The Macroeconomics of Hedging Income Shares

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Introduction

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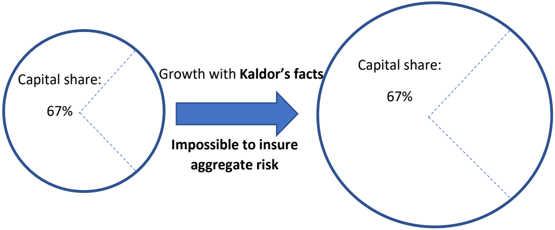
- But now we "know" that they are not constant: many implications.
- **One relevant implication: it affects asset's markets**
  - When income shares move, aggregate shocks can be "insured".
  - These shocks affect capitalists and workers in different ways.
  - Because agents would trade assets to obtain insurance, there could be important effects on financial markets.

Main mechanism at a glance



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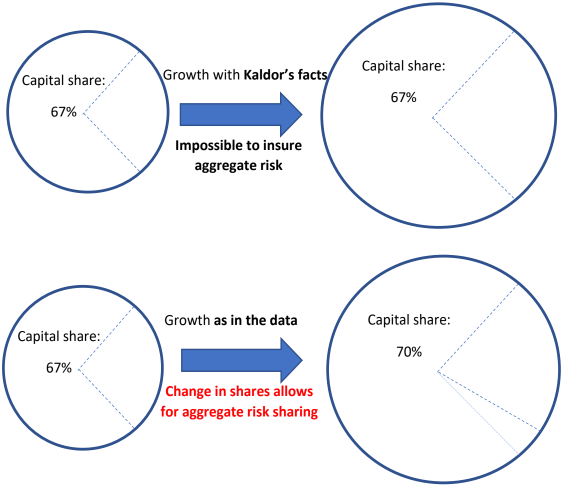

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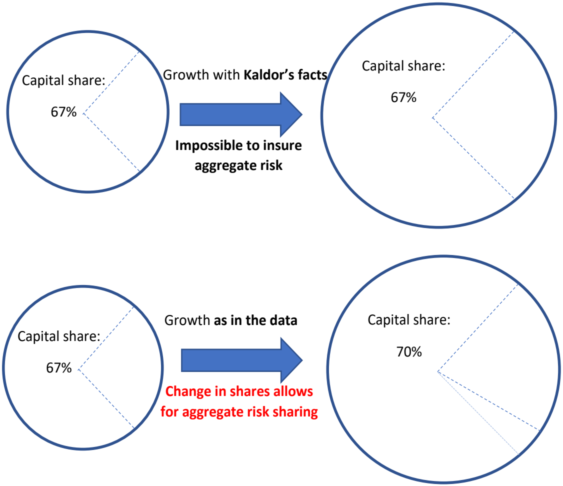

- What are the macro-finance implications? ⇒ **two period model**
- How quantitatively important is the effect? ⇒ **calibrated dynamic model**

This talk

- **What do we do?**
  - We do **NOT** have a theory of why income shares are changing.
    - There are many competing theories.
  - Tractable discrete time model with **varying labor shares using CES**.
  - Similar environment to macro finance, but focused on long run and quantities rather than short run and prices $\Rightarrow$ **Cobb-Douglas is boring**.

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  - Similar environment to macro finance, but focused on long run and quantities rather than short run and prices ⇒ **Cobb-Douglas is boring**.
- **What do we say?** Varying capital/labor shares can generate:
  1) Increased demand for safe assets by firms and risky by households.
  2) Time varying uncertainty.
  3) Decline in risk-free interest rate.
- **Why?** To insure future changes in labor share, workers borrow on the risk free asset from capital holders and buy equity.

Literature (by no means exhausting)

- **Labor share:**
  - Karabarbounis Neiman (2014), Koh Santaeulalia-Llopis Zheng (2017), De-Loecker Eeckhout (2018), León-Ledesma Satchi (2018).
- **Corporate savings glut:**
  - Sanchez Yurdagul (2013), Chen Karabarbounis Neiman (2017), Begenau Palazzo (2017)
- **Business Cycle implications:** Cantore et al (2014,2015). Leon-Ledezma Satchi (2018)
- **Macro Finance:**
  - He and Krishnamurthy (2011), Brunnermeier Sannikov (2014), Di Tella (2013)
  - All things linked: Caballero Farhi Gourinchas (2018) and Karabarbounis et al (2017)

Outline

- Two period version for intuition
  - Complete markets
  - Incomplete markets: constant shares
  - Incomplete markets: varying shares
- Overview of the model for quantitative assessment
- The full model and simulations
- Conclusions

Environment

- Economy with two types of agents.
  - **Consumer-workers:** can work, consume and trade in financial markets.
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  - Entrepreneurs are subject to **skin-in-the-game constraint**: they must keep some idiosyncratic risk arising from production.
  - **Remark 1:** complete set of AD securities does not imply full insurance.
  - **Remark 2:** true, AD securities don't exist, but later I show implementation with known existing assets.
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- Income shares changing: CES with biased technological shocks.

Two period model: intuition

Two periods intuition: **consumers and entrepreneurs**

- At $t = 1$, no uncertainty, but at $t = 2$ many possibilities: $s \in [s^1, .., s^N]$: economy's aggregate state, **with** $Prob(s) = \gamma(s)$

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$$\max_{\{c_1, c_2(s), a(s)\}} \{u(c_1) + \mathbb{E}_s[u(c_2(s))]\}$$

$$st. \;\; c_1 + \sum_s p(s)a(s) \leq a_1 + w_1 L; \quad c_2(s) \leq a(s) + w(s)L, \qquad \forall s$$

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- Heterogeneous entrepreneurs, **who own capital but don't work**, they solve:

$$\max_{\{e_1, e_2(s), E(s)\}} \{u(e_1) + \mathbb{E}_{s,i}[u(e_2(s,i))]\}$$

$$s.t. \ e_1 + \sum_s p(s)E(s) \leq E_1 + \pi_1; \quad e_2(s,i) \leq E(s) + \pi(s,i), \qquad \forall s, \forall i$$

- $E(s)$ and $a(s)$ are Arrow-Debreu securities.
- w is the wage and $\pi(s,i)$ are profits. With $\pi(s,i) = R(s) \times k \times g_i$

Two period model: **closing the model**

- There is no investment $\Rightarrow$ exogenous and random capital: $K(s)$ (later endog.)
- Market clearing

$$a(s) + E(s) = 0; \quad \forall s$$

$$L_t = 1; \quad \forall t$$

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$$\alpha(s) = \alpha \left(\frac{Y(s)}{K(s)} \right)^{\frac{1-\rho}{\rho}}$$
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- Both utility functions: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. No leisure choice.

Two period model: **common equations**

- First order conditions imply:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_{i'}[u'(e_2(s,i'))]}{u'(c_2(s))}; \qquad \forall s$$

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$$\left(\frac{1-x_1}{\underbrace{x_1}_{\text{wealth ratio}}}\right)^{-\sigma} = \frac{\mathbb{E}_i[(\overbrace{-\phi(s)Y_2(s)+\alpha(s)Y_2(s)g_i}^{E(s)+\pi(s,i)})^{-\sigma}]}{(\underbrace{\phi(s)Y_2(s)+(1-\alpha(s))Y_2(s)}_{a(s)+w(s)L})^{-\sigma}}; \quad \forall s$$

Two period, benchmark 1: **complete markets**

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- Efficient allocation: **no wealth effects**. Prices and financial positions are:

$$p(s) = \Pi(s)\left(\frac{x}{x}\right)^{\sigma}\left(\frac{Y_1}{Y_2(s)}\right)^{\sigma}; \qquad \phi(s)^{CM} = x - (1-\alpha(s))$$

  - Security pays positive when $(1-\alpha)$ goes down, negative when it goes up.
  - Asset's prices independent of wealth distribution, and financial asset's holdings stationary (since x is fixed). Inc.

Two period, benchmark 2: **Cobb-Douglas**

- Idiosyncratic risk with **Cobb-Douglas ($\rho = 1$)**, generates:

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- Guess and verify that $a(s) = \phi Y_2(s)$

Note that ϕ solves: $$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} = \frac{\mathbb{E}_{i'}[(-\phi+\alpha g_{i'})^{-\sigma}]}{(\phi+(1-\alpha))^{-\sigma}}$$

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- The last result would break if either:
  - $\mathbb{V}ar(g_i)$ is time varying (volatility shocks: Di Tella, 2016)
  - $\mathbb{E}_i(.)$ is affected by s.

Two period with general technology: **CES**

- Moving $\alpha(s)$ gets both, what matters is $\alpha(s)^2\mathbb{V}ar(g_i) \Rightarrow \phi(s)$ depends on s.
- Replace $\mathbb{E}_i$ with second order Taylor approximation:

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} \simeq \frac{(-\phi(s)^{IM}+\alpha(s))^{-\sigma}}{(\phi(s)^{IM}+(1-\alpha(s)))^{-\sigma}}\left(1+\frac{\sigma(1+\sigma)\alpha(s)^2}{(-\phi(s)^{IM}+\alpha(s))^2}\frac{\mathbb{V}ar(g_i)}{2}\right); \quad \forall s$$

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  2) $\frac{\partial\phi(\alpha)^{IM}}{\partial\alpha} < \frac{\partial\phi(\alpha)^{CM}}{\partial\alpha} = 1.$ **Increasing precautionary savings**
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- **In short: idiosyncratic risk makes insurance a lot harder!**

Implementation with two assets: **complete markets**

structure

- Assume economy with two shocks $s = L, H$.
    - B is consumer's holdings of a risk free asset
    - A is consumer's holdings of a risky security indexed on stock market.
    - R_F is the risk free rate.

- Implied allocations of financial assets

$$R_F^{CM} B^{CM} = -\frac{Y_2(L)Y_2(H)[\alpha(H) - \alpha(L)](1 - x_1)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}$$

$$A^{CM} = 1 - \frac{Y_2(H) - Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}(1 - x_1)$$

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3) The smaller x_1, the larger the positions $\Rightarrow$ constant over time.

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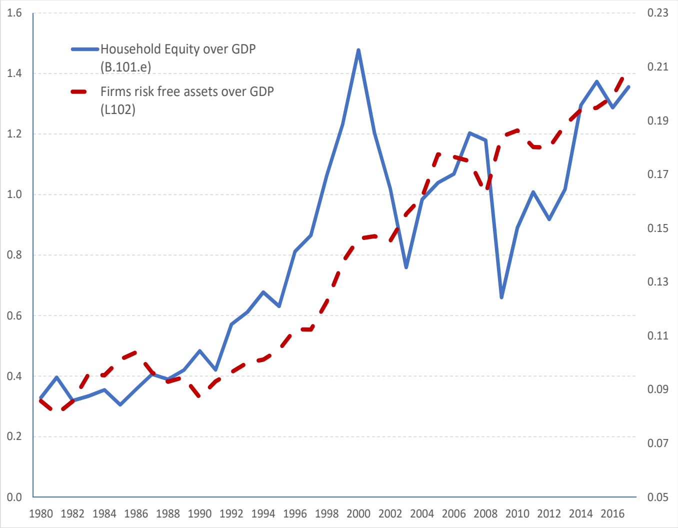
3) As x_1 decreases, growing positions and approaching complete markets.

Some evidence from the data

- Corporate savings glut and households equity glut.



Asset prices volatility

- Finally, $x_2(s) = c_2(s)/Y_2(s)$ no longer constant!: **wealth effects**

- Recall that prices satisfy:

$$p(s) = \Pi(s) \left(\frac{x_1}{x_2(s)} \right)^{\sigma} \left(\frac{Y_1}{Y_2(s)} \right)^{\sigma}$$

- If $\mathbb{V}ar(g_i) \neq 0$ they are more volatile

$$\mathbb{V}ar(p(s)) \propto \mathbb{V}ar(x_2(s)) + \mathbb{V}ar(Y_2(s)) + 2\mathbb{C}ov(x_2(s), Y_2(s))$$

- If α is constant, then $\mathbb{V}ar(x_2(s)) = 0$
- Also, $x_2(s) = \phi(s) + 1 - \alpha(s)$ is decreasing in α.
- Effect on prices depends on correlation between $Y_2(s)$ and $\alpha(s)$, so on EIS.
- Potential increase in volatility of asset prices.

General Environment

Consumers

- Consumer solves

$$V^c(a,s) = \max_{\{c,a(s')\}} \{u(c) + \beta \mathbb{E}_{s'}[V^c(a(s'),s')|s]\}$$

$$st. \quad c + \sum_{s'} p(s'|s)a(s') \leq a + \omega(s)L$$

- $s \in [s^1, .., s^N]$: aggregate state, with $Prob(s'|s) = \Pi(s'|s)$
- $a(s')$ Arrow-Debreu securities. a_1 is initial financial wealth.
- $\omega(s)$ is the wage in state s.
- L is labor. Assumed constant and equal to 1.

Entrepreneurs

- Entrepreneur solves

$$V^e(E, k; s, i) = \max_{\{e, E(s'), k'\}} \{u(e) + \beta \mathbb{E}_{s', i'}[V^e(E(s'), k'; s', i')|s]\}$$

$$st. \quad e + k' + \sum_{s'} p(s'|s)E(s') \leq E + R(s)kg_i$$

- Variables and interpretations are as before.
- $E(s')$ Arrow-Debreu securities. E_1 is initial financial wealth.
- $R(s) = 1 - \delta + r(s)$ is the gross return on capital in state s.
- g_i is the idiosyncratic shock.

Some comments about the economy

- **Idiosyncratic risk:**
  - Use Taylor approximation and assume g_i is i.i.d. $\Rightarrow$ only $\mathbb{V}ar(g_i)$ matters.

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- **Technology and gross returns:**

$$\begin{aligned} y(k, L; s, i) &= \left[\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\ R(s, g_i) &= r(s)g_i + (1-\delta)g_i g_s \end{aligned}$$

- **Aggregate shocks:** $K_t(s) = g_s \times K_t$, where g_s is *i.i.d.* and endogenous K_t

- **Remark:** both $K(s)$ and $k_i(s)$ are **random walks with endogenous drifts.**

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- **About the solution**
  - The economy features aggregation. The only state variables are $s = \{g_S K, x\}$, where $x = \frac{Worker's\ wealth}{Total\ wealth}$
  - Thus, s is endogenous and so it is its transition probabilities.

General solution structure

- Homothetic preferences plus CRS technology $\Rightarrow$ linear policy functions
- Let $h(s)$ human wealth, the consumer policy functions satisfy:

$$\begin{aligned} c(s) &= (1 - \zeta(s))W^c(s) \\ a(s'|s) &= \phi^c(s'|s)\zeta(s)W^c(s) - \omega(s') - h(s') \\ W^c(s) &\equiv a + \omega(s) + h(s) \end{aligned}$$

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- Entrepreneur policy functions satisfy:

$$
\begin{aligned}
e(s,i) &= (1-\vartheta(s))W^e(s,i,k) \\
k'(s,i) &= \nu(s)\vartheta(s)W^e(s,i,k) \\
E(s'|s,i) &= \phi^e(s'|s)(1-\nu(s))\vartheta(s)W^e(s,i,k) \\
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- Linear policy functions: **distribution within entrepreneurs does not matter.**
- Aggregate state variable are: $g_s K$ and $x(s) = \frac{W^c(s)}{W^c(s)+W^e(s)}$
- Let aggregate state be: $s = \{g_s K, x\}$, so $\Pi(s'|s)$ is endogenous.

General solution structure

- Given $p(s'|s)$ and all the aggregates, consumer saving rate solves recursive equation, linear in $(1-\zeta(s))^{-1}$:

$$(1-\zeta(s))^{-1} = 1 + \sum_{s'|s} \left[(\beta \Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma} (1-\zeta(s'))^{-1} \right]$$

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- For entrepreneurs, saving rates solve (independent of i):

$$(1-\vartheta(s))^{-1} = 1 + m^{-1}(s) \sum_{s'|s} \left[(\beta \Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma} (1-\vartheta(s'))^{-1} \right]$$

- Later: what is $m(s)$? For the time being it is important to know that when $\mathbb{V}ar(g_i) = 0$, then $m(s) = 1$, $\forall s$.

- Later: laws of motion of states.

Asset's prices: **infinite horizon**

- At any state s, with wealth distribution x:

$$p(s'|s) = \beta \Pi(s'|s) \left(\frac{\frac{(1-\zeta(s))}{(1-\zeta(s'))} x + \frac{(1-\vartheta(s))}{(1-\vartheta(s'))} \left[\mathbb{R}(s', s, V_g; \phi^e)\right]^{1/\sigma} (1-x)}{\iota(s', s)} \right)^{\sigma}$$

- Where, using second order Taylor approximation, the entrepreneur's expected growth rate of marginal utility is:

$$\mathbb{E}_i o(s', i; \phi^e)^{-\sigma} \simeq \left[(1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')\right]^{-\sigma} \left[1 + \frac{\sigma(1+\sigma)(\nu(s)R(s'))^2 Var(g_i)}{2((1-\nu(s))\phi^e(s'|s) + \nu(s)R(s'))^2}\right]$$

the last term is the risk adjustment $\mathbb{R}(s', s, V_g; \phi^e)$

- Total wealth's growth: $\iota(s'|s)$

Multiplicity of equilibria

- Law of motion of x

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)}{\iota(s',s) + \nu(s)\vartheta(s)(1-x)R(s')} x(s)$$

- The source of potential multiplicity is that s' contains x' itself.
- In general, $R(s')$ and $\iota(s',s)$ depend only on $g_{s'}K'$.
- But, $\phi^c(s'|s)$ depends on x'.

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- But, $\phi^c(s'|s)$ depends on x'.

- Suppose realized distribution is: $\tilde{x}$. Must satisfy:

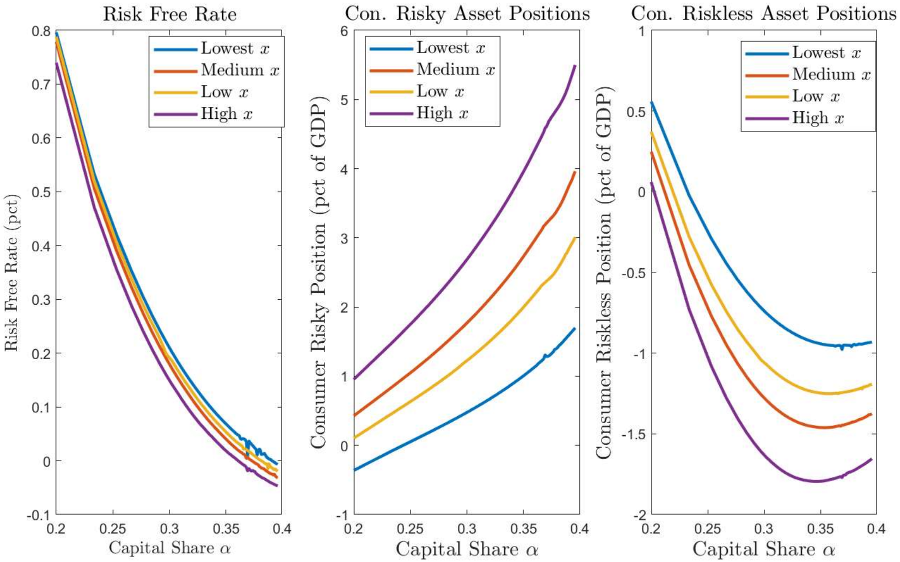
$$\tilde{x} = \frac{\phi^c(\{g_{s'}K', \tilde{x}\}|s)\zeta(s)}{\iota(g_{s'}K', s) + \nu(s)\vartheta(s)(1-x)R(g_{s'}K')} x$$

Calibration

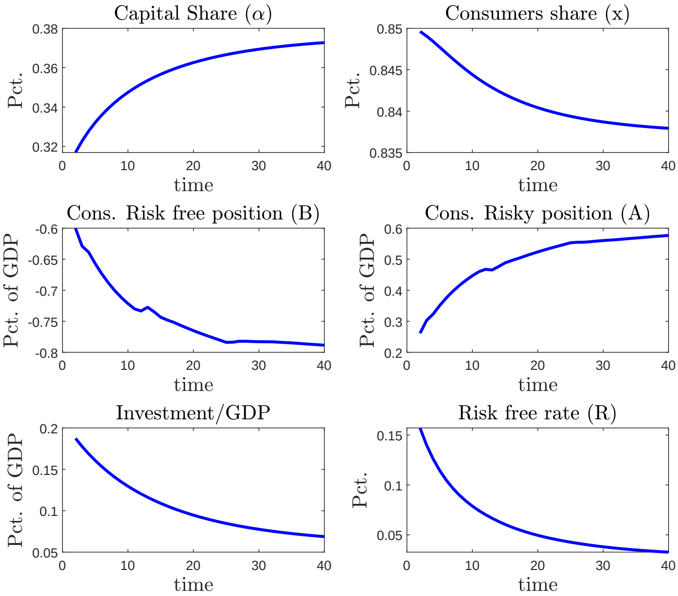
Parameter	Description	Value
σ	Risk aversion	2
β	Discount Factor's	0.94
ρ	Elasticity of Substitution	1.25
α	Capital Share	0.31
δ	Depreciation	0.06
$g_{s,h}, g_{s,l}$	Aggregate Shocks to Capital	1.02, 0.98
p_s	Probability of g_s	1/2
$Var(g_i)$	Id. Shocks to Capital	0.04
ψ	Exposure to Id Risk	0.2
$E[x(s)]$	Unconditional mean of x	0.82

- Most things are standard.
- Match $K/Y = 3$ and average $\alpha = 0.35$
- Aggregate volatility 2% per year. Idiosyncratic volatility $\simeq$ 20%
- Stationary distribution of x: kill agents and replace them with new ones.

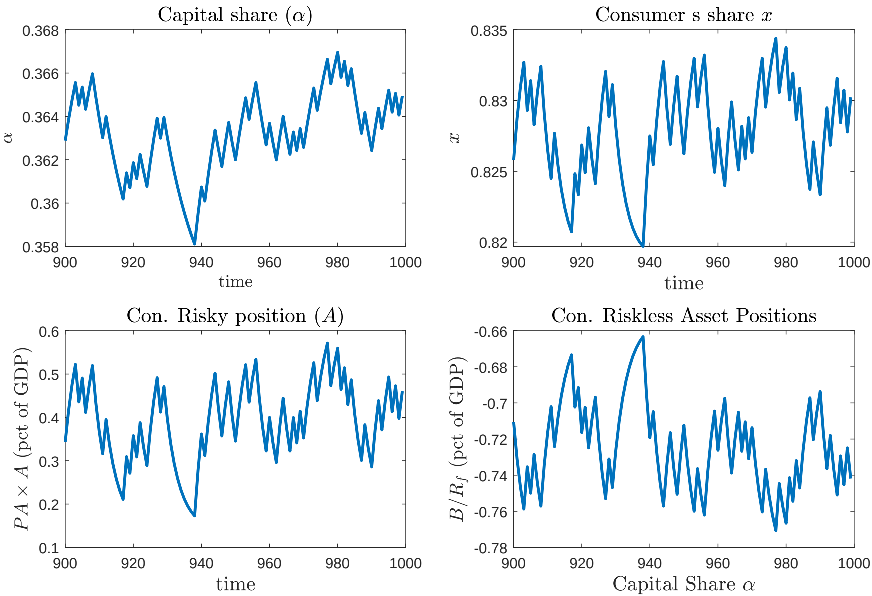
Assets positions and risk free rate



Shocks sequence to replicate α moving from 0.32 to 0.37



Typical stationary path



Concluding remarks

- Changes in the labor share can have **important consequences for financial markets** $\Rightarrow$ and appears quantitatively relevant.
- If entrepreneurship is "more risky" than employment, the increase in the capital share also **increases the uninsured risk** in the economy.
- Insuring idiosy. risk becomes harder, affecting aggregate risk sharing.

Concluding remarks

- Changes in the labor share can have **important consequences for financial markets** $\Rightarrow$ and appears quantitatively relevant.
- If entrepreneurship is "more risky" than employment, the increase in the capital share also **increases the uninsured risk** in the economy.
- Insuring idiosy. risk becomes harder, affecting aggregate risk sharing.
- **Many potential implications to analyze in future research:**
  - Short run vs long run changes in income shares.
  - Asset prices: need adjustment cost (almost done).
  - Implications for long term debt vs short term debt.

Equilibrium with two assets back

- The budget constraints for the worker are:

$$
\begin{aligned}
c_1 + P_A A^c + B^c &\leq a_1 + \omega_1 L \\
c_{2,L} &\leq A^c \pi_L + R_F B^c + \omega_L L \\
c_{2,H} &\leq A^c \pi_H + R_F B^c + \omega_H L
\end{aligned}
$$

- Where R_F is the risk free rate and P_A the price of the risky asset.

- The focs are:

$$
\begin{aligned}
u'(c_1) &= R[\Pi_L u'(c_{2,L}) + \Pi_H u'(c_{2,H})] \\
P_A u'(c_1) &= \Pi_L u'(c_{2,L})\pi_L + \Pi_H u'(c_{2,H})\pi_H
\end{aligned}
$$

Optimal insurance contract. DeMarzo-Fishman (2007)

back 2t general

- Friction: **managers can steal profits from the firm.**
  - If they do it, they can only eat ψ per unit. Cost is $1 - \psi$.
- A **risk neutral intermediary** can provide insurance to the entrepreneur.

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- Suppose there are two shocks $\epsilon_i \in \{\epsilon_L, \epsilon_H\}$, one period contract solves:

$$\max_{\{J,d_i\}} \sum_i Pr(i)u(J+d_i)$$

$$\text{st.} \quad \psi\pi k(\epsilon_H - \epsilon_L) + d_L + J \leq d_H + J$$

$$\sum_i Pr(i)(\pi k \epsilon_i - d_i) - J = 0$$

- Solution is equity contract: $d_i = \psi\pi k\epsilon_i$, and $J = (1-\psi)\pi k \mathbb{E}_i \epsilon_i$
- Residual risk is: $g_i = (1-\psi)\mathbb{E}_i\epsilon_i + \psi\epsilon_i$

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- Residual risk is: $g_i = (1-\psi)\mathbb{E}_i\epsilon_i + \psi\epsilon_i$
- Aggregate uncertainty is irrelevant. It multiplies all constraints simultaneously.
- Same is true if the contract is repeated but CANNOT be history dependent.
- Better commitment: long term contracts, **but hidden savings plays a role.**