 72% of the total spread, respectively. The relative contribution of the default component to the total spread increases during recessions. Specifically, the default and liquidity components make up 25% and 75% of the total spread, respectively, conditional

	(1) Uncond. mean	(2) $y_t > y_{p11}$	(3) $y_t \leq y_{p11}$
(i) Total spreads, cs	0.0816	0.0735	0.1460
(ii) Liquidity component, cs_{LIQ}	0.0229	0.0213	0.0361
Pure liquidity, $cs_{pureLIQ}$	0.0122	0.0122	0.0122
Default driven liquidity, $cs_{DEF\to LIQ}$	0.0107	0.0091	0.0239
(iii) Default component, cs_{DEF}	0.0586	0.0522	0.1099
Pure default, $cs_{pureDEF}$	0.0672	0.0607	0.1191
Liquidity driven default, $cs_{LIQ\to DEF}$	-0.0086	-0.0085	-0.0092
Through debt policy, $cs_{LIQ\to DEF,Debt}$	-0.0118	-0.0113	-0.0160
Through default policy, $cs_{LIQ\to DEF,Def}$	0.0032	0.0027	0.0068

Table 3: **Quantifying the sovereign spread decomposition.** Column (1) reports the unconditional mean for the decomposition of sovereign spreads. Columns (2) and (3) report the mean decomposition conditional on output being above and below the 11th percentile of its unconditional distribution, respectively. The default and liquidity components are defined by equations (3.4) and (3.5), respectively. The default component is further decomposed according to equations (3.8) and (3.11). The liquidity component is further decomposed according to equation (3.14).

on output being in the low regime.

Row (ii) of Table 3 further decomposes the liquidity component. It shows that default risk is an important determinant of the overall liquidity component. In particular, we see that the contribution of default-driven liquidity to the overall liquidity component is 47% on average, and increases to 66% in the low output regime.

Row (iii) of Table 3 further decomposes the default component. Recall that the overall effect of liquidity on default risk depends on the dampening effects of the conservative debt policy response (see panel A of Figure 3.3), and the amplifying effect of the increase in default thresholds (see panel B of Figure 3.3). Interestingly, we see that the former effect is stronger so that default driven liquidity is negative on average. This result highlights the importance of effect debt policy management in the face of liquidity risk—the average total spread would be 14% higher had there not been a debt policy response to liquidity risk (so that the $cs_{LIQ\to DEF,Debt}$ term is zero).

The overall interactions effect between default and liquidity risk can be gauged by the sum of the default driven liquidity and liquidity driven default terms (i.e., $cs_{LIQ\to DEF} + cs_{DEF\to LIQ}$). Table 3 shows that although the overall interaction effect only contributes to 3% of total spreads on average, it becomes non-negligible in the run up to a default. For example, the interaction terms make up 10% of total spreads in the low output regime.

Relationship to the Literature. Combining (3.8) and (3.14), we obtain

$$cs \equiv cs_{pureDEF} + cs_{LIQ \to DEF} + cs_{DEF \to LIQ} + cs_{pureLIQ}. \tag{3.17}$$

This expression is the decomposition reported by He and Milbradt (2014) and Chen et al. (2017) in the context of corporate bonds. Our contribution is as follows: the decomposition from He and Milbradt (2014) and Chen et al. (2017) assumes a fixed debt policy so that liquidity influences default solely through its effect on default policy (i.e., only the $cs_{LIQ \to DEF,Def}$ component of $cs_{LIQ \to DEF}$ is present). In contrast, debt policy is dynamic in our setting so that liquidity can additionally influence default through its effect on debt policy (i.e., $cs_{LIQ \to DEF}$ has an additional $cs_{LIQ \to DEF,Debt}$ term). We find that it is important to additionally account for the response of debt policy to liquidity conditions. This is illustrated by Panel B of 3.4. We see that liquidity-driven changes to debt policy decrease spreads (i.e. $cs_{LIQ \to DEF,Debt} < 0$), which is due to more conservative debt issuances in the face of worsening liquidity conditions. This can offsets the increase in spreads from liquidity-driven changes to default policy (i.e., $cs_{LIQ \to DEF,Def} > 0$).

3.6 Secondary Market Frictions and Welfare

In this section, we examine the implications of secondary market illiquidity for household welfare. We define welfare as the certainty equivalent consumption,

$$c\left(h_c\right) \equiv u^{-1}\left(\mathbb{E}\left[V^C\left(y, b=0; h_c\right)\right]\right), \tag{3.18}$$

obtained by a sovereign with no initial debt ($b = 0$) operating in an economy in which constrained international investors are subject to holding costs h_c. We vary the illiquidity of secondary bond markets (as measured by, for example, bid-ask spreads) by varying the severity of liquidity shocks for international investors and consider the resulting welfare implications for the sovereign.

Table 4 reports the results of this exercise. We see that relative to an economy without liquidity frictions ($h_c = 0$), household welfare decreases by 0.3% when we increase liquidity frictions to its baseline value ($h_c = 0.0062$). This is because higher trading frictions increases the cost of boring and the sovereign borrows less in equilibrium.

To put this welfare decrease into perspective, note that the welfare gain from eliminating the business cycle (for a representative agent with CRRA preferences) is 0.40% in consumption equivalent terms (see Appendix E for details of this welfare calculation). Thus, in the context of our calibrated model, the welfare cost of secondary market fric-

Moment	$h_c = 0$	$h_c = 0.0021$	$h_c = 0.0042$	$h_c = 0.0062$
Mean Debt to GDP	1.03	1.02	1.01	1.00
Mean Spread	0.0727	0.0758	0.0786	0.0816
Vol. of Spread	0.0479	0.0467	0.0453	0.0443
Mean Bid-Ask Spread	0	0.0018	0.0037	0.0056
Welfare	1.016	1.015	1.014	1.013

Table 4: **Secondary market frictions and welfare.** This table reports welfare, defined in equation (3.18), when we vary the severity of secondary market frictions.

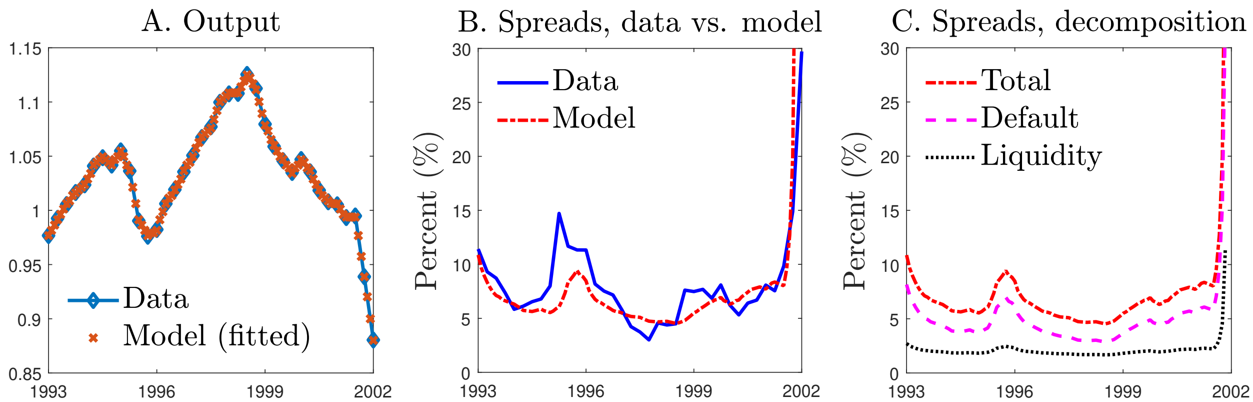

Figure 3.5: **Event Study: Argentina's Default, 2001:IV.** Panels A through C plot output, total spreads, and decomposition (3.6), respectively. The model-implied values in Panels B and C are obtained by initiating the initial debt level at its data counterpart and then feeding in the realized output series into our baseline model.

tions for Argentina is roughly 75% of the cost of its business cycle.

3.7 Case Study: Argentina's Default in 2001

In this section, we use our baseline model to conduct an event study of Argentina's default in December of 2001. Figure 3.5 reports the results. We input Argentina's initial level of debt and then feed the realized path for output (see Panel A) into our model. We then compare our model-implied total spreads to that of the data (see Panel B), and report the corresponding decomposition (3.8) (see Panel C).

First, note that our model can replicate the most salient features of the series of spreads. The first spike is the Tequila crisis in Mexico in 1996, where there is a sharp increase in the spreads coming from a recession. In addition, we can see that for 2001 the model can correctly account for the spike in spreads and default. Note that although the recession started in 1998:III, spreads continued to be below 800 basis points until the beginning of 2001.

Variable	Data	Model	CE (2012), Table 4
$\sigma(c)/\sigma(y)$	1.09	1.10	1.11
$corr(c, y)$	0.98	0.98	0.99
$\sigma\left(\frac{NX}{y}\right)/\sigma(y)$	0.17	0.08	0.20
$corr\left(\frac{NX}{y}, y\right)$	-0.88	-0.44	-0.44
$corr(r - r^f, y)$	-0.79	-0.62	-0.65
$corr(ba, y)$	-0.91	-0.61	-
Debt service	0.053	0.08	0.055

Table 5: **Business cycle.** This table reports business cycle moments from simulations of our baseline model (whose parameters are listed in Table 1).

Second, Panel C decomposes total spreads into default and liquidity components. According to our baseline model, 31% of the total spread is attributable to the liquidity component, on average, over the period of 1993:I to 2001:IV. The contribution of the liquidity component was slightly lower at 26%, on average, over the year leading up to the default in the fourth quarter of 2001.

3.8 Business Cycle Properties

The model's business cycle properties are summarized in Table 5. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model performs well. As in the data, consumption is as volatile as output, and nearly perfectly correlated with it. The volatility of the current account relative to output volatility is 0.08 in the model, which is close to its empirical counterpart of 0.17. In addition, there is a negative correlation between the current account and output both in our model and in the data. The model performs well at capturing counter-cyclical sovereign credit risk, with a correlation of -0.62 between the sovereign spread and output. The model also captures the negative relation between proportional bid-ask spreads and output in the data. Finally, debt service (as a fraction of output) is 8 percent. Overall, the business cycle properties generated by the calibrated model are similar to those generated by the model in Chatterjee and Eyigungor (2012).

	Debt/ GDP	Reco- very	Spread, mean	Spread, vol.	Spread, liq. share	Bid-ask, ND	Bid-ask, D	Turn- over	Def freq
(1) Baseline	1.00	0.300	0.0816	0.0443	0.281	0.0056	0.0292	1.19	0.024
(2) $\overline{\delta} = 0.99$	1.00	0.276	0.0865	0.0507	0.274	0.0057	0.0331	1.19	0.026
(3a) $h_c = 0.003$	1.01	0.291	0.0769	0.0460	0.146	0.0027	0.0116	1.19	0.027
(3b) $h_c = 0.009$	0.99	0.307	0.0854	0.0433	0.380	0.0082	0.0518	1.19	0.022
(4a) $\alpha = 0.4$	0.99	0.303	0.0831	0.0438	0.323	0.0080	0.0451	1.19	0.023
(4b) $\alpha = 0.6$	1.00	0.298	0.0804	0.0447	0.248	0.0039	0.0191	1.19	0.025
(5a) $\lambda = 0.75$	1.00	0.302	0.0826	0.0441	0.324	0.0063	0.0342	1.17	0.023
(5b) $\lambda = 0.95$	1.00	0.299	0.0809	0.0446	0.278	0.0052	0.0263	1.20	0.024
(6a) $\zeta = 0.06$	1.01	0.293	0.0779	0.0456	0.176	0.0061	0.0278	0.67	0.026
(6b) $\zeta = 0.16$	0.99	0.305	0.0844	0.0437	0.353	0.0052	0.0307	1.62	0.022
(7a) $\overline{b} = 0.6$	0.96	0.229	0.0858	0.0467	0.271	0.0057	0.0441	1.19	0.024
(7b) $\overline{b} = 1.0$	1.03	0.347	0.0783	0.0431	0.289	0.0055	0.0236	1.19	0.024

Table 6: **Sensitivity analysis.** Row (1) reports model-implied moments from our baseline calibration (see Table 1). Subsequent rows report model-implied moments when we alter parameter values.

3.9 Sensitivity Analysis

In this section, we conduct a sensitivity analysis to illustrate the role of various parameters of our model. Table 6 shows the results; row (1) reports the results from our baseline calibration (whose parameters are listed in Table 1) for reference. We highlight the main effects of our parameters below.

As discussed towards the end of Section 2.4, having a limit $\overline{\delta}$ for the expected default probability of newly issued bonds is neccesary to obtain finite moments for spreads. Row (2) reports the results when we increase this limit to $\overline{\delta} = 0.99$. The resulting model-implied moments remain similar to their baseline counterparts, although the mean and volatility of spreads, and the default frequency increase.

The secondary market parameters mainly affect bid-ask spreads and turnover. Rows (3a-b) show that increasing constrained investors' holding costs h_c increases bid-ask spreads and the share of total spreads due to the liquidity component. Decreasing investors' bargaining power α (rows 4a-b) or the meeting probability λ (rows 5a-b) generates similar outcomes. Rows (6a-b) shows that turnover is predominantly determined by investors' probability of receiving a liquidity shock ζ—a higher probability results in a higher turnover.

Finally, rows (7a-b) show that the maximal recovery parameter $\overline{b}$ is the main determinant of the average amount recovered following default—increasing $\overline{b}$ increases the recovery rate.

4 Extension: Risk Averse Investors

In this section, we extend our baseline model by relaxing the assumption that international investors price bonds in a risk-neutral manner. Our main finding is that liquidity risk remains an important determinant of sovereign spreads.

Set up. We assume the existence of a stochastic discount factor,[28] $\Lambda_{t,t+1}$, which international investors use to price all asset returns between t and $t+1$, $R_{t,t+1}$, according to the asset pricing relation

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t,t+1} \right]. \tag{4.1}$$

We model the SDF as

$$\Lambda_{t,t+1} = \frac{1}{1+r} \exp\left(-\frac{1}{2} x_t^2 - x_t \varepsilon_{\Lambda,t+1} \right), \tag{4.2}$$

where r is the risk-free rate and x_t is the market price of risk. The shock

$$\varepsilon_{\Lambda,t+1} = \rho_{\Lambda,z} \varepsilon_{z,t+1} + \sqrt{1-\rho_{\Lambda,z}^2} \varepsilon_{\Lambda,t+1}^{\perp} \tag{4.3}$$

is normally distributed and is normalized to have a unit variance. It has correlation $\rho_{\Lambda,z} = corr\left(\varepsilon_{\Lambda,t+1}, \varepsilon_{z,t+1}\right)$ with respect to the output shock $\varepsilon_{z,t+1}$ from equation (3.2); $\varepsilon_{\Lambda,t+1}^{\perp} \sim N(0,1)$ is the component of $\varepsilon_{\Lambda,t+1}$ that is orthgonal to output shocks.

The SDF (4.2) can be interpreted as follows. The shock $\varepsilon_{\Lambda,t+1}$ is the source of risk for which international investors require compensation for bearing. The market price of risk, x_t, is the compensation (risk averse) investors receive for holding an asset whose return has a unit exposure to the source of risk $\varepsilon_{\Lambda,t+1}$.[29] We model the market price of risk as an AR(1) process,

$$x_t = (1-\rho_x)\,\overline{x} + \rho_x x_{t-1} + \sigma_x \varepsilon_{x,t}, \tag{4.4}$$

with mean $\overline{x}$, autocorrelation ρ_x, volatility σ_x, and normally distributed innovations $\varepsilon_{x,t} \sim N(0,1)$ which are orthogonal to $\varepsilon_{\Lambda,t}$. Specifications (4.2) and (4.4) for the SDF and the market price of risk is taken directly from the asset pricing literature (see, e.g., Brennan et

[28] From the Fundamental Theorem of Asset Pricing (see, e.g, Dybvig and Ross (2003) for a review), the existence of a SDF is equivalent to the absence of arbitrage in asset markets. The latter is a necessary requirement for asset markets to be in equilibrium.

[29] That is, for assets whose log returns are of the form $\log R_{t,t+1} = \mathbb{E}_t\left[\log R_{t,t+1}\right] + \varepsilon_{\Lambda,t+1}$, it follows from the asset pricing equation (4.1) that $\mathbb{E}_t\left[\log R_{t,t+1}\right] - \log(1+r) - \frac{1}{2} Var_t\left(\log R_{t,t+1}\right) = -Cov_t\left(\log \Lambda_{t,t+1}, \log R_{t,t+1}\right)$. From equation (4.2), this quantity is just the market price of risk x_t.

al. (2004) and Lettau and Wachter (2007)), and is closely related to the specifications used in affine term structure models (see, e.g., Singleton (2006) for a textbook treatment). We do not take a stance on the microfoundation for the SDF.[30] Instead, we calibrate international investors' SDF (4.2) to US data and study the resulting implications for sovereign debt.

The valuation equations for bond prices are then modified to take the SDF (4.2) into account. For example, equation (2.10) for unconstrained investors' debt valuation becomes

$$\begin{aligned} q_{ND}^{H}\left(x,y,b'\right) = \mathbb{E}_{x',y'|x,y}\Bigg\{ \exp\left(-\frac{1}{2}x^2 - x\varepsilon'_{\Lambda}\right)\Bigg(& \left[\left(1-d\left(x',y',b'\right)\right]\right. \\ & \times \frac{m+(1-m)\left[z+\zeta q_{ND}^{L}\left(x',y',b''\right)+(1-\zeta)q_{ND}^{H}\left(x',y',b''\right)\right]}{1+r} \\ & + d\left(x',y',b'\right)\frac{\zeta q_{D}^{L}\left(x',y',b'\right)+(1-\zeta)q_{D}^{H}\left(x',y',b'\right)}{1+r}\Bigg)\Bigg\}, \end{aligned} \tag{4.5}$$

where an additional state variable for the market price of risk is now included to take the law of motion (4.4) into account. Similarly, the bond valuation equations (2.11), (2.12), and (2.13) are modified to additionally include the market price of risk as a state variable. In turn, the value functions (2.3), (2.5), and (2.6), default policy (2.4), and debt policy (2.9), all additionally depend on the market price of risk.

The baseline model arises as a special case of the extended model in which output shocks are uncorrelated with investors' source of risk (i.e., $\rho_{\Lambda,z} = 0$ in equation (4.3)). In this case, international investors discount bond payoffs at the risk free rate r—they do not assess any risk premium upon sovereign bonds because the output risk $\varepsilon_{z,t}$ inherent within sovereign bonds is orthogonal to the source of risk that investors care about.

Calibration and results. We simulate the extended model at a monthly frequency using the parameters summarized in Panel A of Table 7. We set the risk free rate to $r = 0.0033$, the same as that from our baseline calibration. We calibrate the remaining parameters of the SDF (4.2) under the assumption that US investors are the marginal international investor. We directly use estimates from Lettau and Wachter (2007) who estimate the SDF (4.2) based on US equity returns: $\overline{x} = 0.1804$, $\rho_x = 0.9885$, and $\sigma_x = 0.0693$. In order to estimate the correlation $\rho_{\Lambda,z}$, we proxy for discount rate shocks $\varepsilon_{\Lambda,t}$ using AR(1) innovations to the (log) dividend-price ratio of the aggregate US stock market.[31] We find

[30] See, for example, Cochrane (2017) for a review of leading asset pricing theories that offer interpretations for the source of risk $\varepsilon_{\Lambda,t}$ and shocks to the market price of risk $\varepsilon_{x,t}$.

[31] This identifying assumption implicitly assumes (1) that the source of risk for international investors, $\varepsilon_{\Lambda,t}$, corresponds to aggregate US stock market risk, and (2) that stock return risk is mainly driven by discount rate variation rather than cashflow variation (in line with evidence from the Campbell and Shiller

Parameter	Description	Value
	A. SDF	
r	Risk free rate	0.0033
$\overline{x}$	Market price of risk, mean	0.1804
ρ_x	Market price of risk, autocorrelation	0.9885
σ_x	Market price of risk, volatility	0.0693
$\rho_{\Lambda,z}$	Corr. between output shock and investors' source of risk	-0.033
	B. Moments, extended model	
	Mean Debt to GDP	1.0
	Expected Recovery	0.30
	Mean Sovereign Spread	0.0815
	Vol. Sovereign Spread	0.0443
	Mean Bid-Ask Spread, ND	0.0056
	Mean Turnover (annual)	1.19
	Default frequency (annual)	0.024

Table 7: **Parameters for extended model.** Panel A reports the calibrated values for the SDF; the remaining parameter values are taken from the baseline model (see Table 1). Panel B plots the main moments from the extended model.

a low correlation of $\rho_{\Lambda,z} = -0.033$ in the data.[32] All remaining parameters are taken from our baseline calibration, reported in Table 1.

Panel B of Table 7 reports the moments from the extended model. These moments are near-identical when compared to their baseline counterparts from column (2) of Table 2. This is not surprising, as the calibrated correlation $\rho_{\Lambda,z} = -0.033$ is too insignificant of a deviation from our baseline model in which $\rho_{\Lambda,z} = 0$. All other results for the extended model are also near-identical in comparison to that of the baseline model. For example, the liquidity share of total spreads is 29.6% on average, exactly as in our baseline model. We conclude that, as in our baseline model, liquidity risk remains an important determinant of sovereign spreads when we extend our model to include investor risk aversion.

5 Conclusion

The quantitative literature on sovereign debt has helped us to understand debt capacity, spreads, and welfare when the central friction is lack of commitment. The recent Sovereign Debt Crisis in Europe, however, has highlighted that there is substantial liquidity risk associated with sovereign lending. Motivated by these facts, we study debt

(1988) decomposition).

[32]We similarly compute a low correlation $\rho_{\Lambda,z}$ if we proxy for $\varepsilon_{\Lambda,t}$ using AR(1) innovations to U.S. aggregate consumption growth.

and default policy when the government lacks commitment, and there are search frictions in the secondary market for sovereign bonds.

After proposing a tractable framework in which both credit and liquidity risk constrain the debt and default choices of the government, we proceeded to study the quantitative importance of liquidity in sovereign spreads and welfare. To do so, we calibrated our model to match the main features of the Argentinean experience during the 1990's, one of the most widely studied cases of sovereign default. Our first result is that liquidity risk is a substantial component of sovereign spreads. In particular, in a model-based decomposition, we find that it accounts for 23 percent of total sovereign spreads. Our second result is that liquidity matters for welfare. In particular, through the lens of our model, a representative agent would be willing to pay 42 percent of the cost of income fluctuations to shut down liquidity frictions.

Why should we distinguish between credit and liquidity risk? Why should we incorporate liquidity risk into models of sovereign borrowing? An important reason to distinguish between credit and liquidity risk are the different normative implications of each friction. There are at least three reasons why the introduction of liquidity would matter for debt management policy. First, long-run policies to fight spreads caused by lack of commitment might be different from those to combat spreads due to the lack of liquidity. Second, policies in the short run might also be different; for example, one strategy during bad times could be to capitalize financial intermediaries as opposed to decreasing government spending or repaying debt. Third, maturity and currency management policies might also differ. The benefit of spreading debt across currencies and maturities is to complete the market. However, a possible unintended consequence would be to lower liquidity in the market for these bonds. The increase in the cost of borrowing due to illiquidity could potentially undo the insurance benefits of a more complete debt market. All of these are topics for further research.

References

Admati, Anat R, Peter M DeMarzo, Martin F Hellwig, and Paul Pfleiderer, "The leverage ratchet effect," *The Journal of Finance*, 2018, *73* (1), 145–198.

Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 2006, *69* (1), 64 – 83.

_ and Manuel Amador, "Sovereign debt," in "Handbook of International Economics," Vol. 4, Elsevier, 2014, pp. 647–687.

_ , _ , Hugo Hopenhayn, and Iván Werning, "Take the Short Route: Equilibrium Default and Debt Maturity," *Econometrica*, 2019, *87* (2), 423–462.

_ , Satyajit Chatterjee, Harold Cole, and Zachary Stangebye, "Self-Fulfilling Debt Crises, Revisited: The Art of the Desperate Deal," Technical Report, National Bureau of Economic Research 2017.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 2008, *98* (3), 690–712.

_ and Ananth Ramanarayanan, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 2012, *120* (2), pp. 187–232.

_ and Yan Bai, "Renegotiation policies in sovereign defaults," *American Economic Review*, 2014, *104* (5), 94–100.

_ and _ , "Fiscal austerity during debt crises," *Economic Theory*, 2017, *64* (4), 657–673.

_ , _ , and Luigi Bocola, "Sovereign default risk and firm heterogeneity," *Working Paper*, 2019.

Bai, Jennie, Christian Julliard, and Kathy Yuan, "Eurozone Sovereign Bond Crisis: Liquidity or Fundamental Contagion," *Working Paper*, 2012.

Bai, Yan and Jing Zhang, "Duration of sovereign debt renegotiation," *Journal of International Economics*, 2012, *86* (2), 252 – 268.

Bao, Jack, Jun Pan, and Jiang Wang, "The Illiquidity of Corporate Bonds," *The Journal of Finance*, 2011, *66* (3), 911–946.

Beim, DO. and CO Calomiris, "Emerging Financial Markets," *McGraw-Hill*, 2001.

Benjamin, D and M Wright, "Recovery Before Redemption: A Theory of Sovereign Debt Renegotiation," *Working Paper*, 2009.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez, "International reserves and rollover risk," *American Economic Review*, 2018, *108* (9), 2629–70.

Bocola, Luigi and Alessandro Dovis, "Self-fulfilling debt crises: A quantitative analysis," *American Economic Review*, 2019, *109* (12), 4343–77.

Borri, Nicola and Adrien Verdelhan, "Sovereign risk premia," 2009.

Brennan, Michael, Ashley Wang, and Yihong Xia, "Estimation and Test of a Simple Model of Intertemporal Capital Asset Pricing," *Journal of Finance*, 2004, *59* (4), 1743–1776.

Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler, "Why Do Emerging Economies Borrow Short Term?," *Journal of the European Economic Association*, 2013, *11*, 67–100.

Bulow, Jeremy and Kenneth Rogoff, "Sovereign Debt Repurchases: No Cure for Overhang," *Quarterly Journal of Economics*, 11 1991, *106* (4), 1219–1235.

Campbell, John Y and John H Cochrane, "By force of habit: A consumption-based explanation of aggregate stock market behavior," *Journal of political Economy*, 1999, *107* (2), 205–251.

Campbell, John Y. and Robert J. Shiller, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *The Review of Financial Studies*, 1988, *1* (3), 195–228.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, pp. 2674–2699.

_ **and** _ , "A seniority arrangement for sovereign debt," *The American Economic Review*, 2015, *105* (12), 3740–3765.

Chaumont, Gaston, "Sovereign Debt, Default Risk, and the Liquidity of Government Bonds," *Working Paper*, 2020.

Chen, Hui, Rui Cui, Zhiguo He, and Konstantin Milbradt, "Quantifying liquidity and default risks of corporate bonds over the business cycle," *The Review of Financial Studies*, 2017, *31* (3), 852–897.

Chen, Long, David Lesmond, and Jason Wei, "Corporate Yield Spreads and Bond Liquidity," *The Journal of Finance*, 2007, *62* (1), 119–149.

Cochrane, John H, "Macro-Finance," *Review of Finance*, 03 2017, *21* (3), 945–985.

Cruces, Juan J. and Christoph Trebesch, "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 85–117.

Dovis, Alessandro, "Efficient sovereign default," *The Review of Economic Studies*, 2018, *86* (1), 282–312.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, "Over-the-Counter Markets," *Econometrica*, 2005, *73* (6), 1815–1847.

Dybvig, Philip and Stephen Ross, "Arbitrage, state prices and portfolio theory," in George Constantinides, Milton Harris, and Rene Stulz, eds., *Handbook of Economics and Finance*, Elsevier, 2003.

Eaton, Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 1981, *48* (2), pp. 289–309.

Edwards, Amy, Lawrence Harris, and Michael Piwowar, "Corporate Bond Market Transaction Costs and Transparency," *The Journal of Finance*, 2007, *62* (3), 1421–1451.

Flandreau, Marc, Juan H Flores, Norbert Gaillard, and Sebastián Nieto-Parra, "The end of gatekeeping: underwriters and the quality of sovereign bond markets, 1815-2007," Technical Report, National Bureau of Economic Research 2009.

Fleming, Michael J., "Are Larger Treasury Issues More Liquid? Evidence from Bill Reopenings," *Journal of Money, Credit and Banking*, 2002, *34* (3), pp. 707–735.

Friewald, Nils, Rainer Jankowitsch, and Marti G. Subrahmanyam, "Illiquidity or credit deterioration: A study of liquidity in the {US} corporate bond market during financial crises," *Journal of Financial Economics*, 2012, *105* (1), 18 – 36.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79* (1), 117 – 125.

_ , _ , and Cesar Sosa-Padilla, "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 2016, *124* (5), 1383–1422.

_ , _ , and Horacio Sapriza, "Quantitative properties of sovereign default models: solution methods matter," *Review of Economic Dynamics*, 2010, *13* (4), 919–933.

He, Zhiguo and Konstantin Milbradt, "Endogenous liquidity and defaultable bonds," *Econometrica*, 2014, *82* (4), 1443–1508.

Lagos, Ricardo and Guillaume Rocheteau, "Search in Asset Markets: Market Structure, Liquidity, and Welfare," *The American Economic Review*, 2007, *97* (2), pp. 198–202.

Lettau, Martin and Jessica Wachter, "Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium," *Journal of Finance*, 2007, *62* (1), 55–92.

Lizarazo, Sandra Valentina, "Default risk and risk averse international investors," *Journal of International Economics*, 2013, *89* (2), 317–330.

Longstaff, Francis., Sanjay Mithal, and Eric Neis, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *The Journal of Finance*, 2005, *60* (5), 2213–2253.

Lucas, Robert E, "Macroeconomic priorities," *The American Economic Review*, 2003, *93* (1), 1.

Mendoza, Enrique and Vivian Yue, "A general equilibrium model of sovereign default and business cycles," *Quarterly Journal of Economics*, jan 2012.

Moen, Espen R, "Competitive search equilibrium," *Journal of political Economy*, 1997, *105* (2), 385–411.

Morelli, Juan M, Pablo Ottonello, and Diego J Perez, "Global Banks and Systemic Debt Crises," *Working Paper*, 2019.

Neumeyer, Pablo A. and Fabrizio Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 2005, *52* (2), 345 – 380.

Ottonello, Pablo and Diego Perez, "The currency composition of sovereign debt," *Mimeo*, 2018.

Pelizzon, Loriana, Marti G Subrahmanyam, Davide Tomio, and Jun Uno, "Sovereign credit risk, liquidity, and European Central Bank intervention: Deus ex machina?," *Journal of Financial Economics*, 2016, *122* (1), 86–115.

_, Marti Subrahmanyam, Davide Tomio, and Jun Uno, "The Microstructure of the European Sovereign Bond Market: A Study of Euro-zone Crisis," *Working Paper*, 2013.

Perez, Diego et al., "Sovereign debt, domestic banks and the provision of public liquidity," *Manuscript, New York University*, 2019.

Pouzo, Demian and Ignacio Presno, "Sovereign default risk and uncertainty premia," *American Economic Journal: Macroeconomics*, 2016, *8* (3), 230–66.

Roch, Francisco and Harald Uhlig, "The dynamics of sovereign debt crises and bailouts," *Journal of International Economics*, 2018, *114*, 1–13.

Sanchez, Juan, Horacio Sapriza, Emircan Yurdagul et al., "Sovereign default and the choice of maturity," *Journal of Monetary Economics, Forthcoming*, 2018.

Singleton, Kenneth, *Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment*, Princeton University Press, 2006.

Tauchen, George, "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 1986, *20* (2), 177–181.

Tourre, Fabrice, "A macro-finance approach to sovereign debt spreads and returns," *manuscript, University of Chicago*, 2017.

Yue, Vivian Z., "Sovereign default and debt renegotiation," *Journal of International Economics*, 2010, *80* (2), 176 – 187.

Appendix to "Illiquidity in Sovereign Debt Markets"

Juan Passadore and Yu Xu

A Debt issuance

This section shows that, in simulations of our baseline calibration (Table 1 lists the parameters), in equilibrium the government does not buy back debt. Monthly proportional debt issuance, measured as a fraction of outstanding debt at the start of each period, is given by

$$issue_t = \frac{b_{t+1} - (1-m)\, b_t}{b_t}. \tag{A.1}$$

We simulate a long time series for our baseline calibration and report results. In the simulation, proportional debt issuance has a mean of $\mathbb{E}\left[issue_t\right] = 0.017$ and a standard deviation of $\sigma\left(issue_t\right) = 0.0044$. The mean proportional debt issuance is slightly higher than the debt maturity probability of $m = 0.0167$, which indicates that in most months, the government is simply rolling over maturing debt. The minimum and maximum values for proportional debt issuances in simulations are $min\left\{issue_t\right\} = 0.0051$ and $\max\left\{issue_t\right\} = 0.1093$, respectively. The strictly positive minimum value indicates that there are no debt buybacks on the equilibrium path.

B Jump to Default

In this appendix, we describe and analyze a particular case of our model in which default probabilities are exogenously given. In subsection B.1 we describe the simple model. In subsection B.2 we introduce a definition of the liquidity premium and describe its determinants. In subsection B.3 we discuss the interaction of the credit and liquidity risk premium. In subsection B.5 we clarify the role of maturity and recovery in the interaction of the credit and liquidity risk premium.

B.1 Simple Model

Assume an unconditional constant default probability $\overline{p}_d$ in each period. Then the pricing equations yield a system of 6 equations for 6 bond prices, $\overline{q}^H_{ND}, \overline{q}^L_{ND}, \overline{q}^H_{D}, \overline{q}^L_{D}, \overline{q}^S_{ND}$, and $\overline{q}^S_{D}$.

The system is given by

$$\bar{q}^H_{ND} = \frac{1}{1+r}\left[(1-\bar{p}_d)\left(m+(1-m)\left(z+\zeta\bar{q}^L_{ND}+(1-\zeta)\,\bar{q}^H_{ND}\right)\right)\right.$$
$$\left.+\bar{p}_d\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)\right], \tag{B.1}$$
$$\bar{q}^L_{ND} = \frac{1}{1+r}\left[(1-\bar{p}_d)\left(-h_c+m+(1-m)\left(z+\lambda\bar{q}^S_{ND}+(1-\lambda)\,\bar{q}^L_{ND}\right)\right)\right.$$
$$\left.+\bar{p}_d\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)\right], \tag{B.2}$$
$$\bar{q}^S_{ND} = \bar{q}^L_{ND}+\alpha\left(\bar{q}^H_{ND}-\bar{q}^L_{ND}\right),$$

for bonds not in default,[33] and by

$$\bar{q}^H_D = \frac{1-\theta}{1+r}\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)+\theta\bar{f}\bar{q}^H_{ND}, \tag{B.3}$$
$$\bar{q}^L_D = \frac{1-\theta}{1+r}\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)+\theta\bar{f}\bar{q}^L_{ND}, \tag{B.4}$$
$$\bar{q}^S_D = \bar{q}^L_D+\alpha\left(\bar{q}^H_D-\bar{q}^L_D\right),$$

for bonds in default, where $\bar{f}\in[0,1]$ denotes the fraction recovered after a default (i.e., $\bar{f}$ corresponds to the long run average of the $\mathcal{R}(b)/b$ term in equation 2.12). The solution to this system yields six bond prices each of which depends on the unconditional default probability, $\bar{p}_d$, and other parameters of the model. Now suppose that there is a transition shock whereby the current default probability becomes p_d, and reverts back to the long-run default probability $\bar{p}_d$ after defaulting.[34] Current prices for non-defaulted bonds are

[33] For ease of exposition, the equations do not include the free asset disposal conditions that guarantee non-negative prices. Bond prices in the presence of free asset disposal can be characterized as the solution to a linear complementarity problem.

[34] This captures the idea of mean reversion in default hazard rates, which is common in the literature on credit risk modeling (see, for example Longstaff et al. (2005)).

then given by:

$$
\begin{aligned}
q_{ND}^{H} &= \frac{1}{1+r}\left[(1-p_d)\left(m+(1-m)\left(z+\zeta q_{ND}^{L}+(1-\zeta)\,q_{ND}^{H}\right)\right)\right.\\
&\quad \left.+p_d\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)\right],\\
q_{ND}^{L} &= \frac{1}{1+r}\left[(1-p_d)\left(-h_c+m+(1-m)\left(z+\lambda q_{ND}^{S}+(1-\lambda)\,q_{ND}^{L}\right)\right)\right.\\
&\quad \left.+p_d\left(-h_c+\lambda q_{D}^{S}+(1-\lambda)\,q_{D}^{L}\right)\right],\\
q_{ND}^{S} &= q_{ND}^{L}+\alpha\left(q_{ND}^{H}-q_{ND}^{L}\right),\\
q_{D}^{H} &= \frac{1-\theta}{1+r}\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)+\theta f\overline{q}_{ND}^{H},\\
q_{D}^{L} &= \frac{1-\theta}{1+r}\left(-h_c+\lambda q_{D}^{S}+(1-\lambda)\,q_{D}^{L}\right)+\theta f\overline{q}_{ND}^{L},\\
q_{D}^{S} &= q_{D}^{L}+\alpha\left(q_{D}^{H}-q_{D}^{L}\right),
\end{aligned}
\tag{B.5}
$$

where the post-default bond prices $\overline{q}_{ND}^{H}, \overline{q}_{ND}^{L}, \overline{q}_{D}^{H}, \overline{q}_{D}^{L}$ are defined in equations (B.1) to (B.4) and incorporate the reversion of the default probability to its long-run value $\overline{p}_d$.

B.2 The Liquidity Premium and its Determinants

Rewrite the bond price q_{ND}^{H}, given by the first equation of (B.5), as:

$$
q_{ND}^{H} = \frac{(1-p_d)\left(m+(1-m)\left(z+q_{ND}^{H}\right)\right)+p_d q_{D}^{H}}{1+r+\ell_{ND}},
\tag{B.6}
$$

where the liquidity premium

$$
\ell_{ND} \equiv (1-p_d)\,(1-m)\,\zeta\frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}}+p_d\zeta\frac{q_{D}^{H}}{q_{ND}^{H}}\frac{q_{D}^{H}-q_{D}^{L}}{q_{D}^{H}},
\tag{B.7}
$$

is the additional discount that is needed to equate the market price q_{ND}^{H} to the valuation of an investor who is not subject to liquidity concerns (i.e., the right hand side of equation B.6). The liquidity premium (B.7) is a function of (1) the frequency at which investors encounter liquidity shocks, through the ζ term; (2) the valuation loss conditional on receiving a liquidity shock, through the bid ask spreads, which identifies the $\left(q^H-q^L\right)/q^H$ terms; (3) the recovery rate, through the term q_D^H/q_{ND}^H, which is the ratio of the valuation of the defaulted bonds to the non defaulted bonds; (4) the maturity of the bonds issued, through the term $(1-m)$; (5) the default probabilities, through the p_d terms.

B.3 The Co-movement of Default and Liquidity Risk

In this subsection, we investigate how the default and liquidity risk terms interact. From equation (B.7), we see that the liquidity premium depends on distance to default (or p_d). We can group terms in p_d and we arrive to

$$\ell_{ND} = (1-m)\,\zeta \frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}} + p_d \zeta \left[\frac{q^H_D}{q^H_{ND}} \frac{q^H_D - q^L_D}{q^H_D} - (1-m) \frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}} \right]. \tag{B.8}$$

In our simulations the second term of (B.8) is approximately equal to zero (more precisely, second order in comparison to the first term), and we show it is exactly equal to zero in the continous time limit in subsection B.6, thus we can approximate (B.8) as:

$$\ell_{ND} \approx (1-m)\,\zeta \frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}}. \tag{B.9}$$

Equation (B.9) shows that there is a co-movement between the liquidity premium and default risk as long as the (pre default) bid ask spreads *endogenously* respond to the economic conditions (summarized by b, y in the full model).

B.4 Endogenous Maturity Extension Channel

From equation (B.9), it is clear that for our model to generate comovement between credit and liquidity risk, we need pre-default bid-ask spreads to increase as the probability of default increases. Now we explain why this is the case. In our model, bid-ask spreads during default are higher due to an *endogenous maturity extension channel*. The intuition is that defaulted bonds have a longer expected maturity than the bonds while the country is repaying debt. This longer maturity results from the fact that sovereigns do not pay coupons nor principal while the bond is defaulted, which lasts (approximately) on average $1/\theta$ months. Therefore, receiving a liquidity shock is worse during default because the higher discounting changes the price more of a longer maturity instrument.

To see the endogenous maturity extension channel more formally, consider the expected time to repayment for an individual unit of debt:

$$T^i \equiv \mathbb{E}\left[\tau_m \,|i\right],\; i \in \{D, ND\}, \tag{B.10}$$

where τ_m denotes the random time of maturity whose distribution depends on the default status of the government $i \in \{D, ND\}$. As in Section B, we distinguish between the long-run and short-run expected maturities, $\overline{T}^i$ and T^i, respectively. In particular, the long-run expected maturities, in and out of default, satisfies

$$\begin{aligned}\overline{T}^D &= 1+\overline{\theta}\overline{T}^D+(1+\overline{\theta})\overline{T}^D,\\ \overline{T}^{ND} &= 1+\overline{p}_d\overline{T}^D+(1-\overline{p}_d)(1-m)\overline{T}^{ND},\end{aligned}$$

and takes into account the long-run probabilities of default $\overline{p_d}$ and reentry $\overline{\theta}$. Similarly, short-run expected maturities satisfy

$$\begin{aligned}T^D &= 1+\theta\overline{T}^{ND}+(1+\theta)T^D,\\ T^{ND} &= 1+p_dT^D+(1-p_d)(1-m)T^{ND},\end{aligned}$$

and incorporate the short run probabilities of default p_d and reentry θ. The solution to the systems of equations above is given by

$$\overline{T}^{ND}=\frac{\overline{\theta}+\overline{\theta}_d}{m\overline{\theta}(1-\overline{\theta}_d)},\quad \text{and}\quad \overline{T}^D=\frac{m+\overline{\theta}+(1-m)\overline{\theta}_d}{m\overline{\theta}(1-\overline{\theta}_d)},$$

for long-run expected maturities, and by

$$T^{ND}=\frac{1+p_dT^D}{m+(1-m)p_d},\quad \text{and}\quad T^D=\frac{1}{\theta}+\overline{T}^{ND},$$

for short-run expected maturities. It is instructive to consider the case where the long-run default probability is small (i.e., $\overline{p}_d\approx 0$). In this case, the expected maturity prior to defaulting in the short run becomes

$$T^{ND}\approx\frac{1}{m+(1-m)p_d}+\frac{p_d}{m+(1-m)p_d}\left(\frac{1}{\theta}+\frac{1}{m}\right),$$

which is increasing in the default probability p_d.

Figure B.1 illustrates the resulting implications of the maturity extension channel for the liquidity premium. Panel A holds the default probability fixed at $p_d=0.4$ and shows that the valuation component of liquidity premium (B.9), $(q^H_{ND}-q^L_{ND})/q^H_{ND}$, is increasing as a function of the expected duration of autarky $(1/\theta)$, which parameterizes the strength of the maturity extension channel. Panel B plots the valuation component as a function of default probability p_d. We see that the relation between the liquidity premium and the default probability becomes steeper when the expected duration of autarky is longer (solid line).

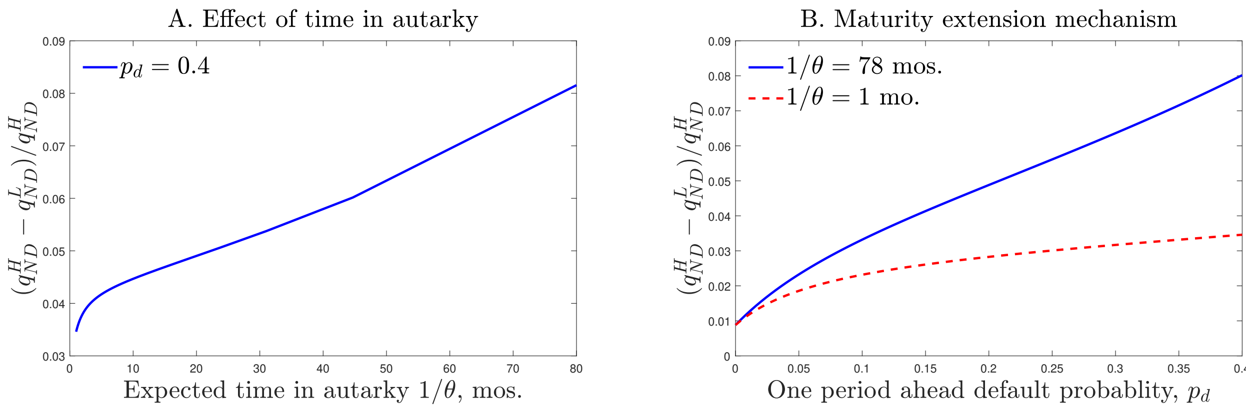

Figure B.1: **Maturity extension mechanism.** This figure illustrates the maturity extension mechanism in the context of the jump to default model. Panel A plots the proportional valuation difference between unconstrained and constrained investors, $(q^H_{ND} - q^L_{ND})/q^H_{ND}$, as a function of the duration of autarky (the default probability is held fixed at $p_d = 0.4$). Panel B plots the proportional valuation difference as a function of the short-run default probability p_d for short (dashed line) and long (solid line) durations of autarky.

B.5 The role of long-term debt and debt recovery

We now illustrate the roles of long-term debt and debt recovery in the context of the jump to default model outlined in subsection B.1.

First, Panel A of Figure B.2 plots the relation between the liquidity premium (B.7) and the default probability p_d for a short debt maturity of a month (dashed-line), and for a long debt maturity of sixty months (solid line). We see that the relation is almost flat when debt maturity is short. Intuitively, constrained investors care little about liquidity risk if debt matures in a period—they will simply collect the principal from the government when debt comes due.

Second, Panel B of Figure B.2 illustrates the relation between the liquidity premium (B.7) and the default probability p_d when fractional recovery is (i) constant ($f = \overline{f}$), and (ii) a decreasing function of the default probability ($f = f(p_d)$). The latter is a feature of our full model in which the fractional recovery $\mathcal{R}(b)/b$ negatively depends on the default probability through its dependence on the debt level b. We see that, compared to the case of a constant recovery, the relation between liquidity and default is steeper when the recovery rate is a decreasing function of the default probability. This result can be understood from the maturity extension channel: in present value terms, recovering a smaller amount is the same as recovering a (fixed) larger amount at a later date. Therefore, a decreasing recovery function amplifies the maturity extension channel, which is why the default-liquidity relation is steeper.

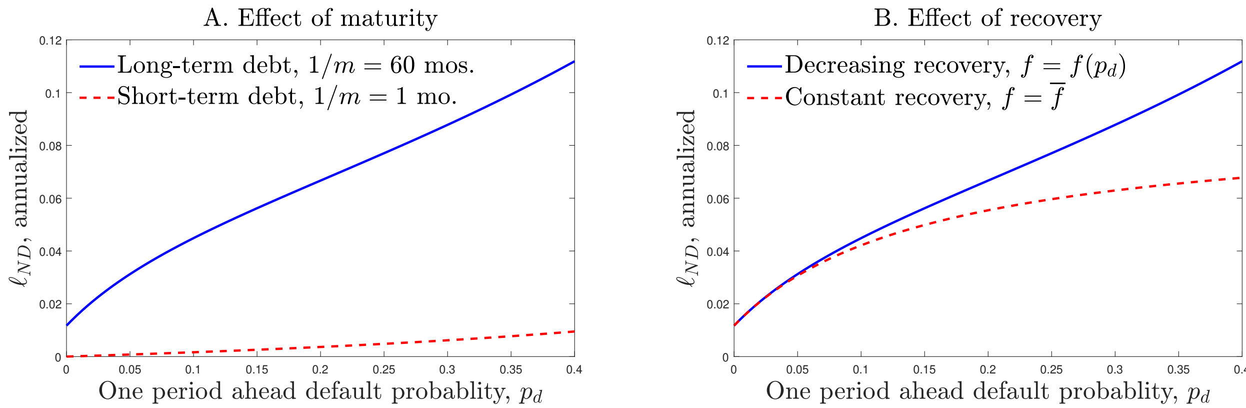

Figure B.2: **The role of long-term debt and debt recovery.** Panel A plots the liquidity premium, defined in equation (B.7), as a function of the default probability p_d, when debt maturity is one month (dashed line) and when debt maturity sixty months (solid line). Panel B plots the liquidity premium in the case in which recovery is constant (red dotted line) and in when recovery depends on the default probability (as in our baseline model). The particular functional form for the plot is $f(p_d) = \overline{f}e^{-p_d}$.

B.6 Continuous Time Limit

Now we show that equation (B.9) holds in the continuous time limit. To do so, we transform probabilities into hazard rates through

$$x_\Delta = 1 - e^{-\tilde{x}\Delta},\ x \in \{\zeta, p_d, m, \lambda\},$$

and convert the per-period interest rate into its continuously compounded counterpart through

$$1 + r_\Delta = e^{\tilde{r}\Delta}.$$

The bond price equation (B.5) for the valuation of an unconstrained investor becomes

$$\tilde{r}q^H_{ND} = \tilde{z} + \tilde{m}\left(1 - q^H_{ND}\right) + \tilde{p}_d\left(q^H_D - q^H_{ND}\right) + \tilde{\zeta}\left(q^L_{ND} - q^H_{ND}\right). \tag{B.11}$$

in the continuous time limit $\Delta \to 0$. The liquidity premium in the continuous time limit $\tilde{\ell}_{ND}$ then solves:

$$\left(\tilde{r} + \tilde{\ell}_{ND}\right) q^H_{ND} = \tilde{z} + \tilde{m}\left(1 - q^H_{ND}\right) + \tilde{p}_d\left(q^H_D - q^H_{ND}\right). \tag{B.12}$$

Equalizing (B.12) and (B.11), and solving for $\tilde{\ell}_{ND}$, results in

$$\tilde{\ell}_{ND} = \tilde{\zeta}\frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}},$$

which is the analog of equation (B.9).[35]

C Numerical Method

It is well-known that numerical convergence is often a problem in discrete-time sovereign debt models with long-term debt. To circumvent this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). This involves altering total output to be: $y_t + \varepsilon_t$, where $\varepsilon_t \sim \mathit{trunc}\, N(0, \sigma_\varepsilon^2)$ is continuously distributed. As shown in Chatterjee and Eyigungor (2012), the noise component ε_t guarantees the existence of a solution of the pricing function equation. The government's repayment problem (2.6) is altered as follows:

$$V^C(y,b,\varepsilon) \;=\; \max_{b'} \left\{ (1-\beta)\, u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}(y', b') \right] \right\},$$

where the budget constraint is now given by,

$$c \;=\; y + \varepsilon - b\left[m + (1-m)\, z\right] + q_{ND}^H(y, b') \left[b' - (1-m)\, b\right]$$

and the maximum default probability is given by

$$\delta(y,b') = \mathbb{E}_{\varepsilon', y'|y}\left[d(y', b', \varepsilon')\right] \leq \bar{\delta}.$$

Debt choice is denoted as $b'(b, y, \varepsilon)$. We impose that $\varepsilon_t \equiv 0$ during the autarky regimes, meaning that the expression for the value to default remains the same; i.e.,

$$V^D(y,b) \;=\; (1-\beta)\, u(y - \phi(y)) + \beta \mathbb{E}_{y'}\left[\theta V^{ND}(y', \mathcal{R}(b)) + (1-\theta)\, V^D(y,b)\right].$$

The default decision is given by:

$$d(y,b,\varepsilon) \;=\; 1_{\left\{V^C(y,b,\varepsilon) \geq V^D(y,b)\right\}},$$

and depends on the randomization component. The continuation value is adjusted as follows:

$$V^{ND}(y,b) \;=\; \mathbb{E}_\varepsilon\left[\max\left\{V^D(y,b), V^C(y,b,\varepsilon)\right\}\right],$$

[35] Note that the probability m in equation (B.9) converges to zero as the time interval goes to zero.

to take into account the randomization component. Finally, bond prices are also adjusted to take into account the additional randomization variable:

$$
\begin{aligned}
q^H_{ND}\left(y,b'\right) &= \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(y',b',\varepsilon')}{1+r}\left[m+(1-m)\left[\begin{array}{c} z+\zeta q^L_{ND}\left(y',b'\left(b',y',\varepsilon'\right)\right) \\ +\left(1-\zeta\right)q^H_{ND}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right] \\ +\frac{d(b',y',\varepsilon')}{1+r}\left[\zeta q^L_D\left(b',y'\right)+\left(1-\zeta\right)q^H_D\left(y',b'\right)\right]\end{array}\right\} \\
q^L_{ND}\left(b',y\right) &= \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(y',b',\varepsilon')}{1+r}\left[-h_c+m+(1-m)\left[\begin{array}{c} z+\left(1-\lambda\right)q^L_{ND}\left(y',b'\left(b',y',\varepsilon'\right)\right) \\ +\lambda q^S_{ND}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right] \\ +\frac{d(y',b',\varepsilon')}{1+r}\left[-h_c+\left(1-\lambda\right)q^L_D\left(y',b'\right)+\lambda q^S_D\left(y',b'\right)\right]\end{array}\right\} \\
q^H_D\left(y,b\right) &= \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q^H_D\left(y',b\right)+(1-\zeta)q^L_D\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q^H_{ND}\left(y,\mathcal{R}(b)\right) \\
q^L_D\left(y,b\right) &= \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c+\lambda q^S_D\left(y',b\right)+(1-\lambda)q^L_D\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q^L_{ND}\left(y,\mathcal{R}(b)\right) \\
q^S_{ND}\left(y,b\right) &= (1-\alpha)q^L_{ND}\left(y,b\right)+\alpha q^H_{ND}\left(y,b\right) \\
q^S_D\left(y,b\right) &= (1-\alpha)q^L_D\left(y,b\right)+\alpha q^H_D\left(y,b\right)
\end{aligned}
$$

The rest of the numerical scheme can be summarize the scheme in four steps:

(1) Start by discretizing the state space. This involves choosing grids $\{y_i\}_{i=1}^{N_y}$ and $\{b_j\}_{j=1}^{N_b}$ for output and debt. The grid points and transition probabilities for output are chosen in accordance with the Tauchen (1986) method and encompass ± 3 standard deviations of the unconditional distribution of output. The grid points for debt values are uniformly distributed over the range $[0, b_{max}]$, with the upper limit set to $b_{max} = 6$ so as to be never binding in simulations. In the baseline model, we choose a dense grid with $N_y = 200$ and $N_b = 450$ to ensure numerical accuracy.

(2) Next, perform value function iteration. Given bond prices, update value functions V^C and V^D. The debt and default policies, $b'(\cdot)$ and $d(\cdot)$, are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, interpolation is used to obtain terms involving $\mathcal{R}(b)$.

(3) Given the debt and default policies, bond prices are then updated.

(4) The above steps are iterated until both value functions and bond prices converge.

We implement the above algorithm in CUDA and numerically compute the model on a Nvidia GeForce RTX 2080 Ti GPU.

D Distribution of bond holder types and turnover

The fraction of outstanding debt held by constrained investors, f_t^L, has law of motion

$$f_{t+1}^L = \begin{cases} (1-\lambda) f_t^L + \zeta \left(1 - f_t^L\right) & \text{if gov. is in default} \\ \frac{\left[(1-\lambda) f_t^L + \zeta\left(1-f_t^L\right)\right](1-m) b_t + \zeta [b_{t+1} - (1-m) b_t]}{b_{t+1}} & \text{if gov. is not in default.} \end{cases} \tag{D.1}$$

When the government is in default, the law of motion (D.1) reflects changes in the composition due to high types receiving liquidity shocks (occurring with probability ζ) as well as low types offloading their position to high types through the market maker (this occurs with probability λ). When the government is not in default, the law of motion (D.1) reflects changes in the composition of bond holder types for previously issued debt, and the fraction of newly issued bonds held by low type investors one period after issuance. In addition, expression (D.1) reflects the fact that the government does not buy back bonds in equilibrium (see Appendix A), so that the $b_{t+1} - (1-m) b_t$ term is always positive. Turnover, or the fraction of outstanding debt transacted in secondary markets each period, is then given by

$$Turnover_t = \lambda f_t^L, \tag{D.2}$$

and is the product of the fraction of potential sellers, f_t^L, and the probability that each potential seller is able to offload his position in a period, λ.

In simulations of the baseline model, we find that both f_t^L and $Turnover_t$ is approximately constant over time. This is because consumption smoothing by the government results in a steady debt level as the government mostly rolls over debt by issuing new debt to replace maturing debt (i.e. $b_{t+1} - (1-m) b_t \approx m b_t$). For example, proportional debt issuance is 0.017 on average, which is approximately equal to the debt maturity rate $m = 0.0167$. As a result, both f_t^L and $Turnover_t$ are approximately equal to their stationary values in a setting where the government only rolls over debt (i.e. $b_{t+1} = b_t$). That is, $f_t^L \approx \zeta / [m + (1-m)(\lambda + \zeta)]$ and $Turnover_t \approx \lambda\zeta / [m + (1-m)(\lambda+\zeta)]$. We confirm these approximations in simulations of baseline model which fully implements the exact law of motion (D.1). For example, the volatility of turnover is $\sigma(Turnover_t) = 2 \times 10^{-5}$.

E Welfare Cost of Business Cycles

In order to put the welfare cost of liquidity frictions into perspective, we follow Lucas (2003) and compute how much a representative agent in Argentina would pay in order to avoid fluctuations in consumption. We assume that the representative agent has consumption

$$c_t = y_t^{\kappa} \tag{E.1}$$

where y_t is given by equations (3.1) and (3.2), and $\kappa = 1.1$ so that $\sigma(c)/\sigma(y) = 1.1$ as in the data. The certainty equivalent value of the consumption stream (E.1) is then

$$c^E = u^{-1}\left(\mathbb{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t)\right]\right). \tag{E.2}$$

How much consumption is the agent willing to forgo in order to avoid the business cycle? In the absence of shocks, $c_t = 1$ for all t so that the cost of the business cycle is $1 - c^E$. This value is 0.4% when we use our baseline model's output process to compute c^E.

F Bid Ask Spread for Argentinean bonds

In this section we report bid-ask spreads for Argentinean bonds for the period 1993 to 2004. This period includes our sample period, 1993-2001, so that all of our calibration targets (interest rates, turnover, bid ask spreads, and debt levels) are extracted from the same period. It additionally includes the post-default period, 2002-2004, which allows us to check our model's predictions for post-default bid-ask spreads against their data counterpart. We find an average bid-ask spread before default (a target value in our calibration) of 56 basis points. In addition, the maximum and average post-default bid-ask spread are 301 basis points and 170 basis points, respectively.

Main Target: Bid Ask Spread. We focus on the global issues (as classified by Bloomberg) for Argentinean bonds that took place between M01 1993 and M12 2004. Denote by $\mathcal{I}$ the set of all bonds that were issued in that period. Denote by $B(t) \subset \mathcal{I}$ the bonds that are still active (non-matured) in date t (we obtain bid ask prices at a daily frequency). The bid ask spread for bond i at date t is

$$BA_{i,t} := \frac{Ask_{i,t} - Bid_{i,t}}{\frac{1}{2}\left(Ask_{i,t} + Bid_{i,t}\right)},$$

where $Ask_{i,t}$ and $Bid_{i,t}$ are the ask and bid prices of bond $i \in B(t)$ on day t. The bid ask spread for Argentina at the aggregate level is defined as

$$BA_t := \sum_{i \in B(t)} \omega_t^i BA_{i,t},$$

where we weight the individual bid ask spreads by the corresponding outstanding face value of the individual bond, which is given by:

$$\omega_t^i := \frac{\text{Value Outstanding}_{i,t}^{USD}}{\sum_{i \in B(t)} \text{Value Outstanding}_{i,t}^{USD}}.$$

We then compute the bid ask spread at the simple average of

$$\overline{BA}^{ND} = \frac{1}{T} \sum_t BA_t,$$

where the dates included are from January 2, 1995 to December 3, 2001 (the latter is the date that Fitch declares Argentina in default in 2001, with a rating of DDD). The average bid ask spread is equal to 56 basis points, which is our calibration target in Table 2. This target is robust to alternative weighting schemes (equal weighting), period of analysis, and dropping stale observations. The default period is from December 4, 2001 to December 31, 2004, and the maximum and average bid-ask spread is equal to 301 basis points and 170 basis points, respectively, over this period.