The Macroeconomics of Hedging Income Shares

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July 2023

Abstract

The debate about the falling labor share has brought attention to the income-shares trends, but less attention has been devoted to their variability. We analyze how their fluctuations can be insured between workers and capitalists, and the corresponding implications for financial markets. We study a neoclassical growth model with aggregate shocks that affect income shares and financial frictions that prevent firms from fully insuring idiosyncratic risk. We examine theoretically how aggregate risk sharing is shaped by the combination of idiosyncratic risk and moving shares. In this setting, accumulation of safe assets by capitalists and risky assets by workers emerges naturally as a tool to insure income shares' risk. Then, in a quantitative exploration we show that low interest rates, rising capital shares, and accumulation of safe assets by firms and risky assets by households can be rationalized by persistent shocks to the labor share.

JEL classification: E20, E32, E44, G11.
Keywords: Income shares fluctuation. Risk sharing. Corporate savings.

*We thank Loukas Karabarbounis (the editor), two anonymous referees, Vasco Carvalho, Sebastian Di Tella, Claudio Michelacci, Dejanir Silva, and seminar participants at the European Central Bank, EIEF, Paris School of Economics, Universidad Di Tella, Cambridge University, Universidad Carlos III, University of Cologne, CORE 2018 meetings, SED 2017, Bank of Italy, Chicago Fed, and the University College of London for helpful comments and suggestions. We thank Marco Castelluccio, Roberto Saitto and Maria Tiurina for excellent research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank, the Eurosystem, or any of its member countries. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management. All remaining mistakes are ours.

1 Introduction

For several decades the ubiquity and the robustness of the Kaldor facts led to the dominant belief that capital and labor income shares are roughly constant over time. An important implication of this paradigm is the impossibility of insurance between workers and capitalists. Because aggregate shocks affect both agents equally, even if markets existed, aggregate risk would be uninsurable. However, many recent studies find that income shares are moving far more than in Kaldor's original predictions.[1] This opens new possibilities: if aggregate shocks have different impacts on capitalists and workers, these shocks can be insured. If so, several questions arise: How do these insurance possibilities affect the financial markets? Which kinds of assets could be affected?

The cyclical properties of income shares are key factors in determining how fluctuations are insured. Our focus is not on explaining the reasons for these fluctuations; instead, we analyze whether there are implications for the financial markets and the macroeconomy. We first present a theoretical argument that a countercyclical labor share can be insured between capitalists and workers. This is achieved through capitalists accumulating risk-free assets and lending them to workers. The workers, in turn, use these loans to leverage and purchase risky assets.[2] Next, we analyze business-cycle dynamics, which yields interesting predictions. As capitalists are more exposed to idiosyncratic risk, upward changes in the capital share reduce capitalists' risk absorption, hindering aggregate risk sharing. As a consequence, the demand for precautionary savings increases, decreasing the risk-free interest rate and increasing the risk premium. Finally, we show that these qualitative predictions are also quantitatively sizable.

The channel that we analyze is simple and intuitive and has been overlooked despite being consistent with several seemingly unconnected findings. A growing literature addresses what is known as the *Corporate Saving Glut*, shifting the view of corporations from net borrowers to net lenders.[3] Our theory is not only consistent with this fact, but also with the observed changes in the labor income share, the declining interest rates and the rising risk premium.[4] Furthermore, the theory provides precise implications regarding the necessary changes in the financial portfolios of the involved agents.

[1] See, for instance, Karabarbounis and Neiman (2014) and Rodriguez and Jayadev (2010).

[2] A large literature shows that the labor share is countercyclical. See for example Gomme and Greenwood (1995), Rotemberg and Woodford (1999), Ríos-Rull and Santaeulàlia-Llopis (2010), Karabarbounis (2014) and Nekarda and Ramey (2020).

[3] Chen et al. (2017) document the global increase in corporate savings.

[4] See Del Negro et al. (2017b) (for the U.S. economy) and Del Negro et al. (2017a) (globally) document the trend in the interest rate. These two papers attribute most of the fall in the risk-free rate to an increase in the convenience yield.

We build on the neoclassical growth model, allowing for income shares that fluctuate persistently over time. The economy is populated by a continuum of capitalists with different endowments of capital and workers who supply labor inelastically. Capitalists rent labor and carry out the production. Workers consume and fund firms through the financial markets, but they do not own capital directly. Production is subject to both aggregate and idiosyncratic risk. Moreover, there is a contracting friction; as in DeMarzo and Fishman (2007), the capitalists' returns cannot be verified, because they can privately divert resources for consumption. Firms would like to pool the idiosyncratic risk and obtain funding, but they are subject to a "skin in the game" constraint: the lenders force firms to keep a fraction of their investment. Nevertheless, enough financial instruments are available such that both capitalists and workers can perfectly insure against aggregate risk. Yet, the contracting friction prevents capitalists from fully insuring the idiosyncratic risk, which affects the agents' willingness to bear aggregate risk.[5]

Our key departure from the literature is that we move away from constant-shares technologies (Cobb-Douglas or AK production functions). We assume that the labor share is countercyclical (the capital share is procyclical) so that capitalists benefit more in booms and suffer more in recessions. Our purpose is to analyze how *exogenous* fluctuations in income shares affect financial markets. The reverse channel, although potentially interesting, is left for future research.

We begin by characterizing asset prices and quantities in a simplified two-period economy. We then extend the results to a richer infinite-horizon economy. The simple environment helps us understand the main trading patterns of financial assets and how the presence of idiosyncratic risk is key to generating the observed trends in the financial markets. To do so, we assume that agents have access to a complete set of Arrow-Debreu (AD) securities, contingent on the realization of the aggregate shock but not on the realization of the idiosyncratic one.

In Proposition 1, we show that to insure against aggregate risk, workers and capitalists engage in trading AD securities. For instance, if the capital share increases, capitalists compensate workers with contingent transfers, and vice versa. However, the predicted trends crucially depend on the presence of idiosyncratic risk. In the absence of such risk, the economy becomes memoryless and trendless: neither the shock's realizations nor their history affect consumption, AD positions, or asset-price patterns.

In contrast, the introduction of uninsurable idiosyncratic risk creates an additional de-

[5]We assume that workers are not subject to idiosyncratic risk. Thus, the fact that capitalists are exposed to idiosyncratic risk must be interpreted in relative terms throughout this paper. There is ample evidence that firms are more exposed to idiosyncratic risk than workers; see, for example, Guiso et al. (2005).

mand for precautionary savings among capitalists. They meet this demand by adjusting their positions in AD securities, moving away from complete insurance of income shares. Consequently, aggregate shocks *alter the relative wealth* of capitalists and workers, prompting further portfolio rebalancing. This transforms the economy into a history-dependent system. We refer to this additional channel as the "wealth effects" channel.

We also characterize the enduring effects of aggregate shocks, presenting well-defined and testable empirical predictions. We show that *larger capital shares are correlated with an increase in capitalist's net savings, lower risk-free rates, and greater risk premia*. Intuitively, a higher share of total output held by capitalists not only increases profits but also amplifies their total variance, given the susceptibility of profits to idiosyncratic risk. Consequently, states with higher capital shares also exhibit higher levels of idiosyncratic risk, leading to an increased demand for insurance. This, in turn, gives rise to the observed outcomes.

We then show that in the presence of two possible aggregate shocks, the optimal insurance contract can be implemented using only a risk-free asset and a risky asset. This equilibrium is achieved through firms taking a long position on the risk-free asset (saving), while households take a long position on the risky asset (investing in equity). Since markets must clear, a positive net position by one sector in a particular asset implies a corresponding negative net position by the other. Intuitively, workers leverage (borrow from firms) to purchase shares and partake in changes in the capital share. Thus, while the optimal insurance contract robustly predicts increased savings by capitalists (under any implementation of the contract), this implementation adds sharper predictions regarding which assets would be affected and how.

This market allocation is reminiscent of a corporate savings glut, paired with a *households' equity glut*. Absent idiosyncratic risk, financial positions and asset prices would remain constant and independent of history. However, in the presence of idiosyncratic risk, the inclination of capitalists to accumulate risk-free assets and workers to accumulate equity persists. Yet, the wealth effects introduce an additional channel that magnifies portfolio rebalancing over time. As the capital share increases, so does the demand for precautionary savings, leading to firms to increase savings, tilted towards larger long positions in risk-free assets. Concurrently, workers increase their leverage, borrowing from capitalists to increase their equity holdings. All this unfolds as the risk-free rate is falling and the equity premium rising.

Considering the slow movement of income shares, a concern might arise regarding the quantitative relevance of these predictions. To assess magnitudes, we calibrate the economy with typical parameter values whenever possible, and we set our model specific parameters to replicate standard moments for financial quantities and prices. We find that, given a labor share variance of 0.5%, workers on average ought to borrow the equivalent to around 1.6 times the GDP and hold equity amounting to 80% of GDP. This scenario unfolds with a risk-free rate of 1% or less (depending on the labor share) and an equity premium ranging between 5% and 6%.

The rest of this paper is organized as follows. Section 1.1 reviews the literature. Section 2 highlights the main mechanisms in a tractable two-period model. In Section 3, we present a general model and generalize most results. In Section 4, we calibrate and numerically evaluate the general model. Section 5 concludes. All proofs are in the Appendices.

1.1 Literature Review.

This paper is motivated by the recent literature emphasizing changes in the labor share. Since Karabarbounis and Neiman (2014), several studies have pointed to the apparent downward labor share trend. The potential reasons for this trend include a fall in the price of investment, the growing importance of housing (Rognlie, 2015), rising market power and concentration (De Loecker et al., 2020 and Barkai, 2020), demographics (Hopenhayn et al., 2022), and a productivity slowdown (Grossman et al., 2017), as well as the possibility that the labor share is not falling and it is just a measurement issue (Koh et al., 2020). We present our theory first assuming exogenous variations to income shares, and then we extend the results to an economy with CES technology and capital-augmenting productivity shocks. Thus, these results may not hold in environments where the labor share changes are due to other motives. For instance, if the income shares are responding to demographic changes, these changes could also have a direct impact on the households portfolio choices. Similarly, we abstract from the potential (and interesting) feedback from asset markets to income shares.

Our theory relies on the existence of cyclical fluctuations, since the insurance channel would be muted if the movements were driven by a deterministic trend. The theory allows for both procyclical and countercyclical movements in the labor share, although reverting the predicted portfolio choices. There are many studies estimating the cyclical properties of the labor, for instance see Ríos-Rull and Santaeulàlia-Llopis (2010), León-Ledesma and Satchi (2019), and Cantore et al. (2019). In particular, Karabarbounis (2014) and Nekarda and Ramey (2020) focused on the cyclicality of income shares. The former uncovers a clear inverse relationship between the firm component of the labor wedge and the labor share; and argues that the lack of strong procyclicality of the labor share refutes the firm's theories of the labor wedge. The latter shows that most measures of markups are procyclical conditional on a technology shock, and either procyclical or acyclical conditional on demand shocks. In their baseline analysis, markups are measured as the reciprocal of the labor share.

The main robust implication of our theory is an increase in corporate savings, which has been documented and risen many questions. Chen et al. (2017) document the corporate savings glut globally and relate it to the decline in labor share, which they argue is driven by a combination of changes in the real interest rate, the price of investment goods, corporate income taxes, and the increase in markups. In turn, Armenter and Hnatkovska (2017) argue that a combination of tax structure and borrowing limits adds concavity to firm's objective function that can lead to higher savings. Instead, we propose a different channel relying on the interaction of the idiosyncratic and income shares risk. In addition, in our setup the interest rate is endogenous; hence, its change is not a cause but rather another implication of the theory.

In addition to the increase in firm's savings and drop in the interest rate, our proposed implementation of the optimal insurance contract has implications for the firm's capital structure: increase in equity issuances and savings in risk-free assets. The rise in corporate risk-free assets holdings has been documented by Foley et al. (2007) and Bates et al. (2009), among others, who stress the important role played by tax incentives. Instead, we abstract from tax incentives. In this sense, we see our channel as reinforcing these clearly relevant forces.[6]

Our paper is also relate to the recent literature that connects low risk-free rates, risk premia, and changes in the labor share. Caballero et al. (2017) proposes an accounting framework that connects falling short-term real rates, a constant marginal product of capital, the labor-share decline, and a stable earnings yield from corporations. Farhi and Gourio (2018) and Eggertsson et al. (2021) document and link the simultaneous patterns of a decreasing labor share and risk-free rates with an increasing savings supply and risk premia. Eggertsson et al. (2021) argue that these trends are mostly due to rising markups. In contrast, Farhi and Gourio (2018) use a different methodology and find that even though markups could be playing an important role, the risk premia and unmeasured intangibles are key.

Our paper is also related to the literature on the financial amplification of aggregate shocks, following the seminal work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). We build on the recent contributions of He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014) and Di Tella (2017), where financial frictions and het-

[6] Taxes also play a key role in the predicted changes in the capital structure by Armenter and Hnatkovska (2017)

erogeneity play a key role. We depart from the previous studies by introducing human capital and income shares correlated with the business cycle. These two assumptions allow us to study positive and normative implications of changes in labor and capital shares over the business cycle.

Carvalho et al. (2016) and Auclert et al. (2019) provide an explanation for low interest rates based on demographics. The channel through which demographics imply a lower interest rate is that a longer life span implies a higher supply of safe assets for retirement, and lower demand for investment. Our paper focuses on changes in the labor share and in idiosyncratic risk that increase firms' precautionary savings, which in turn depresses the real interest rate.

2 Hedging Income Shares

In this section, we study a two-period economy with exogenous capital to obtain a sharp characterization of the implications on asset prices and quantities of changes in income shares. In Section 3, we develop an infinite-horizon economy with endogenous investment and show that all findings in this section hold in the general model.

2.1 Simplified environment

There are two types of agents: workers and capitalists. The economy lasts for two periods, $t = 1, 2$. There are two sources of uncertainty – aggregate shocks, indexed by $s \in \mathbb{S}$, and idiosyncratic production shocks, indexed by $i \in \mathbb{I}$ – which occur with probability $\Pi(s, i)$. In this section, for simplicity, we assume that there is no time discounting.

Workers. Workers are endowed with initial assets A_1 and can supply one unit of labor at no utility cost. Labor income in period one is certain and given by ω_1, which denotes the wage rate. In period two, they receive $\omega(s)$ as labor income, which is contingent on the realization of the aggregate shock. To insure against variations in wages, workers have access to a complete set of Arrow-Debreu (AD) securities, denoted by $A_2(s)$, contingent on state s. Each asset can be traded at price $p(s)$.

Notice that we allow for as many aggregate financial assets as possible aggregate states. We made this modeling choice for two reasons. First, it is a standard setup in the literature (e.g., He and Krishnamurthy (2012), Di Tella (2017), Di Tella (2019)), so we can easily relate our findings to previous contributions. Second, it is a reasonable approximation of reality, in which there are multiple types of financial assets, whose payoffs neither rely on nor are constrained by individual moral hazard or commitment problems, and that help workers and entrepreneurs insure aggregate shocks. By properly combining them, one can replicate the same allocations as with AD securities.[7] Whether complete insurance is achieved ultimately depends on the assets' prices.

The worker maximizes expected utility:

$$\max_{\{c_1, c_2(s), A_2(s)\}} u(c_1) + \mathbb{E}_s(u(c_2(s)))$$

$$s.t. \quad c_1 + \sum_s p(s) A_2(s) \leq A_1 + \omega_1 \tag{1}$$

$$c_2(s) \leq A_2(s) + \omega_2(s) \tag{2}$$

The worker uses initial assets A_1 and income ω_1 to consume and buy AD securities. In the second period, consumption is given by the income realization and the payoff of the assets acquired in the first period, $A_2(s)$.

Capitalists. Firm's owners are endowed with initial financial assets E_1 and exogenous capital income $\{\pi_1, \pi_2(s,i)\}$, which is a function of aggregate and idiosyncratic shocks. To highlight the insurance mechanism, we start by assuming that capital income is exogenous.

Capitalists would like to share the idiosyncratic risk but are prevented from doing so due to a financial friction: they could divert income to a private account. As a result, they must retain some idiosyncratic risk. This feature can be rationalized as the result of an optimal risk-sharing contract with moral hazard between the entrepreneur and a principal (the market), as in DeMarzo and Fishman (2007) and Di Tella (2017).[8] Capitalists can buy a complete set of AD securities $E(s)$, which are contingent on s but not on i. The capitalist's problem is

$$\max_{\{e_1, e_2(s,i), E_2(s)\}_{s \in S}} u(e_1) + \mathbb{E}_{s,i}(u(e_2(s,i)))$$

[7] In discrete-time models, it is well known that to complete the market there must be as many nonstate contingent assets,with imperfectly correlated prices, as possible states. When the time interval becomes infinitesimal and the underlying risk is characterized by a Brownian motion (in continuous time), only two assets are needed. See for example Merton (1992).

[8] Because of moral hazard, the optimal contract provides only partial insurance of idiosyncratic risk: entrepreneurs must keep some "skin in the game." See the online Appendix D and Section 3.2 for additional details on the optimal contract for the entrepreneur. On the other hand, due to the lack of contracting friction on the side of consumers, the consumers who hold equity in the firm can diversify the risk by pooling their ownership, and for that reason, they do not hold idiosyncratic risk.

$$s.t. \quad e_1 + \sum_s p(s)E_2(s) \leq E_1 + \pi_1$$
$$e_2(s,i) \leq E_2(s) + \pi_2(s,i)$$

for all (s,i). The capitalist can use initial assets E_1 to consume and buy AD securities. In the second period, consumption is given by the realization of the return to capital, $\pi_2(s,i)$, and the payoff of the assets acquired in the first period, $E_2(s)$.

Profits and wages. Profits and wages are given by

$$\pi(s,i) = g_i \alpha(s) Y(s) \tag{3}$$
$$\omega(s) = (1-\alpha(s))Y(s) \tag{4}$$

where $g_i > 0\ \forall i$, $\mathbb{E}(g_i) = 1$, and is independent identically distributed (i.i.d.) in the cross section. Equations (3) and (4) stress the sources of income variations. In addition to the capitalists' exposure to idiosyncratic risk, capitalists' and workers' income will vary after an aggregate shock. The shock changes both aggregate output and the relative claims to it. In the quantitative section, we generate time-varying income shares with a CES production function and shocks to the capital quality.

Markets. Market clearing implies

$$c_1 + e_1 = Y_1 \tag{5}$$
$$c_2(s) + \mathbb{E}_i(e_2(s,i)) = Y_2(s) \qquad \forall s \tag{6}$$
$$A_2(s) + E_2(s) = 0 \qquad \forall s \tag{7}$$

where $Y_1 = \pi_1 + \omega_1$ and $Y_2(s) \equiv \int y_2(s,i)di$, $\forall s$. Equation (5) is the market clearing condition for goods in period 1. It also implies that the initial asset holdings are such that $A_1 + E_1 = 0$. The second condition, equation (6), is market clearing for goods in period 2. The idiosyncratic *i.i.d.* shocks cancel out in the aggregate. Finally, equation (7) specifies that asset markets clear.

Definition: A *Competitive Equilibrium* is a consumption allocation $\{c_1, e_1, c_2(s), e_2(s,i)\}_{s\in S}^{i\in \boldsymbol{I}}$, asset holdings $\{A_2(s), E_2(s)\}_{s\in S}$, and asset prices $\{p(s)\}_{s\in S}$ such that (1) given prices, the worker maximizes utility by choosing asset holdings and consumption; (2) given prices, the capitalist maximizes utility by choosing financial asset holdings and consumption; and (3) markets clear.

2.2 Equilibrium characterization

We now derive the optimality conditions for workers and capitalists. From the individual problems' first-order conditions we obtain

$$
\begin{aligned}
p(s)u'(c_1) &= \Pi(s)u'(c_2(s)) \\
p(s)u'(e_1) &= \Pi(s)\mathbb{E}_i[u'(e_2(s,i))].
\end{aligned}
$$

A key element of the above equations is that, due to the existence of a complete set of AD securities for the aggregate state, the Euler equations hold state by state. The two first-order conditions together imply

$$
\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[u'(e_2(s,i))]}{u'(c_2(s))} \tag{8}
$$

for all s. Equation (8) states that the ratio of future average marginal utilities is constant across states. Define the holding of state s AD securities as a fraction of state s output as $\phi(s) := \frac{A_2(s)}{Y_2(s)}$. Market clearing implies that

$$
\begin{aligned}
A_2(s) &= \phi(s)Y_2(s) \\
E_2(s) &= -\phi(s)Y_2(s).
\end{aligned}
$$

Then, from market clearing in the goods market in the first period, and assuming constant relative risk aversion (CRRA) preferences with parameter σ, we can rewrite (8) as

$$
\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[(-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i)^{-\sigma}]}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\sigma}}. \tag{9}
$$

For future reference, define the worker's wealth share in periods 1 and 2 as

$$
\begin{aligned}
x_1 &= \frac{A_1 + \omega_1 + \sum_s p(s)\omega_2(s)}{Y_1 + \sum_s p(s)Y_2(s)}, \\
x_2(s) &= \frac{A_2(s) + \omega_2(s)}{Y_2(s)}.
\end{aligned} \tag{10}
$$

The numerator is the worker's total wealth, and the denominator is the economy's total wealth. In period 1, the worker's wealth is the initial assets plus the present value of wages. Total wealth in the economy is the sum of the initial output and of the present value of future total output. For period 2, the share of total wealth may depend on the aggregate shock. Both shares are endogenous and determined in equilibrium.

In the next section, we characterize the equilibrium as a function of the wealth share and discuss the conditions under which it is constant. We say that aggregate shocks are amplified whenever the share of wealth is state-dependent.

2.3 Positive implications

Define the "certainty equivalent" $g^{ce}(\alpha, \phi; s)$ as the function satisfying:

$$(-\phi(s) + \alpha(s) g^{ce}(\alpha, \phi))^{-\sigma} = \mathbb{E}_i[(-\phi(s) + \alpha(s) g_i)^{-\sigma}], \quad \forall s \in \mathbb{S}. \tag{11}$$

This function depends on $\alpha(s)$, which is a primitive of the problem, and $\phi(\cdot)$, which is the choice of AD securities. Because for CRRA preferences marginal utility is convex (i.e. $u''' > 0$), we have as a result that $g^{ce}(\alpha, \phi; s) \leq 1\ \forall s$, with equality only if $\mathbb{V}ar(g_i) = 0$.[9] The main result of the subsection, Proposition 1, characterizes the asset prices and asset holdings in the Competitive Equilibrium.

Proposition 1. *If $\mathbb{V}ar(g_i) > 0$, and $\mathbb{V}ar(\alpha(s)) > 0$, then, in a competitive equilibrium:*

(a) Prices and financial positions satisfy

$$p(s) = \Pi(s) \left[1 + \alpha(s)(g^{ce}(s) - 1)\right]^{-\sigma} g_s^{-\sigma}, \tag{12}$$

$$\phi(s) = x_1 - (1 - \alpha(s)) + \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce}), \tag{13}$$

and for $\Gamma(\cdot)$:$\mathbb{R} \to \mathbb{R}_+$, which is determined in equilibrium, it holds that $\Gamma(1) = 0$.

(b) Wealth shares evolve according to

$$x_2(s) - x_1 = \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce}). \tag{14}$$

(c) Moreover, let $p(s)^{CM}$ and $\phi(s)^{CM}$ be the solutions to (12) and (13) when $\mathbb{V}ar(g_i) = 0$. Then, there is precautionary savings:

$$-\sum_s p(s)\phi(s) > -\sum_s p(s)^{CM} \phi(s)^{CM}.$$

[9] Though slightly corrupting the terminology, we refer here to g^{ce} as the "certainty equivalent", even though we are working with marginal utilities rather than utilities. Technically speaking, any utility function that has a positive coefficient of *prudence* would generate the same outcome. Further, equation (11) also points out the relevance of the minimum realization of the idiosyncratic shock, i.e., $\underline{g}_i$. If $\underline{g}_i = 0$, only solutions with $\phi(s) \leq 0$ are admissible, and thus the entrepreneur cannot borrow. Instead, if $\underline{g}_i = 1$, the entrepreneur can borrow up to the full expected value of future income. For the rest of this paper, we assume that $\underline{g}_i > 0$ is sufficiently large such that both borrowing and lending are feasible in equilibrium.

Proof. See Appendix B.2.

We will now analyze the different cases of Proposition 1, depending on whether markets are complete or incomplete, and whether the income shares are varying. Discussing these cases will help us to build intuition of the forces that drive the allocations in the infinite-horizon model of Section 3.

Complete markets: $\mathbb{V}(g_i) = 0$. When there is no idiosyncratic risk, Proposition 1 characterizes the insurance arrangement to hedge aggregate risk. Since $\mathbb{V}ar(g_i) = 0$ implies $g^{ce}(s) = 1$ and therefore $\Gamma(1) = 0$, prices and asset holdings given by equations (12) and (13) for each s simplify to

$$p^{CM}(s) = \Pi(s)\left(\frac{Y_2(s)}{Y_1}\right)^{-\sigma} \tag{15}$$

$$\phi^{CM}(s) = x_1 - (1 - \alpha(s)) \tag{16}$$

$$x_2(s) = x_1 \tag{17}$$

Three features of this allocation are noteworthy. First, regarding asset prices, the fact that prices are given by (15) is a standard result in a Lucas (1978) economy. The price to transfer consumption to states that have higher probabilities or feature lower endowments is higher. More importantly, because agents can fully share risk, the state s security price depends on the aggregate endowment, not its distribution.

Second, regarding asset holdings given by (16), if the income shares are constant, there is no need for insurance between workers and entrepreneurs. Aggregate shocks equally hit both types of agents, so $\phi(s)$ is constant. But, when income shares are stochastic, there are gains from trade in financial assets because aggregate shocks affect profits and wages differently. For example, from (16), we can observe that workers will buy insurance against states in which the capital (labor) share is higher (lower).

Third, regarding the evolution of the wealth shares, from equation (17) it follows that they are constant. We denote it as $x^{CM} := x_2(s) = x_1$ for all s, which is (58) evaluated at the complete market prices, which are given by (15).[10] Hence, the wealth shares are constant over aggregate states and across periods, which is an expression of full insurance. Intuitively, capitalists fully compensate workers with contingent payments when the capital income share increases, and vice versa. This compensation through AD securities is such that both types of agents consume a constant proportion of the aggregate resources, which is independent of the current income shares and the history of shocks: the economy is memoryless.

[10] Recall that $g^{ce}(s) = 1$ and $\Gamma(1) = 0$.

Incomplete markets: $\mathbb{V}(g_i) > 0$**.** Things are different when capitalists are subject to idiosyncratic risk. First, regarding prices, from equation (12) and (15) it is evident that $p(s) > p^{CM}(s)$ for all s, as long as there is idiosyncratic risk, which implies $g^{ce}(s) < 1$ for all s. How different these prices are depends on the factor $[1 + \alpha(s)(g^{ce}(s) - 1)]^{-\sigma}$, which is increasing in $\alpha(s)$. A larger capital share realization in state s implies that capitalists bear more idiosyncratic risk in that state, since the variance of profits is increasing in both output and the capital share.[11] For this reason the capitalist wants to increase its insurance against the realization of that state, and as a result, insuring aggregate risk becomes more expensive. This is one of the key intuitions of this paper.

Second, from (13), we can observe opposing forces regarding asset positions. Recall that $E(s) = -\phi(s)Y(s)$, hence the capitalist increases her position in state s if and only if $-\phi(s) > -\phi(s)^{CM}$. This happens whenever

$$\phi(s)^{CM} - \phi(s) \;=\; x_1^{CM} - x_1 - \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce}(s))] - \Gamma(g^{ce}(s)) > 0$$

We show in Appendix B.4 that $x_1^{CM} > x_1$ when $\frac{A_1}{Y_1} \geq \alpha_1$. Intuitively, due to the higher price of insurance, workers are relatively poorer: the net present value (NPV) of total output increases more than the worker's wealth. As capitalists have higher relative wealth with incomplete markets, they demand relatively *more* insurance. The second term, $-\alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})]$, is also positive because $g^{ce}(s) < 1$. This is a hedging demand due to idiosyncratic risk. Finally, the last term is negative. Unlike the previous term, which depends on s, this last term captures the "average" shift in the demand for AD securities. Quantitatively, the first two terms dominate. As a result, capitalists increase asset positions overall, with a more pronounced increase on states with higher capital share $\alpha(s)$.

Intuitively, capitalists move away from the full insurance of aggregate risk to accumulate precautionary savings and hedge against idiosyncratic risk. As we discussed before, states with higher capital shares also feature higher variance of profits; to insure part of that risk, capitalists buy insurance. The insurance is imperfect, though, because they still need to bear idiosyncratic risk. Because the insurance is imperfect, wealth shares vary as a result of changes in the income shares, as is apparent from (14).

Crucially, part (c) shows that the total amount capitalists spend on insurance is now higher: *the total capitalists' savings increase.* Since in this simplified setting investment is exogenously set to zero, it implies that non-financial business savings net of investment

[11] The variance of profits given idiosyncratic risk is $\mathbb{V}(\pi(s,i) \mid g_i) = g_i^2[\alpha(s)^2\mathbb{V}(Y_2(s)) + Y_2(s)^2\mathbb{V}(\alpha(s)) + 2Cov\,(\alpha(s), Y_2(s))]$ and $\mathbb{V}(\pi(s,i) \mid s) = \mathbb{V}(g_i)\alpha(s)^2 Y_2(s)^2$.

rise, resembling the findings by Chen et al. (2017). These results stress the relevance of incorporating idiosyncratic capital-income risk, which diminishes the power of the income shares insurance motive and, as is evident from equation (14), renders the economy no longer memoryless. Both elements would be important in our quantitative analysis where we analyze the quantitative power of the theory and the potential history-dependent effects of income shares shocks. For the rest of this paper, we call this additional channel "wealth effects," i.e., the induced changes in quantities and prices due to movements in the wealth share x.

Implementation with two assets

Proposition 1 helps us to build intuition regarding allocations. However, it is instructive (and useful for positive analysis) to map those predictions to assets that are observed in reality. In Proposition 2, we study a decentralization with two shocks and two assets. In particular, suppose there are only two aggregate states, $s_L < s_H$, and two financial assets, a risk-free bond B and a stock-market-indexed risky asset A with payoffs $A \times \pi_2(s)$ for $s = L, H$. The (gross) risk-free rate is denoted by R, and P_A denotes the price of the risky asset. We focus on the case in which the labor share is procyclical; i.e., $\alpha(H) > \alpha(L)$ and $Y_2(H) > Y_2(L)$. $\{A^w, B^w\}$ is the portfolio allocation of workers, and $\{A^e, B^e\}$ is the capitalists' aggregate portfolio allocation.

Proposition 2. *The position in risk-free debt and equity of the worker is given by*

$$R_L B^w = -\left(\frac{\alpha(H) - \alpha(L)}{\pi_2(H) - \pi_2(L)}\right) Y_2(L)Y_2(H)(1 - x_1) - \frac{Y_2(L)Y_2(H)\alpha(H)\alpha(L)}{\pi_2(H) - \pi_2(L)} \left(g^{CE}(H) - g^{CE}(L)\right) x_1 + \mathbf{\Psi} \tag{18}$$

where $\mathbf{\Psi} = \Gamma \times \frac{\alpha(H)\alpha(L)\left(g^{CE}(L) - g^{CE}(H)\right) + \alpha(H) - \alpha(L)}{(Y_2(L)Y_2(H))^{-1}(\pi_2(H) - \pi_2(L))}$

$$A^w = 1 - \left(\frac{Y_2(H) - Y_2(L)}{\pi_2(H) - \pi_2(L)}\right)(1 - x_1) + x_1 \left[\frac{\alpha(H)Y_2(H)g^{CE}(H) - \alpha(L)Y_2(L)g^{CE}(L)}{\pi_2(H) - \pi_2(L)} - 1\right] + \mathbf{\Xi} \tag{19}$$

$\mathbf{\Xi} = \Gamma \times \frac{Y_2(H)\left(\alpha(H)\left(g^{CE}(H) - 1\right) + 1\right) - Y_2(L)(\alpha(L)\left(g^{CE}(L) - 1\right) + 1\right)}{\pi_2(H) - \pi_2(L)}$

Proof. See Appendix B.3. □

As in the previous subsection, we begin by discussing the case of complete markets, which will help us understand the case of incomplete markets. In the case of *complete* *markets*, equations (18) and (19) become

$$R^{CM}B^{CM} := -\left(\frac{\alpha(H)-\alpha(L)}{\pi_2(H)-\pi_2(L)}\right) Y_2(L)Y_2(H)\left(1-x_1^{CM}\right) \tag{20}$$

$$A^{CM} := 1-\left(\frac{Y_2(H)-Y_2(L)}{\pi_2(H)-\pi_2(L)}\right)\left(1-x_1^{CM}\right) \tag{21}$$

where x_1^{CM} is the wealth ratio evaluated at $p^{CM}(s)$.

First, workers take an active position on the risk-free asset only if $\alpha(H) \neq \alpha(L)$. In particular, with a Cobb-Douglas production function, $\alpha(H) = \alpha(L)$ which means that the risk-free asset is not traded in equilibrium.

Second, whether the position is positive or negative depends on the correlation between the income shares and output. If positive output shocks are associated with higher $\alpha(s)$, workers borrow on the riskless asset and invest on the risky asset. By market clearing, this in turn means that capitalists are issuing equity to increase their positive holdings of the risk-free asset to mitigate idiosyncratic risk.

Third, regarding the risky asset, if output is constant over time, then $A^{\mathrm{CM}} = x^{\mathrm{CM}}$, so workers (and capitalists) hold the risky asset proportional to their relative level of wealth. If output is not constant, then workers and capitalists transfer consumption over time using the risky asset, holding positions that are more or less proportional to their wealth.

Now, consider the impact of the *market incompleteness*. As in Proposition 1, asset positions change for two reasons: changes in relative wealth, and changes due to idiosyncratic risk due to $g^{CE}(\cdot)$. The difference in positions, which we obtain from equations (18) to (21), is given by

$$RB^{w} - R^{CM}B^{CM} = -\left(\frac{\alpha(H)-\alpha(L)}{\pi_2(H)-\pi_2(L)}\right) Y_2(L)Y_2(H)\left(x^{CM}-x_1\right) - \frac{\prod_s \alpha(s)Y_2(s)\left(g^{CE}(H)-g^{CE}(L)\right)}{\alpha(H)Y_2(H)-\alpha(L)Y_2(L)}x_1$$

$$A^{w} - A^{CM} = -\left(\frac{Y_2(H)-Y_2(L)}{\pi_2(H)-\pi_2(L)}\right)\left(x^{CM}-x_1\right) + \left[\left(g^{CE}(L)-1\right) + \frac{\alpha(H)\alpha(L)\left[g^{CE}(H)-g^{CE}(L)\right]}{\alpha(H)Y_2(H)-\alpha(L)Y_2(L)}\right]x_1$$

We refer to these *differences* in allocations as “distortions" relative to the complete markets allocations, and analyze their determinants. It is easy to see that, because $x^{CM} - x_1 > 0$ and $g^{CE}(H) - g^{CE}(L) > 0$, debt (principal plus interest) of workers is larger. As a result, capitalists, who take the other side of the trade, are increasing their position in the risk-free asset. This result is magnified when the capital share changes more across states of nature, due to the term $\alpha(H) - \alpha(L)$. Moreover, the distortion to the holdings of the risky asset stems from three sources. The first, captured by the term $g^{\mathrm{ce}}(L) < 1$, arises just because of the existence of uninsured idiosyncratic risk, and it remains even when α is constant. The second, captured by the term $\frac{\alpha(H)\alpha(L)\left[g^{CE}(H)-g^{CE}(L)\right]}{\alpha(H)Y_2(H)-\alpha(L)Y_2(L)}$, arises because of the presence of "time-varying" uncertainty. Thus, the presence of uninsured idiosyncratic risk interacts with the stochastic income shares, amplifying the difference between allocations. The third is due to the already discussed changing wealth shares.

Discussion: interpretation and caveats

The main takeaway from this section is that varying income shares opens a wide range of new implications for financial markets, absent in theories that rely on the standard Cobb-Douglas and AK technologies. Nevertheless, there are some important questions regarding the positive implications. So far, we have referred to these findings as attributable to the non-financial business sector, in general, and to the corporate sector in particular. However, this mapping is by no means obvious; further clarifications are in order.

Although our capitalists can naturally be thought of as entrepreneurs, which points toward a non-corporate business interpretation, we think that an equally good interpretation is to regard them as corporations. The main reason for this approach is precisely the implementation that we just showed: as part of their insurance contract, these firms issue equity and thus become corporations.

However, the "corporate" interpretation raises two important concerns. The first is behavioral: do public companies behave the same way as firms owned by a single risk-averse individual? The second is empirical: is it possible to measure the portfolio implications? If so, how?

There is ample evidence that publicly traded firms are owned by a concentrated pool of investors. See, for example, Shleifer and Vishny (1986) or more recently Holderness (2009). The levels of concentration rarely make it to a majority control by a single individual, but in any case, it reflects a small number of controlling entities, which can be individual or institutional investors (see Bebchuk et al. (2017)), or a combination of the two. Both types of owners are risk-averse and can have considerable influence on the firm's management.[12] Thus, interpreting observed corporate decisions as the capitalists in our model appears is a reasonable path. Hence, and since it does not depend on the particular implementation, that risk-averse corporations would increase their savings when facing a combination of idiosyncratic and income share risks is a sensible implication.

When interpreting the implication regarding the agents' portfolio choices, some additional caveats arise, especially when considering how the transactions are imputed. For instance, a standard source of information for agents' economic behavior is the Flow of

[12]Moreover, these decisions would be implemented by risk-averse managers.

Funds Tables, which makes a clear separation between the household and corporate sector. Who are the workers and who are the capitalists in this imputation? There is a natural mapping from our model-workers to the "household" sector in this accounting framework. Households supply labor, save, and invest in financial assets, including equity, as the workers in our model. But capitalists are also households; hence, their insurance arrangements would be imputed to the portfolio choices of the measured household sector, rather than to the corporate sector. For instance, when a capitalist sells part of its equity to a worker, it would be a transfer of equity from one household to another, generating zero impact on the aggregate. From this point of view, only an increase in the share of corporate relative to private businesses would generate a measurable aggregate impact on households' public equity holdings.[13]

Similarly, if capitalists decide to optimally insure against the composite of income share and idiosyncratic risk as individuals, their transactions would be recorded in the household sector only, without any measurable effect on the corporate sector. However, as investors own these companies, they can choose whether to insure as individuals or through the firm, in which case the transactions would be recorded in the corporate sector accounts. Our working assumption is that the second alternative is the dominant one. For large individual investors, relinquishing control in their business to engage in risk-hedging transactions could be too costly when it can easily be done inside the firm. Moreover, the necessary transactions and the resulting portfolio to insure as an individual can have important taxation consequences. In turn, institutional investors, such as Mutual Funds, Hedge Funds, Pension Funds, etc., would be more prone to offer their clients securities that are already partially hedged by the issuer. As long as this motive to hedge "inside" the corporation is, at least partially, present, out theory has also implication for the corporation's portfolio allocations.

2.4 Normative implications

The allocation in the competitive equilibrium with incomplete markets features redistribution of wealth as a result of aggregate shocks. Is this redistribution a market failure? Can a planner alter the demand for insurance to improve the allocation? The answer is no, when the planner is constrained in a similar way as the agents.

[13] With microeconomic data it could still be possible to test the theory in terms of intra-household equity transfers of already incorporated companies. This would require information regarding the behavior of agents whose wealth is mainly composed of human capital and compare it with those whose wealth is mostly equity. We would like to thank an anonymous referee for pointing these caveats out and suggesting alternative strategies to overcome them.

In this section, we show that the equilibrium in the two-period model is constrained-efficient. The notion of constrained efficiency, which follows Stiglitz (1982), and Geanakoplos and Polemarchakis (1986), provides the planner with the same instruments as the market. The planner can intervene, redistributing consumption across aggregate states with a lump-sum transfer $T(s)$. Consumption for the consumer and the entrepreneur are given by

$$c_2(s) = T(s) + (1 - \alpha(s))Y_2(s)$$
$$e_2(s,i) = -T(s) + \alpha(s) g_i Y_2(s)$$

Without loss of generality, define $T(s) := \phi(s)Y_2(s)$. In Appendix D.2, we set up the planner's problem, and we show that the first order conditions from the problem of maximizing the welfare of the consumers (given Pareto weights) subject to the technology constraints yields

$$\frac{e_1^{-\gamma}}{c_1^{-\gamma}} = \frac{\mathbb{E}_i \left(-\phi(s)Y_2(s) + \alpha(s) g_i Y_2(s)\right)^{-\gamma}}{\left(\phi(s)Y_2(s) + (1 - \alpha(s))Y_2(s)\right)^{-\gamma}}$$

These make up the same set of conditions as the ones in equation (9) for the Competitive Equilibrium, which implies that the Competitive Equilibrium is constrained-efficient.[14]

3 General Model

In this section, we present the general model with: (a) an arbitrary arbitrary number of aggregate states $s \in [s^1, s^2, ..., s^N]$, (b) infinite horizon and, (c) endogenous investment decision.

Markov Equilibrium. To simplify notation, we characterize the solutions in a recursive fashion. In the two-period economy, there was no investment, and given that after the second period there was no choice to be made, keeping track of the exogenous aggregate shock was enough. However, we also showed that the allocations depend on initial wealth distribution. In the infinite-horizon economy, the distribution of wealth will be changing along the business cycle. Thus, we will need to keep track of it, together with the effective stock of capital to determine the equilibrium. The redefined state space is $s = \{g_s K, x\}$, where x is the ratio of workers' wealth to total wealth. We formally show in Section 3.3 that these two state variables are enough to characterize the equilibrium.

[14]Things would be different if the criteria for efficiency were Pareto Optimality. If a planner has enough instruments to perfectly control consumption in both aggregate and idiosyncratic states, she will choose the same allocation as in the Complete Markets solution.

Since both K and x are endogenous, the transition function $\Pi(s'|s)$ is an equilibrium object. However, when solving the individual problems, in Subsection 3.1 and Subsection 3.2, the composition of s and how its transition is determined are irrelevant, because each individual takes them as exogenous.

3.1 Workers

In this section, we maintain the assumption that workers supply labor inelastically, but we extend the analysis to allow for Epstein-Zin preferences separating the intertemporal elasticity of substitution (IES) from the risk-aversion parameter. We do so building on Angeletos (2007).[15] Let σ be the inverse of the IES and γ the parameter governing risk aversion, then the worker solves

$$
\begin{aligned}
V^w(a,s) \quad &= \quad \max_{\{c(s),a(s'|s)\}} \left\{ \frac{c(s)^{1-\sigma}}{1-\sigma} + \beta \left(\mathbb{E} V^w \left(a \left(s' \mid s \right), s' \right)^{\frac{1-\gamma}{1-\sigma}} \right)^{\frac{1-\sigma}{1-\gamma}} \right\} \\
st. \quad & \quad c(s) + \sum_{s'} p(s'|s) a(s'|s) \leq a(s) + \omega(s); \qquad \forall s, s'
\end{aligned}
$$

where $\omega(s)$ is the wage and $a(s'|s)$ denote the AD securities bought by the consumer in state s, which pay off in the next period contingent on the realization of s'. The initial financial wealth $a_1 \equiv a(s_0)$ is given. To characterize the solution, some definitions are in order. Denote by $h(s) = \sum_{s'|s} p(s'|s)[\omega(s') + h(s')]$ the worker's present value of future income (human wealth) and by $W^w(s) \equiv a + \omega(s) + h(s)$ her total wealth. The solution is linear in total wealth, in the sense that both consumption and contingent asset holdings can be expressed as linear functions of $W^w(s)$.[16] They satisfy the following:

$$
c(s) = (1 - \zeta(s))W^w(s) \tag{22}
$$

$$
a(s'|s) = \phi^w(s'|s)\zeta(s)W^w(s) - \omega(s') - h(s') \tag{23}
$$

where $\zeta(s)$ is the savings ratio out of total wealth and $\phi^w(s'|s)$ is the optimal wealth growth factor.

Using first-order conditions with respect to $a(s'|s)$, the decision rules and value function guesses, the envelope theorem and, the worker's budget constraint, we show that the pair $\{\zeta(s), \phi^w(s'|s)\}$ satisfies the following system of equations:

[15] Allowing for an endogenous labor supply would not affect the bulk of our analysis. The conclusions would remain valid as long as the level and fluctuations of the income shares are driven by technology.

[16] See Online Appendix E where present all the derivations of this section with more detail.

$$\phi^w(s'|s) = \left[\left(\frac{\zeta(s)}{(1-\zeta(s))} \right)^{\frac{1-\gamma}{1-\frac{1}{\sigma}}} \tilde{\beta}(s',s)^{\gamma} (1-\zeta(s'))^{\frac{1-\gamma}{1-\frac{1}{\sigma}}} \right]^{\frac{1}{\gamma}}; \qquad \forall s, s' \tag{24}$$

$$\zeta(s)^{-1} = 1 + \left[\sum_{s'} p(s'|s) \tilde{\beta}(s',s) (1-\zeta(s'))^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}} \right]^{\frac{\gamma(1-\frac{1}{\sigma})}{1-\gamma}}; \qquad \forall s \tag{25}$$

where $\tilde{\beta}(s'|s)$ is given by:

$$\tilde{\beta}(s'|s) = \frac{\beta^{\frac{(1-\gamma)}{\gamma(1-\sigma)}} \Pi(s'|s)^{\frac{1}{\gamma}}}{p(s'|s)^{\frac{1}{\gamma}}}; \qquad \forall s, s' \tag{26}$$

Clearly, taking prices and probabilities $p(s'|s)$ and $\Pi(s'|s)$ as given, equation (25) is a fixed point in $\zeta(s)$. Once we find $\zeta(s)$, equation (24) solves for the state-contingent asset holdings. Finally, aided with (22) and (23), we recover $\{c(s), a(s'|s)\}$.[17]

3.2 Capitalists

Technology. Capitalists combine labor and capital to produce using a constant returns to scale (CRS) technology:

$$y(g_s, g_i, k, l) = F(g_i g_s k, l) + (1-\delta) g_i g_s k$$

where k is stock of capital, l is labor, δ is the depreciation rate, and g_i, g_s represent the idiosyncratic and aggregate shocks, respectively. Both are assumed to be *i.i.d* over time. Denote by $k(s,i) = g_i g_s k$ the *effective capital stock*. The firm hires labor in competitive markets. In Appendix B.1, we show that the income from capital, $F(g_i g_s k, l) + (1-\delta) g_i g_s k - \omega(s) l(s)$, can be written as

$$\pi(s,i) = g_i R(s) k(s) \tag{27}$$

where $R(s) = (1-\delta) g_s + r(s)$, with $r(s) = \frac{\partial y(s, \mathbb{E} g_i, k, l)}{\partial k}$. Since we assume that $E(g_i) = 1$, the aggregate capital income share is affected only by the aggregate shock. The linearity of profits in the capital stock and the idiosyncratic shock is instrumental in characterizing the equilibrium, because it allows for linear decision functions.

[17] In the special case in which $\gamma = \sigma$, that is, when the utility function is CRRA, the system is linear in $(1-\zeta(s))^{-1}$, greatly simplifying the solution. In a previous draft, we show how to solve this setting by simple matrix inversions.

Contracting. Capitalists are subject to idiosyncratic risk, so they will try to insure it. Following DeMarzo and Fishman (2007), we assume that they have access to risk-neutral intermediaries who can provide insurance. However, due to moral hazard, there is a limit to how much idiosyncratic risk can be offloaded. To be precise, we model moral hazard as endowing capitalists with the possibility of diverting resources from the firm to their private accounts at a cost $0 < 1 - \psi < 1$. For each unit of profit that they divert, only ψ units are transformed into consumption goods (or savings). The contract stipulates that the capitalist must hand over to the financial intermediary a given proportion of her risky profits, receiving an average of the profits of all firms in return.

Since capitalists can misreport their profits and consume a proportion ψ of the misreported profits, in Appendix D we show that the optimal contract implies that she must retain (or be exposed to) a proportion ψ of the idiosyncratic risk. This is known in the literature as a "skin in the game" constraint.[18] As a result, we can write the exposure to the idiosyncratic risk in a simple reduced form. Let $\tilde{g}_i \geq 0$ be the productivity shock to which the firm is exposed. Then, an economy with idiosyncratic risk $\tilde{g}_i$ and restricted insurance is equivalent to an alternative economy in which individual risk is not insurable and firms are subject to idiosyncratic risk g_i satisfying

$$g_i = (1-\psi)\mathbb{E}_i\tilde{g}_i + \psi\tilde{g}_i \geq 0 \tag{28}$$

Program. Consistent with the worker's problem, we also assume that capitalists are endowed with Epstein-Zin preferences and the same parameters as workers.

$$\begin{aligned} V^e(E,k;s,i) = &\max_{\{e(s,i),E(s'|s),k'(s,i)\}} \left\{ \frac{e(s,i)^{1-\sigma}}{1-\sigma} + \beta\left(\mathbb{E}V^e\left(E',k',s',i'\right)^{\frac{1-\gamma}{1-\sigma}}\right)^{\frac{1-\sigma}{1-\gamma}} \right\} \\ &s.t. \quad e(s,i) + k'(s,i) + \sum_{s'} p(s'|s)E(s'|s) \leq E(s) + g_i R(s)k; \qquad \forall i,s,s' \end{aligned}$$

where $E(s'|s)$ are AD securities bought by the capitalist in state s, with payoffs contingent on the realization of state s'. The initial financial wealth $E_1 \equiv E(s_0)$ is given. In this section, we show that, despite being subject to idiosyncratic risk, the consumption

[18] DeMarzo and Fishman (2007) assume that the principal can sign long-term contracts (there is commitment) and that both the principal and the agent are risk-neutral. In contrast, we consider a risk-averse agent who can commit only to short-term contracts. For similar setups and results in continuous time, see DeMarzo and Sannikov (2006). We also show that as long as insurance contracts are not history-dependent, this is the best-possible insurance, independent of whether the entrepreneurs have access to hidden savings. This contract is akin to an equity contract in which the entrepreneur creates a company, issues equity for a proportion $1-\psi$ of its ex ante value, and retains a proportion ψ of the value of the company. See Di Tella (2019) for an example of how a social planner could improve the allocations using taxes.

and savings ratios are simple and akin to those of the workers. Due to homothetic preferences, savings ratios are linear in wealth, and thus total savings are independent of the distribution of wealth. There is aggregation: knowing the average net worth is enough to forecast future aggregate capital. For this result, it is crucial that individual returns are a linear function of the individual holdings of capital, as shown in Appendix B.1.

In Appendix E, we show that, analogously to the worker's problem, the capitalist's optimal choices are linear in wealth. Her total wealth is $W^e(s,i,k) = E(s,i) + R(s)g_i k$ (recall that $R(s)$ is a gross rate), which allows us to write the optimal decisions as:

$$e(s,i) = (1-\vartheta(s))W^e(s,i,k) \tag{29}$$
$$k'(s,i) = \nu(s)\vartheta(s)W^e(s,i,k) \tag{30}$$
$$E(s'|s,i) = \phi^e(s'|s)\vartheta(s)(1-\nu(s))W^e(s,i,k) \tag{31}$$

where $\vartheta(s)$ is the entrepreneur's savings ratio, $\phi^e(s'|s)$ is the optimal financial wealth growth factor, and $\nu(s)$ is the portion of savings invested in capital. In what follows, we will refer to $\nu(s)$ as the investment rate.

Simple manipulations of the budget constraint, together with (29), (30) and (31), imply that that law of motion of individual wealth is:[19]

$$W^e(s',i',k') = \vartheta(s)o(s',i;\phi^e,\nu)W^e(s,i,k) \tag{32}$$

where

$$o(s',i';\phi^e,\nu) \equiv (1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')g_{i'}$$

Again, the solution to the problem can be characterized by a system of equations solving for $\vartheta(s)$, $\phi^e(s'|s)$, and $\nu(s)$, which is given by:

$$\vartheta(s)^{-1} = 1 + \left[\sum_{s'} p(s'|s)\tilde{\beta}(s',s)(1-\vartheta(s'))^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}} \frac{\mathbb{R}(s',s)^{\frac{1}{\gamma}}}{1+Prem(s)}\right]^{\frac{\gamma(1-\frac{1}{\sigma})}{1-\gamma}} \tag{33}$$

$$o(s',i';\phi^e,\nu) = \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}} \tilde{\beta}(s',s)\mathbb{R}(s',s)^{\frac{1}{\gamma}} \tag{34}$$

$$\mathbb{E}_{s',i'|s}\left[(1-\vartheta(s'))^{\frac{1-\gamma}{1-1/\sigma}}\left[o(s',i';\phi^e,\nu)\right]^{-\gamma}\left(R(s')g_{i'} - \frac{1}{\sum_{s'|s} p(s'|s)}\right)\right] = 0 \tag{35}$$

[19] See Appendix E.2 for details.

where $Prem(s) \geq 0$ is the risk premium and $\mathbb{R}(s', s) \geq 1$ is a risk-adjustment factor shown in the Appendix, so that when $\mathbb{V}ar(g_i) = 0$, then $Prem(s) = 0$ and $\mathbb{R}(s', s) = 1$.[20]

Some features about the system are worth noting. First, all three objects are independent of individual wealth and the current idiosyncratic shock. This independence of the equilibrium from the underlying distributions is due to the linearity of the decision functions and greatly simplifies the analysis.

Second, the main difference between (25) and (33) is $\frac{\mathbb{R}(s',s)^{\frac{1}{\gamma}}}{1+Prem(s)}$. When there is no idiosyncratic risk, this term is equal to 1, so capitalists choose the same savings ratios as workers. However, in general, the term is bigger than 1. As a result, in equilibrium, for any price function $p(s)$, it must be true that $\vartheta(s) > \zeta(s)$: on average, capitalists save more than workers. This creates a downward drift on the workers' wealth ratio, x. As we showed in the two-period economy, this wealth effect has important quantitative implications, generating large changes in the position of financial assets.[21]

Third, equation (34) pins down the wealth growth factor. Comparing (24) and (34), we see that the capitalist's savings ratio is affected by the additional term $\mathbb{R}(s', s)^{\frac{1}{\gamma}}$, which depends on both risk aversion and the exposure to uninsured idiosyncratic risk. In the absence of uninsured idiosyncratic risk, both agents would react equally to aggregate shocks.

3.3 Equilibrium

The allocations must satisfy the assets' and goods' market clearing conditions, which pin down the equilibrium prices $p(s'|s)$. Furthermore, $\Pi(s'|s)$ must be consistent with the laws of motion generated by individual decisions. The assets' and goods' market clearing conditions are:

$$a(s'|s) + E(s'|s) = 0 \qquad \forall s, s' \tag{36}$$

$$c(s) + e(s) + K'(s) = y(s) \quad \forall s \tag{37}$$

[20]See equation (92) in Appendix E, where we provide a definition of $\mathbb{R}(s', s)$, making explicit the dependency of it on both $\mathbb{V}ar(g_i)$ and $\phi^e(s', s)$.

[21]The drift also implies that, as time goes to infinity, the workers end up with zero wealth. This may seem like an odd prediction, but it is the natural outcome of combining agents with heterogeneous exposure to risk. To construct equilibria with nondegenerate wealth distributions, the literature has resorted to alternative strategies. One solution is to introduce different β's, with capitalists discounting the future more, as in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012). Alternatively, one can assume that with some exogenous probability, capitalists and workers switch "functions" while maintaining their wealth.

where $e(s) = \int_i e(s,i,k,E)$, $K'(s) = \int_i k'(s,i,k,E)$, $y(s) = \int_i y(s,i,k,E)$, and $E(s'|s) = \int_i E(s'|s,i,k,E)$. We have avoided the dependency of the allocations on individual wealth because the savings and investment ratios are independent of it.

Let the total wealth be $W^T(s) = W^w(s) + W^e(s)$ and define the worker's wealth ratio as $x = W^w(s)/W^T(s)$. Using the asset market clearing condition, and asset holdings for the entrepreneurs and workers, we find that the AD prices satisfy:[22]

$$\frac{p(s'|s)}{\beta^{\frac{(1-\gamma)}{(1-\sigma)}}\Pi(s'|s)} =$$
$$\left[\left(\frac{\zeta(s)(1-\zeta(s'))}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\zeta(s)x + \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\vartheta(s)\mathbb{R}(s',s)^{1/\gamma}(1-x)\right]^{\gamma}\left(\frac{W^T(s)}{W^T(s')}\right)^{\gamma} \tag{38}$$

All of the elements in this equation are endogenous, which complicates the interpretation. However, in the following sections we show how this equation changes under different assumptions, clarifying the economic mechanisms at play.

It is simple to show that the state variables satisfy the following laws of motion:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s) \tag{39}$$

$$x(s'|s) = \phi^w(s'|s)\zeta(s)\frac{W^T(s)}{W^T(s')}x \tag{40}$$

Both are Markovian, so it is possible to compute their transition probabilities. As a result, (39) and (40), together with the exogenous probability distribution over g_s, determine the transition probabilities $\Pi(s'|s)$.

3.4 Benchmark economies

In this section, Proposition 3, we show that the insights presented in Proposition 1 extend to the infinite-horizon economy with endogenous investment. For simplicity, we consider the case in which $\sigma = \gamma$, but our results extend to the general case of Epstein-Zin (EZ) preferences. Define $\tilde{g}(s',s) := \frac{Y(s')}{Y(s)}$.

Proposition 3. *Suppose that $\sigma = \gamma$. Then*

(a) If $\mathbb{V}ar(g_i) = 0$, then $x(s'|s) = x$ for all (s,s').

(b) If $\mathbb{V}ar(\alpha(s)) = 0$, and output growth is state-independent, then there exists a $\beta^e < \beta$: (i) $x(s'|s) = x$ for all (s,s') and (ii) $p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s',s)^{-\gamma}$ for all (s,s').

[22]See equation (111) in Appendix E

Proof. See Appendix G. □

Complete markets: $\mathbb{V}ar(g_i) = 0$**.** Part (a) of Proposition 3 extends, to a more general environment, the result in part (b) of Proposition 1. After any aggregate shock s, and after any history of shocks, the portfolio holdings are such that all the implied payments leave both agents with the same *relative* wealth. In Proposition 1, we were able to obtain a sharp characterization of the evolution of wealth shares because the period 2 wealth was equal to income. Thus, the compensation consisted of only the difference between the income share and the wealth ratio x. In the general setup, the compensation embeds not only the current difference in income but also the present value of all the expected future changes.

The analogy is also evident in the implied asset prices. Using equation (38), because with complete markets $\zeta(s) = \vartheta(s)$ and $\mathbb{R}(s', s) = 1$, the AD prices are given by

$$p(s'|s) = \beta\Pi(s'|s)\left(\frac{1-\vartheta(s)}{1-\vartheta(s')}\right)^{\gamma}\left(\frac{W^T(s)}{W^T(s')}\right)^{\gamma}$$

The differences with respect to Proposition 1 are (1) the price depends on the ratio of wealth rather than the ratio of period 2 output, and (2) the ratio of the marginal propensity to consume also appears. This is because in Proposition 1 there was no investment decision, while here the resources diverted to investment change over the business cycle. If the consumption ratios were constant, the prices would just reflect the random growth in total wealth. This happens because the changes in $W^T(s)$ reflect the common component of the shock, and therefore cannot be insured.

Constant income shares: $\mathbb{V}ar(\alpha(s)) = 0$**.** When the capitalist and the workers share the same discount factor, because capitalists are exposed to idiosyncratic risk and due to the precautionary savings motive, they tend to accumulate more assets than workers. As time goes on, this force pushes the proportion of wealth in workers' hands toward zero. To compensate for this downward drift, one can assign a smaller discount factor β^e to capitalists.[23] This alternative discount factor satisfies

$$\beta^e = \beta\frac{(1+D)^{-\sigma}}{\mathbb{E}_i(1+Dg_i)^{-\sigma}}$$

where $D > -1$ is the ratio of risky physical investment to financial assets in the capitalist's portfolio, capturing the capitalist's exposure to idiosyncratic risk. Only when $\mathbb{V}ar(g_i) = 0$ then $\beta^e = \beta$, while for any strictly positive variance, $\beta^e < \beta$. With this adjustment,

[23] This result has been known since Yaari (1965) and Blanchard (1985). More recent discussions appear in Gârleanu and Panageas (2015) and Di Tella (2017)

Proposition 3 generates the same prices and allocations as an analogous economy with complete markets ($\mathbb{V}ar(g_i) = 0$) and $\beta = \beta^e$. As we discussed in Section 2.4, *fluctuations in the income shares are key for generating nondegenerate financial portfolios, which interact with the shocks amplifying its effects through changes in the quantities*.

Similar results can be found in the literature. Proposition 3 is a generalization of Di Tella (2017), who presents a similar result in a continuous time environment with an AK technology (and hence no labor supply) and aggregate productivity shocks that follow a geometric Brownian motion. We extend the result to a discrete time environment, allowing for any CRS technology and an additional factor of production (labor), as long as the income shares are constant. Also, Bocola and Lorenzoni (2020) provide a similar result in a discrete time environment, but they maintain the AK assumption and assume that the aggregate productivity shock is *i.i.d.* (in levels) over time.

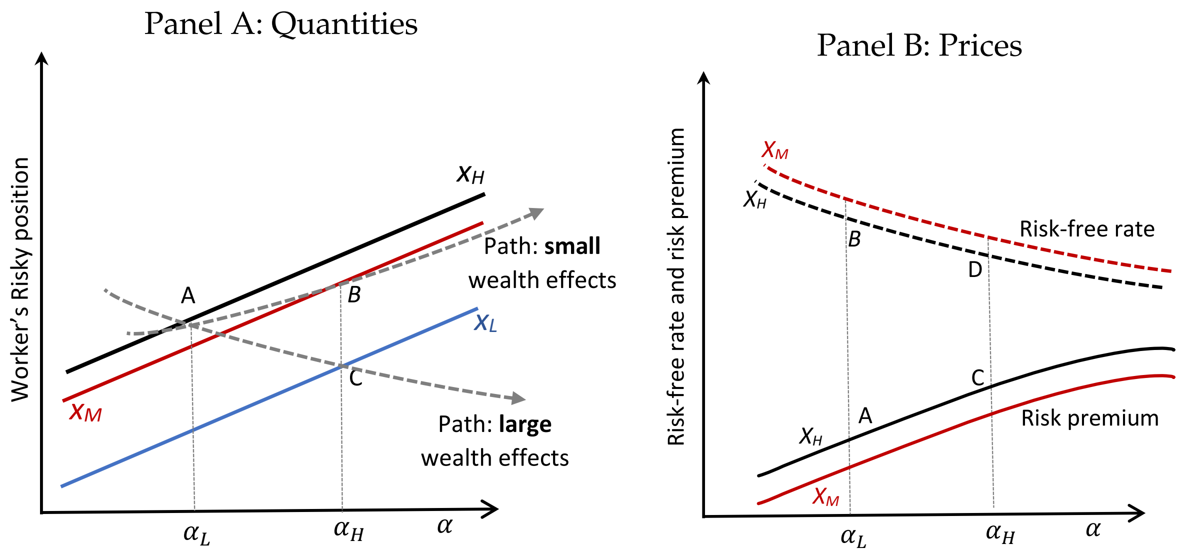
3.5 The economy with moving shares

Obtaining analytical results with varying shares and idiosyncratic risk is less straightforward in the general model. However, using the insights from Proposition 1 and the equilibrium equations, we can provide some intuition for the expected outcomes.

We start by analyzing the effects of an increase in the capital share on the desired financial positions. The assets' positions widen with α, so that a larger α leads workers to a larger positive position in the risky asset, leveraging the risk-free asset. For instance, suppose that the economy is initially in a state with capital share α_L and distribution of wealth x_H. In Panel A of Figure 1, we depict the hypothetical worker's risky position at point A. Now suppose there is a shock that increases the capital share to $\alpha_H > \alpha_L$. Absent wealth effects, the worker's position would move along the black line labeled x_H. In this case, a positive α shock unambiguously increases the worker's position in the risky asset. The same argument applies for the worker's debt. This would be the outcome with complete markets, since x would remain constant after any shock.

However, when markets are incomplete, a positive α shock would increase the capitalist's wealth, and therefore x would fall. Since a larger $1 - x$ implies that more resources must be devoted to insure the idiosyncratic risk, capitalists are less willing to trade on the insurance of the income shares' risk. This effect dampens the increase in the trading of the risky asset; it does so in such a way that after the shock the worker's risky position may end up being smaller. If the wealth effect is "small", say x moves to a new level $x_M < x_H$, such that the demand now lies on the red line labeled x_M, then the new position would be located at a point such as B in Panel A of Figure 1. Despite the dampening effect, the

Figure 1: Wealth Effects and Financial Markets



worker's risky positions would be positively correlated with the capital share. But if the wealth effect is large enough, e.g., the wealth distribution moves to $x_L < x_M < x_H$, then the economy would end up at point C, where the worker's risky position and α are negatively correlated. Thus, the extent of uninsured idiosyncratic risk and its implications for the wealth effects are crucial components of the quantitative implications.

In Panel B of Figure 1, we show the expected patterns for asset prices. To this end, keep in mind equation (38). In the previous section, neither idiosyncratic risk nor the distribution of wealth played any role, but now these two components matter. In our setup, the equivalent to a risk-free rate is given by $1/\sum_{s'} p(s'|s)$. An increase in the "average" price is equivalent to a fall in the risk-free rate. As in the two-period model, an increase in α acts as an increase in uncertainty, which is reflected in a larger factor $\mathbb{R}(s', s)$ in equation (38). This direct impact is the main component generating a decreasing risk-free rate as shown in Panel B. Ceteris paribus, the new rate would move from point B to point D. However, there are two additional effects. First, x drops, say, to $x_M < x_H$. Then the weight on $\mathbb{R}(s', s)$ increases, which also raises the average prices. However, because of the increased risk, the capitalists' consumption slows down, which puts downward pressure on prices. Taken together, these simultaneous effects could dampen the fall in the risk-free rate, as shown in Panel B with the line labeled x_M, or reinforce it.

Moreover, in Appendix E (see equation (99)), we show that the risk premium satisfies

$$Prem(s) = \sum_{s'|s} p(s', s) \left[\frac{\sigma v(s)^2 r(s')^2}{o(s', 1)\mathbb{R}(s', s)} \mathbb{V}ar(g_i) \right] \tag{41}$$

In this case, the larger α has a direct and sizeable impact on increasing the risk premium. Without wealth effects, the risk premium should increase as depicted in the shift from A to C in Panel B. But there are two additional indirect effects related to the wealth distribution. First, because the wealth share of workers who are not exposed to idiosyncratic risk drops, it becomes increasingly harder to insure it, and therefore the risk premium could rise further. Second, the higher exposure to the idiosyncratic risk generates portfolio rebalancing, in which the capitalist invests less in capital so that $\nu(s)$ falls. Thus, the risk premium may fall, as indicated in Figure 1. Which effect dominates is a quantitative question.

4 Quantitative Implications

We have shown that income shares' risk alone generates nontrivial portfolio allocations, but due to the lack of wealth effects, the allocation is invariant to the state of the economy. Uninsured idiosyncratic risk alone delivers relevant wealth effects, but the allocations are still invariant to aggregate shocks and imply degenerate portfolios. Only when both, income shares and idiosyncratic, risks coexist there are well defined portfolio allocations with interesting business cycle properties. However, a natural question arises: how significant could these effects be in quantitative terms? To address this question, this section provides an illustration using a plausible calibration, revealing that the effects can indeed be substantial.

It is important to note that we do not attempt to replicate asset position patterns or asset prices in the model. Our abstraction overlooks various factors that influence financial markets. For instance, households might seek to accumulate risk-free assets for reasons of liquidity, or insurance needs could be impacted by other shocks beyond total factor productivity (TFP) changes. We have not integrated any of these additional relevant features. As a result, we calibrate the model to replicate "untargeted" standard moments and then evaluate its predictions for the financial markets. Accurate alignment with financial moments is a task left for future research.

4.1 Calibration

To construct the mapping to the intuitive risk-free and risky asset positions, we assume that the aggregate shock can take on two values, g_H and g_L, each occurring with probability $1/2$. The *i.i.d.* structure of the shock simplifies the state space without losing realism, since due to the permanent effect on the capital stock, the generated output is close to a random walk. In addition, with only two possible realizations of s, we can construct the straightforward mapping from the economy with AD securities to that with only two assets: a risk-free asset and a risky asset. Adding more realizations would have minimal quantitative effects and would make this mapping less clear.

To discipline the relationship between output and the capital share, with meaningful variation, we use a CES production function with parameters $\{\rho, \alpha_k\}$, as follows:

$$F(K,L) = \left[\alpha_k K^{\frac{\rho-1}{\rho}} + (1-\alpha_k) L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

Thus, two key parameters are α_k, which determines the average capital income share, and ρ, which pins down the correlation between output and the capital share. As a result, we need to calibrate 10 parameters, which we group into two sets: (1) those borrowed from previous studies, $\{\beta, \sigma, \gamma, \psi, \mathbb{V}ar(g_s)\}$, and (2) those chosen to replicate aggregate moments $\{\beta^e, \delta, \alpha_k, \rho, \mathbb{V}ar(g_i)\}$. We now discuss them sequentially.

Regarding the parameters borrowed from the literature, we set the worker's discount factor to $\beta = 0.96$ and the inverse of the IES to $\sigma = 2$. As shown by Crump et al. (2015), most empirical studies point toward an IES of 0.5.[24] Also, following the literature, we set $\gamma = 5$, which together with the other sources of risk determines the risk premium.

We choose $\beta^e = 0.8725$ to obtain an implied average x of around 0.82.[25] Since the worker discounts the future less than the capitalist, the worker would be the agent determining the average risk-free rate. This fact, together with the presence of risk, implies a risk-free rate of around 1% in the stationary equilibrium.

As our model does not include additional frictions, we rely on ρ to obtain income-shares variation. We set $\rho = 1.5$ as in Karabarbounis and Neiman (2014) and Koh et al. (2020), and we choose α_k and δ to jointly target the average capital-output ratio and the average capital share. This generates $\alpha_k = 0.265$ and $\delta = 0.075$. The resulting average capital share is 0.37. We obtain a capital-output ratio of around 2.78, which is in line with most estimates of roughly 2.7. Moreover, since the aggregate shock can take on only two values, we set $g_H = 1.02$ and $g_L = 0.98$. The variance of the assumed process is

[24] This would affect the resulting equity premium. A large literature has explored alternative solutions, such as long-run risk (Bansal and Yaron, 2004 and Hansen et al., 2008) and disaster risk (Barro, 2009 and Gourio, 2012), as possible explanations for the premium between equities and safe bonds.

[25] Computing x is not trivial. We offer one method using the Flow of Funds tables. Since $x = \frac{W^w}{W^T} = \frac{A/y+(1-\alpha)+h/y}{1+(1-\delta)k/y+h/y}$, we use households' financial assets over GDP as a measure of A/y. We approximate h/y as $h/y = (1+E(r))E(1-\alpha)/E(r)$, which is the exact value for the ratio of human wealth to GDP in a deterministic economy. For the risk-free rate, we use the Fred AAA 10-year corporate bond. Then using the capital-to-GDP ratio from Fred in every period, we obtain that the average x is around 0.82.

Table 1: Baseline Calibration

Parameter	Description	Value
γ	Risk aversion	5
σ	IES inverse	2
β	Workers' discount factor	0.96
β^e	Capitalists' discount factor	0.8725
ρ	Elasticity of substitution	1.50
α_k	Capital share parameter	0.265
δ	Depreciation	0.075
$g_{s,h}, g_{s,l}$	Aggregate shocks to capital	1.02, 0.98
p_s	Probability of g_s	0.5
$Var(g_i)$	Variance of idiosyncratic shocks to capital	0.0182
ψ	Exposure to idiosyncratic risk	0.20

$\mathbb{V}ar(g_s) = p(1-p)(g_H - g_L)^2 = \frac{1}{4}0.04^2 = 0.0004$, which is in line with the medium-long-term variation of GDP in the U.S. economy.

There is ample evidence that firms are more exposed to idiosyncratic risk than workers. Following He and Krishnamurthy (2012), we set $\psi = 0.2$ to match the 20% share of profits that hedge funds charge. Thus, given the risk-aversion parameter and aggregate volatility, the risk premium is determined by the exposure to idiosyncratic risk. That is, what matters for capitalists is the residual risk $\psi^2\mathbb{V}ar(g_i)$. Since we are assuming that workers are not subject to idiosyncratic risk, this risk must be interpreted in relative terms. To this end, we target a risk premium of 6%, which generates $\mathbb{V}ar(g_i) = 0.04$, and thus the total idiosyncratic risk borne by entrepreneurs is given by $\psi^2\mathbb{V}ar(g_i) = 0.728 \times 10^{-3}$. We summarize the calibrated parameters in Table 1.

4.2 Quantitative results: increased firms' savings and equity issuances

Table 2 displays several moments of the calibrated economy. The last two columns of Panel A show the targeted data values and the corresponding model results. On average, the capital share is around 0.37, the capital-output ratio is 2.78, and the workers' share of wealth is 0.82, very close to the targeted moments. Panel B displays the values for the risk-free rate, the investment rate, and the workers' portfolio allocations. Though these are all non-targeted statistics, the calibrated economy delivers sensible predictions for each quantity. The risk-free rate is on average 1%, investment is 21% of GDP, and, on average, the workers hold a positive amount of risky assets (equities) and finance this position by borrowing on the risk-less asset. We constructed the assets' positions using

Table 2: Simulated Moments

Quantity	Description	Data	Model
Panel A: Targeted Moments – Means			
$K(s)/Y(s)$	Capital-output ratio	2.50	2.78
$\alpha(s)$	Capital income share	0.37	0.37
$x(s)$	Workers' wealth share	0.82	0.82
Panel B: Nontargeted Moments – Means			
$B(s)/Y(s)$	Workers' riskless asset position		-1.60
$PA(s)A(s)/Y(s)$	Workers' risky asset position		0.80
$r(s)$	Risk-free rate		0.01
$I(s)/Y(s)$	Investment		0.21
$Prem(s)$	Risk premium		0.05

the equilibrium laws of motion, like we did in Section 2.

The main takeaway from Table 2 is that our mechanism can generate large and reasonable financial positions with apparently low variations in the income shares, with plausible underlying risk-free rate and risk premium.

The main results are illustrated in Figure 2, which shows the calibrated version of Figure 1, Panel A. The top panel displays the risk-free rate (left) and the risk premium (right) as a function of the capital share. The bottom panel presents investment as a proportion of GDP (left) and capitalists' financial savings, revenue net of investment, as a proportion of GDP (right), both plotted as a function of the capital share. This figure depicts the primary implication of our paper: as the capital share increases, firms raise their savings, accumulating more financial assets, and reduce investment. These increased precautionary savings by firms have a substantial impact on financial prices, depressing the risk-free rate and increasing the risk premium.

Although the accumulation of financial assets is unambiguously increasing in the capital share, the impact on investment crucially depends on the distribution of wealth in the economy. If the share of worker's wealth (x) is large, the investment rate monotonically decreases with the capital share, while if x is sufficiently small, the investment exhibits a non monotonic relationship with the capital share, decreasing when α is small and increasing when α is large.

These quantitative results stress two important issues. First, seemingly small variation on income shares can have a sizeable impact on firms' savings and investment decisions and financial prices. Second, the wealth distribution can have a drastic impact on macroeconomic aggregates, potentially reverting the predicted patterns of firms' savings and investment.

The results in Figure 2 do not relay on the particular implementation of the optimal contract. Yet, one may wonder what would be the required changes in the financial assets positions under the implementation discussed in Section 2.3, bearing in mind the caveats discusses there. In the left panel of Figure 3, we plot the risky asset positions, $PA(s) \times A(s)/Y(s)$, for different values of x. As discussed in Section 3.5, the position on the risky asset widens as $\alpha(s)$ increases, and the wealth effects become more prominent. For instance, when x falls from 0.84 to 0.58, the risky position decreases by around 3% of GDP (the difference between the yellow dashed line and the red dotted line). If x is sufficiently low, the worker will also be borrowing in the risky asset.

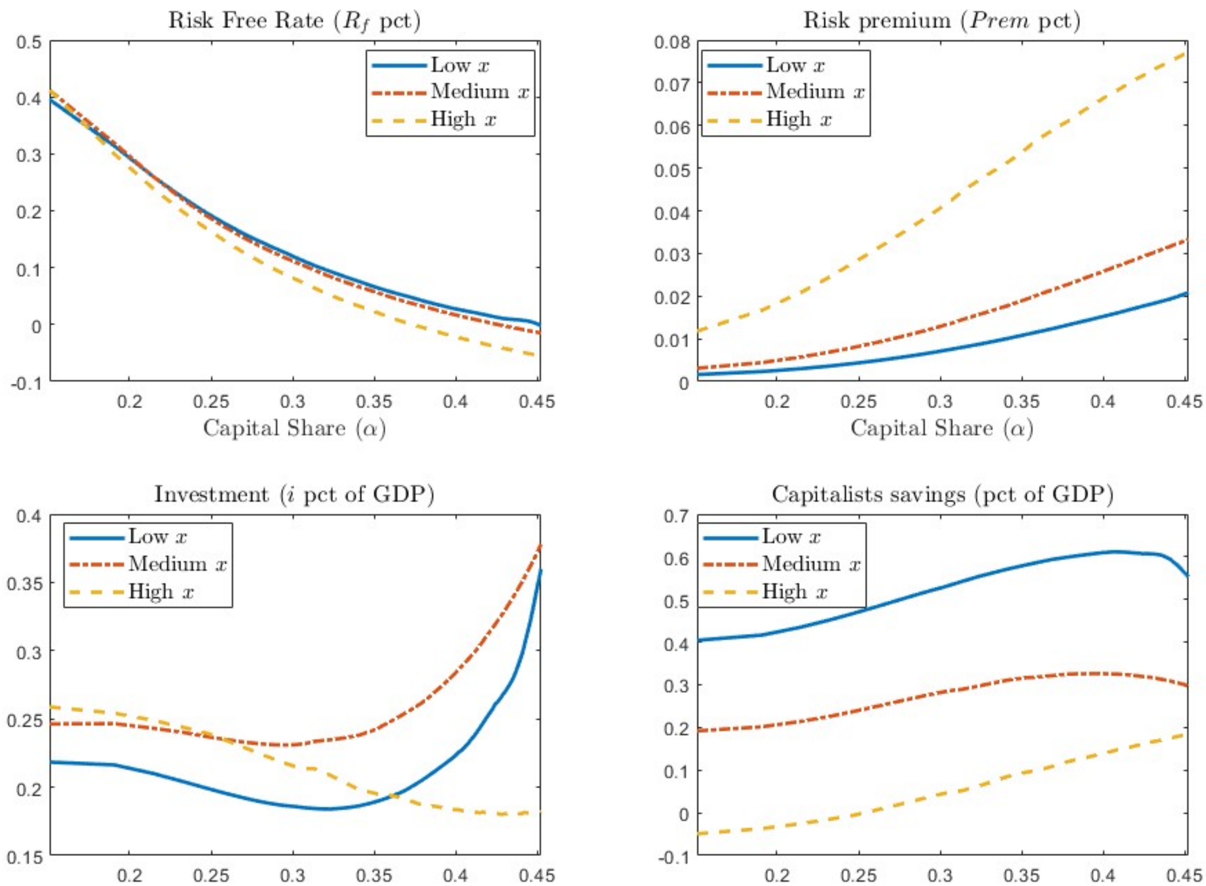
The flip side of the risky-assets accumulation is the borrowing in the risk-free asset. In the right panel of Figure 3, we depict the implied patterns for the workers' holdings of risk-free assets. The pattern for $A(s)$ is mirrored by $B(s)$ with the opposite sign. As the capital share increases, the financial positions widen, increasing the leverage that is used to accumulate risky assets. Because of market clearing, the capitalists' financial positions are the negative of workers' positions. Thus, if workers are borrowing to buy equity, it must be that capitalists are increasing the issuance of equity and accumulating risk-free assets.

Given the discussion in Section 3.5 and the large wealth effects observed in Figure 3, one may wonder if the time paths implied by the model generate a positive or negative correlation between the capital share and the capitalists' holdings of risk-free assets. To shed light on this issue, we simulate an increasing path for α. We plot the implied paths for the worker's wealth share, risk-free assets, risky assets, risk-free rate and the risk premium.

Figure 4 shows the results. With a standard calibration, the wealth effects are not enough to overturn the patterns predicted by the complete markets economy. As the labor share falls, capitalists accumulate more risk-free assets, and lend these funds to workers who, leveraging, invest in equity to insure against changes in the income shares. At the same time, as the capital share increases, it becomes harder for capitalists to insure the idiosyncratic risk, which put downward pressure on the risk-free rate. Because capital is a risky asset, capitalists also invest less, to decrease their exposure to risk.

The bottom half of Figure 3 illustrates the asset-price implications of the model economy. This is the calibrated version of Figure 1, Panel B. First, in the lower left panel, it is evident that the risk-free rate sharply falls as the capital share increases and that the wealth effect is mild. The combination of high exposure to idiosyncratic risk and large quantities of accumulated capital pushes the return on capital downward. As expected

Figure 2: Equilibrium Policy Functions

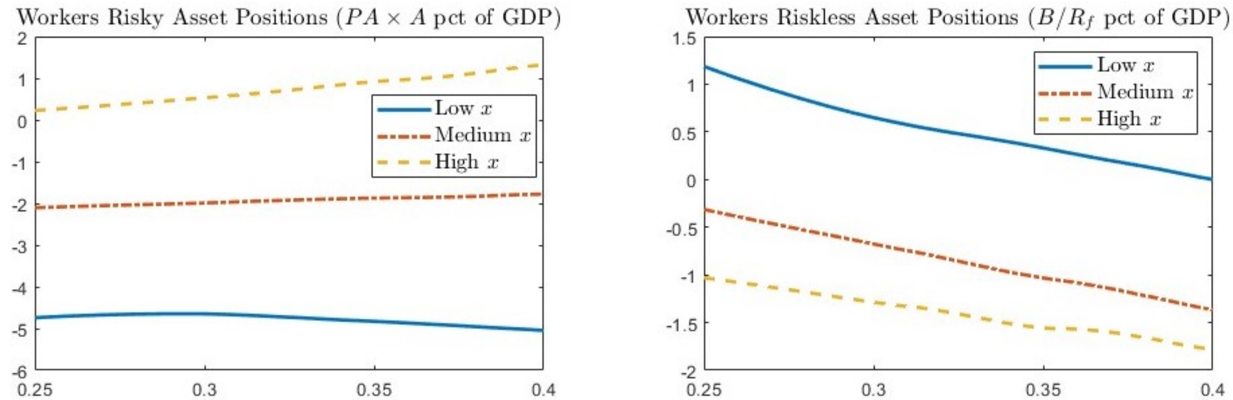


Note: This figure plots (clockwise from upper left) the risk-free rate, the risk premium, investment as a share of GDP, and the capitalists flow of financial savings ($\pi - i$) as a function of the capital share. The low, medium, and high values of x are 0.31, 0.58, 0.84, respectively.

from Section 3.5, the risk premium increases as the capital share rises, as depicted in the bottom right panel of Figure 3. Unlike the risk-free rate, here the wealth effects are more relevant. For a given capital share, as the risk premium moves higher, so does the consumer's wealth share.

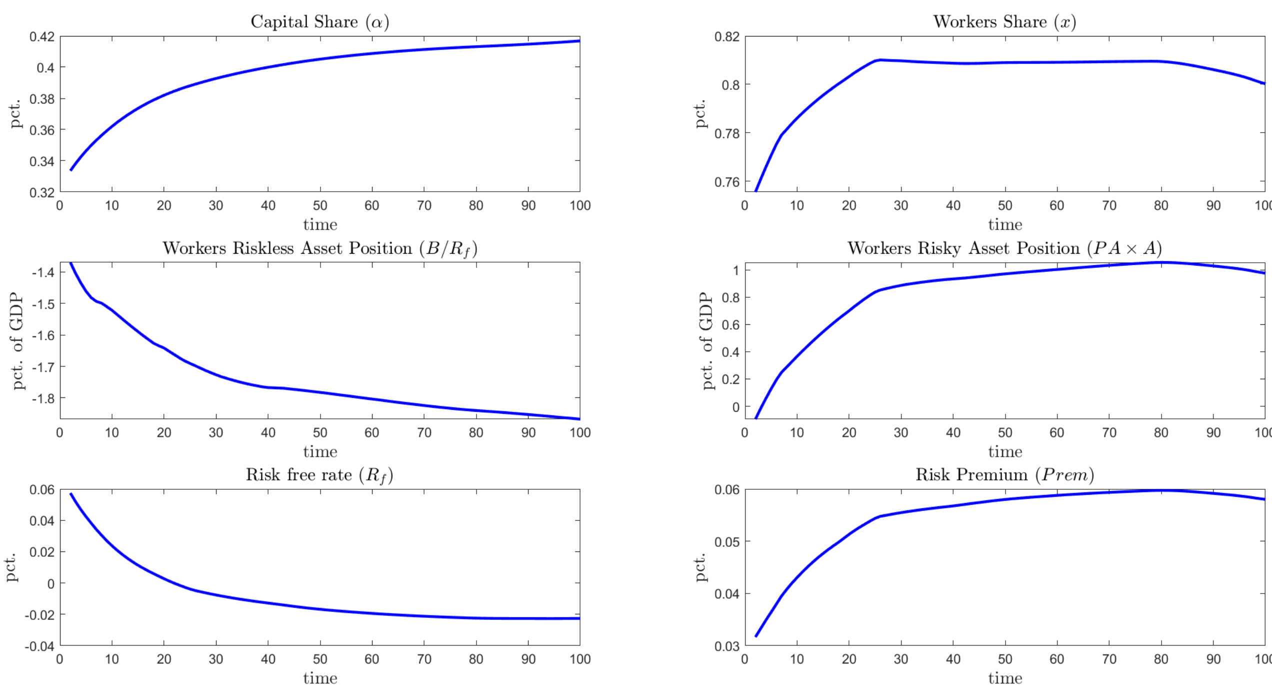
Two issues are worth discussing. First, the risk-free rate varies widely, ranging from 40% for very low capital shares to negative rates for capital shares above 0.4. Though it may appear puzzling and exaggerated, this is a natural implication of the CES production function together with the arbitrage condition between assets. Because $\rho > 1$, low capital shares are only consistent with very low levels of capital. Thus, in that region, due to the decreasing marginal productivity, the return on capital is large, to the point that as $\lim K \to 0$ the marginal productivity of capital approaches infinity. Then, by arbitrage,

Figure 3: Equilibrium Policy Functions



Note: This figure plots (clockwise from upper left) the risky-asset and riskless-asset positions, as a function of the capital share. The low, medium, and high values of x are 0.31, 0.58, 0.84, respectively.

Figure 4: Model-implied Paths after a Sequence of Positive Productivity Shocks

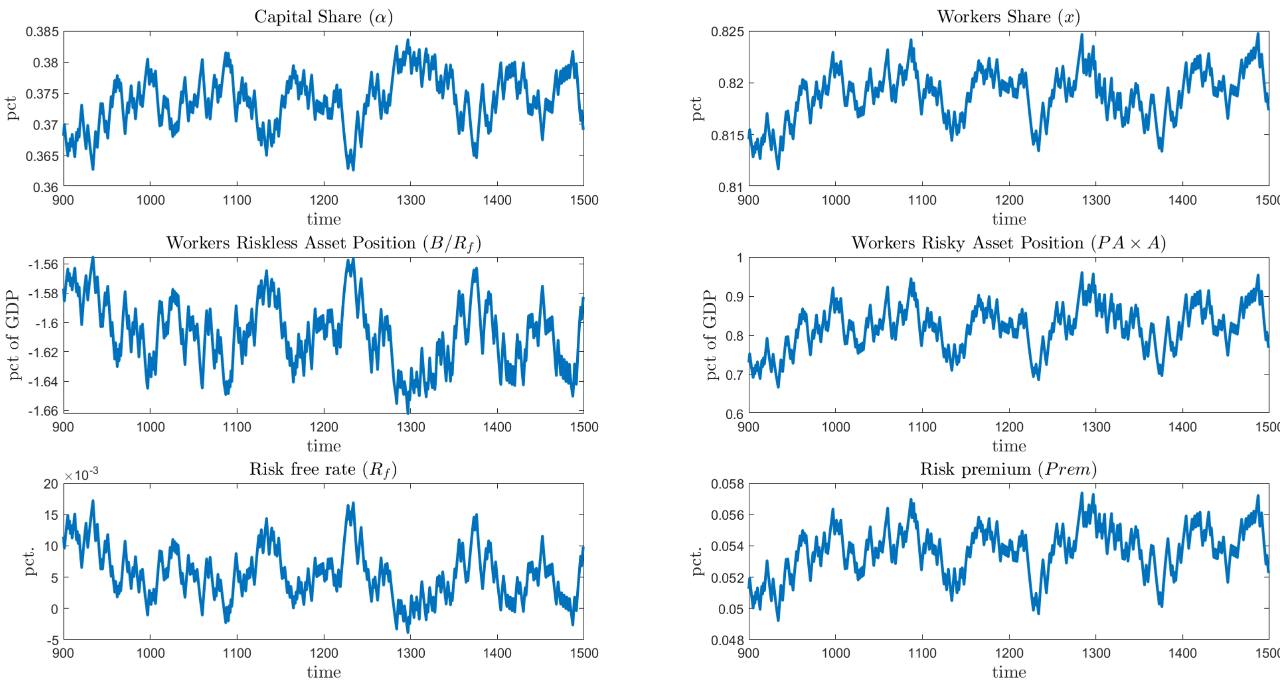


Note: This figure displays key quantities for our calibrated model, generating an increasing path for α by feeding the model economy with a path of productivity shocks with all values equal to s_H.

the risk-free rate must also be large. Still, for empirically relevant levels of the capital share, the risk-free rate remains around the expected levels.[26]

[26]This effect shows up only when one solves the model economy globally. If we had approximated the solution log linearizing around the steady state, the risk-free rate would always remain around the observed

Figure 5: Model-implied Paths after a Sequence of *i.i.d.* Productivity Shocks



Note: This figure displays key quantities for the calibrated model after a sequence of computer-generated random shocks to the productivity of capital s.

Second, the risk premium sharply reacts to changes in the capital share and the wealth ratio. The steep upward-sloping shape is due the "direct" effect generated by a larger α. As discussed in Section 2.3's two-period model (equation (12)) and in Section 3.5's general characterization (equation (97)), an increase in α is akin to an increase in the supply of risk. Since this increase comes at the expense of a lower absorbtion capacity by agents not exposed to risk (workers), the risk premium increases. This effect tends to generate a positive correlation between the risk premium and the capital share and, therefore, with GDP when $\rho > 1$. Moreover, keeping α constant, the risk premium is also increasing on the worker's wealth share. This happens due to "indirect" wealth effects. A lower capitalist's wealth share, on the (capital) investment rate requires a larger compensation to sustain the capital stock. Since positive aggregate shocks decrease x, this second effect could generate a negative correlation between the risk premium and GDP. As a result, the opposite forces between the direct and indirect effects could generate either a positive or a negative correlation between the risk premium and output, depending on the strength of the wealth effects.

values between $0 - 10\%$. Indeed, during our stochastic simulations, the risk-free rate is never above 5%. See Figures 4 and 5.

The path chosen for α in Figure 4 may appear arbitrary within the structure of the model. Could random paths with *i.i.d* aggregate shocks generate a similar pattern? As mentioned in Section 3.3, the g_s shock is such that the *growth rates* are *i.i.d.*, so that the *levels* are close to random walks. Figure 5 depicts a typical path from a simulation of the model. Although the shocks are *i.i.d.*, the model generates long time spans of an increasing capital share (top left panel) and a decreasing consumers' share of wealth (top right panel). In addition, the workers' risky and risk-free asset positions (middle panels) are almost perfectly negatively correlated. This can be seen in detail in Table 3, which displays the model-implied correlations.[27] Most predictions derived analytically in Section 2, reaffirmed in Section 3.3, and clearly depicted in Figure 4 are part of the typical random paths of this economy.

Finally, our model economy cannot generate a negative correlation between the risk premium and GDP, which is the focus of a large fraction of the quantitative studies in macro finance (e.g., Brunnermeier and Sannikov, 2014, Gârleanu and Panageas, 2015, Di Tella, 2017). In our setting, the direct effect due to the increase in α cannot be overcome by opposing forces generated by changes in relative wealth.[28] Given that the focus of our paper is the medium-long term, we believe that the correlation between output and the risk premium does not invalidate our analysis.[29]

In addition, some variations of our model would yield a negative correlation. For instance, one could think that the increase in the risk premium during recessions could be due to the temporary increase in uncertainty, as in Bloom (2009) and Di Tella (2017). Incorporating these elements into our environment would allow us to improve the model fit to financial variables. However, we believe that including these variations is beyond the scope of this paper, so we leave them for future research.

5 Conclusions

The Kaldor facts led to the prevailing belief that the capital and labor income shares were, aside from some small short-run variations, roughly constant. An important implication

[27] Additional correlations for other variables appear in Table 4 in Appendix A.

[28] In online Appendix F, we show how to significantly reduce the short-term correlation by maximizing the impact of the wealth effects. We simulate the economy for "only" 1,500 periods and start the simulation with a significantly low initial $x_0 = 0.37$. By doing so, we are able to replicate similar averages to those in Table 2, but by including the transition paths in the calculations (with x_t always increasing), we reduce the correlation between $\alpha(s)$ (and GDP) and the risk premium to zero. This exercise stresses the role of changes in relative wealth (not powerful enough around the steady state), and the significantly low convergence speed of x.

[29] We thank an anonymous referee for suggesting this point.

Table 3: Model-implied Correlations

Capital Share	Risk-Free Position	Risky Position	Risk-Free Rate	Risk Premium
1.00	-1.00	0.90	-0.99	0.92
-1.00	1.00	-0.91	0.99	-0.93
0.90	-0.91	1.00	-0.95	1.00
-0.99	0.99	-0.95	1.00	-0.97
0.92	-0.93	1.00	-0.97	1.00

of this belief is the impossibility for workers and capitalists to insure each other. With constant income shares, aggregate fluctuations affect both sectors equally, and only common uninsurable shocks are left. Recent studies, however, have shown that income shares move.

In this paper, we argue that variations in the income shares create an important motive to share risk between capitalists and workers. Since both are differentially affected by aggregate shocks, they have incentives to trade in the financial markets to insure changes in their relative income. When the labor share is countercyclical, the optimal insurance contract can be implemented by workers borrowing in risk-free assets and buying equity to participate in capitalists' gains.

The presence of uninsured idiosyncratic risk decreases the capitalists' willingness to trade, hampering the implementation of the optimal contract. Nevertheless, we show in a calibrated model that this channel is quantitatively large, to the extent that it can by itself account for some observed, long-term patterns in the financial markets: the corporate savings glut, the falling interest rates, and the increased risk premium. Thus, although other factors are certainly shaping these patterns, the mechanism proposed here cannot be ignored.

We focus on the medium-long run. However, our model would also lend itself naturally to the study of how income shares exacerbate or mitigate fluctuations in the short run. Our setup is also suitable for analyzing questions linking inequality and asset pricing. In particular, ours is a two-factor asset pricing model in which the capital share and the relative wealth of financial intermediaries are factors pricing the "cross-section" of assets. Business cycles and asset prices are both topics for further research.

Finally, we have abstracted from many frictions that can either directly affect financial markets, such as inflation, liquidity concerns, and default risk, or indirectly affect the financial sector through links to the real economy, such as wage and price rigidities.

The interactions of these frictions with our mechanism are interesting paths for future research.

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A Additional Tables

Table 4: Model-implied Correlations - Extended Table

Capital Share	Risk Free Position	Risky Position	Risk Free Rate	Workers Share	Investment	Risk Premium
1.00	-1.00	0.90	-0.99	0.66	-0.38	0.92
-1.00	1.00	-0.91	0.99	-0.69	0.39	-0.93
0.90	-0.91	1.00	-0.95	0.93	-0.48	1.00
-0.99	0.99	-0.95	1.00	-0.77	0.42	-0.97
0.66	-0.69	0.93	-0.77	1.00	-0.48	0.90
-0.38	0.39	-0.48	0.42	-0.48	1.00	-0.47
0.92	-0.93	1.00	-0.97	0.90	-0.47	1.00

B Proofs

B.1 Profits are linear in the effective capital stock.

This result is valid for both the two-period model and the infinite-horizon economy. Recall that the technology is

$$y(s,i,k,l) = F(g_i g_s k, l) + (1-\delta) g_i g_s k_i$$

where F is homogeneous of degree one. The profits are given by

$$\pi(s,i,k,l) = \max_l \{F(g_i g_s k, l) + (1-\delta) g_i g_s k_i - \omega(s) l\}$$

Because the technology is CRS, we can write it as

$$\begin{aligned}
\pi(s,i,k,l) &= \left[\max_{\frac{l}{g_i g_s k_i}} \left\{F(1, \frac{l}{g_i g_s k_i}) + (1-\delta) - \frac{\omega(s) l}{g_i g_s k_i}\right\}\right] g_i g_s k \\
&= \left[\max_{\tilde{l}} \left\{F(1,\tilde{l}) + (1-\delta) - \omega(s)\tilde{l}\right\}\right] g_i g_s k \\
&= r(\omega(s)) g_s g_i k \\
&= r(s) g_i k_i
\end{aligned}$$

where we have defined

$$r(\omega(s)) \equiv \left[\max_{\tilde{l}} \left\{F(1,\tilde{l}) + (1-\delta) - \omega(s)\tilde{l}\right\}\right]$$

Thus, the shock to capital, including the effect in depreciation, renders the problem linear in individual capital and the idiosyncratic shock. As mentioned before, the gross return $r(s)$ includes the depreciation rate. The net return on capital is $r^n(s) \equiv r(\omega(s)) - (1-\delta)$.

B.2 Proof of Proposition 1

For part (a), we first characterize the equilibrium of the two-period model as the functions $\{\phi(s), \hat{p}(s)\}$ where $\hat{p}(s) = p(s)g_s$ is defined as growth-adjusted Arrow-Debreu prices. We then characterize the welfare shares in the equilibrium. Finally, we solve for asset prices and quantities.

Part (a): Asset Prices and Quantities. *Step 1: Characterization of the equilibrium.* Recall the Euler equation (9):

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[(-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i)^{-\sigma}]}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\sigma}}$$

Replacing the budget constraint in the period 1 by the individual's consumption, dividing all the components in period 1 by Y_1, cancelling $Y_2(s)$ in the right-hand side of (9), and using the definition of $\hat{p}(s)$, the last equation can be written as

$$\left(\frac{y_1 + \sum_s \hat{p}(s)\phi(s)}{1 - y_1 - \sum_s \hat{p}(s)\phi(s)}\right)^{-\sigma} = \frac{\mathbb{E}_i[(-\phi(s) + \alpha(s)g_i)^{-\sigma}]}{(\phi(s) + (1-\alpha(s)))^{-\sigma}}$$

where $y_1 = \alpha_1 + E_1/Y_1$ is the share of resources in hands of entrepreneurs in period 1. Using the definition of consumption equivalent in (11), and taking into account that the Arrow-Debreu prices satisfy $p(s) = \Pi(s)\frac{\mathbb{E}_i u'(e_2(s,i))}{u'(e_1)}$, we can characterize the equilibrium as the functions $\{\phi(s), \hat{p}(s)\}$ satisfying

$$\frac{y_1 + \sum_s \hat{p}(s)\phi(s)}{1 - y_1 - \sum_s \hat{p}(s)\phi(s)} = \frac{-\phi(s) + \alpha(s)g^{ce}(s)}{\phi(s) + (1-\alpha(s))} \tag{42}$$

$$\hat{p}(s) = \Pi(s)\left(\frac{y_1 + \sum_s \hat{p}(s)\phi(s)}{-\phi(s) + \alpha(s)g^{ce}(s)}\right)^{\sigma} g_s^{1-\sigma} \tag{43}$$

Step 2: Welfare shares. Notice that $\sum_s \hat{p}(s)\phi(s)$ are the total entrepreneur's savings in units of time 1 output. Thus $y_1 + \sum_s \hat{p}(s)\phi(s)$ is the normalized consumption of the capitalist. With these definitions, the total economy's wealth in units of period 1 output is $1 + \sum_s \hat{p}(s)$. Similarly, the worker's present value of resources, normalized by Y_1, is

$1-y_1+\sum_s \hat{p}(s)(1-\alpha(s))$. Therefore, the equilibrium initial wealth ratio is

$$x_1 = \frac{1-y_1+\sum_s \hat{p}(s)(1-\alpha(s))}{1+\sum_s \hat{p}(s)}. \tag{44}$$

Step 3: A system for $p(s)$ and $\phi(s)$. First, we derive the equation for $p(s)$. To simplify notation, define

$$P = \sum_s \hat{p}(s) \in \mathbb{R}_+; \quad \text{and} \quad P_\phi = \sum_s \hat{p}(s)\phi(s) \in \mathbb{R}$$

Operating with equation (42) we can write

$$\phi(s) = \alpha(s)g^{ce}(s)[1-y_1-P_\phi] - (1-\alpha(s))[y_1+P_\phi]; \;\; \forall s \tag{45}$$

Multiplying the last by $\hat{p}(s)$ and adding up, we obtain

$$P_\phi = \sum_s \hat{p}(s)\alpha(s) - P[y_1+P_\phi] + \sum_s \hat{p}(s)\alpha(s)[g^{ce}(s)-1][1-y_1-P_\phi]$$

Adding y_1 in both sides of the last equation and reorganizing generates

$$\begin{aligned} y_1 + P_\phi &= \frac{y_1+\sum_s \hat{p}(s)\alpha(s)}{1+P} + \frac{\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s)-1][1-y_1-P_\phi]}{1+P} & (46) \\ &= 1-x_1+\hat{c}\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s)-1] & (47) \end{aligned}$$

where $\hat{c} = \frac{1-y_1-P_\phi}{1+P} \geq 0$. Hence, equations (45) and (47) generate the solution for the quantities of transacted assets.

Step 4: Asset Prices. To solve for the prices, equation (45) can be rewritten as

$$[y_1+P_\phi][(1-\alpha(s))+\alpha(s)g^{ce}(s)] = -\phi(s)+\alpha(s)g^{ce}(s)$$

Using the last relationship in the price equation (43) and recalling that $\hat{p}(s) = p(s)g_s$, we obtain

$$p(s) = \Pi(s)\left(1+\alpha(s)[g^{ce}(s)-1]\right)^{-\sigma} g_s^{-\sigma} \tag{48}$$

Step 5: Asset Quantities. For $\phi(s)$, reorganize (45) as follows:

$$\phi(s) = \alpha(s)g^{ce}(s) - (y_1+P_\phi) + \alpha(s)[1-g^{ce}(s)][y_1+P_\phi]$$

Plugging (47) into the last

$$\phi(s) = \alpha(s)g^{ce}(s) + [\alpha(s)(1 - g^{ce}(s)) - 1][1 - x_1 + \hat{c}\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1]]$$

Define

$$\Gamma(g^{ce}) := -\hat{c}\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1]] > 0$$

This holds because $\mathbb{V}ar(g_i) > 0$ implies that $g^{ce}(s) < 1$ for all s, then

$$\phi(s) = \alpha(s)g^{ce}(s) + [\alpha(s)(1 - g^{ce}(s)) - 1][1 - x_1 - \Gamma(g^{ce})]$$

$$\phi(s) = x_1 - (1 - \alpha(s)) + \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce}) \tag{49}$$

which is (13) in Proposition 1.

Part (b): Evolution of the Wealth Shares. Since the second period is the last one, the wealth ratio in that period equals the income ratio, which is $x_2(s) = \phi(s) + 1 - \alpha(s)$. Plugging (49) into this expression and substracting x_1 gives

$$x_2(s) - x_1 = \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce})$$

which is (14) that describes the evolution of wealth shares.

Part (c): Precautionary Savings. Recall that $y_1 + P_\phi$ is the capitalist's normalized consumption and also its consumption share out of output. When $\mathbb{V}ar(g_i) = 0$, i.e., $g^{ce} = 1; \forall s$, from (47), we have $y_1 + P_\phi = 1 - x_1$ so that the consumption share is equal to the wealth ratio. When $\mathbb{V}ar(g_i) > 0$, we have $g^{ce} < 1; \forall s$. Therefore, $y_1 + P_\phi < 1 - x_1$. Due to the presence of uninsured idiosyncratic risk, capitalists consume a smaller proportion of their wealth, hence they save more. Using (47), we can state this formally. Let $\sum_s p(s)\phi(s)$ and $\sum_s p(s)^{CM}\phi(s)^{CM}$ be P_ϕ in $\mathbb{V}ar(g_i) > 0$ and $\mathbb{V}ar(g_i) = 0$ cases respectively. Then

$$\sum_s p(s)\phi(s) - \sum_s p(s)^{CM}\phi(s)^{CM} = \hat{c}\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1] < 0$$

which proves part (c) of the proposition.

B.3 Proof of Proposition 2

Recall that the equation for asset quantities from Proposition 1 is

$$\phi(s) = x_1 - (1-\alpha(s)) + \alpha(s)(g^{ce}(s)-1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce}) \tag{50}$$

In equilibrium $A^w + A^e = 0$ and $B^w + B^e = 0$. There is a one-to-one mapping between period 2's payoffs of the AD securities $\phi(s)$ and the payoffs of a portfolio with the following assets:

$$\phi(L)Y_2(L) = R_L B^w + A^w \alpha(L) Y_2(L)$$
$$\phi(H)Y_2(H) = R_L B^w + A^w \alpha(H) Y_2(H)$$

The latter implies positions and prices given by

$$A^w = \frac{\phi(H)Y_2(H) - \phi(L)Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{51}$$
$$R_L B^w = \frac{Y_2(L)Y_2(H)(\phi(L)\alpha(H) - \alpha(L)\phi(H))}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{52}$$
$$R_L = \frac{1}{\sum_s p(s)} \tag{53}$$
$$P_A = \sum_s p(s)\alpha(s)Y_2(s) \tag{54}$$

Where $p(s)$ is the price of the AD securities, then both the worker and the capitalist are optimizing. Evaluating (50) in high and low states and plugging the respective values into their respective places in (51) and (52), we have

$$A^w = 1 - \left(\frac{Y_2(H) - Y_2(L)}{\pi_2(H) - \pi_2(L)}\right)(1-x_1) + x_1\left[\frac{\alpha(H)Y_2(H)g^{CE}(H) - \alpha(L)Y_2(L)g^{CE}(L)}{\pi_2(H) - \pi_2(L)} - 1\right] + \mathbf{\Xi} \tag{55}$$

where $\mathbf{\Xi} = \Gamma \times \frac{Y_2(H)\left(\alpha(H)\left(g^{CE}(H)-1\right)+1\right) - Y_2(L)(\alpha(L)\left(g^{CE}(L)-1\right)+1)}{\pi_2(H)-\pi_2(L)}$.

$$R_L B^w = -\left(\frac{\alpha(H) - \alpha(L)}{\pi_2(H) - \pi_2(L)}\right) Y_2(L)Y_2(H)(1-x_1) - \frac{Y_2(L)Y_2(H)\alpha(H)\alpha(L)}{\pi_2(H) - \pi_2(L)}(g^{CE}(H) - g^{CE}(L))x_1 + \mathbf{\Psi} \tag{56}$$

where $\mathbf{\Psi} = \Gamma \times \frac{\alpha(H)\alpha(L)\left(g^{CE}(L)-g^{CE}(H)\right)+\alpha(H)-\alpha(L)}{(Y_2(L)Y_2(H))^{-1}(\pi_2(H)-\pi_2(L))}$.

B.4 Lemma 1

Lemma 1 *If $\frac{A_1}{Y_1} \geq \alpha_1$ then $x_1^{IM} \leq x_1^{CM}$.*

Proof. Recall that:

$$x_1^{IM} = \frac{A_1 + (1-\alpha_1)Y_1 + \sum_s p(s)(1-\alpha(s))Y_2(s)}{Y_1 + \sum_s p(s) Y_2(s)} \tag{57}$$

$$= 1 - \frac{\sum_s p(s)\alpha(s) g_s + \alpha_1 - A_1/Y_1}{1 + \sum_s p(s) g_s}. \tag{58}$$

Define,

$$D(s) : = \left[1 + \alpha(s)\left(g^{ce}(s) - 1\right)\right]^{-\sigma}. \tag{59}$$

Then, from Proposition 1, $p(s)$ can be written as $p(s) = \Pi(s)D(s)g_s^{-\sigma}$. As a result, we can rewrite (58) as:

$$x_1^{IM} = 1 - \frac{\sum_s \Pi(s)D(s)\alpha(s)g_s^{1-\sigma} + \alpha_1 - A_1/Y_1}{1 + \sum_s \Pi(s)D(s)g_s^{1-\sigma}}. \tag{60}$$

Under complete markets, $(g^{ce}(s) - 1)$, which implies that $D(s) = 1\ \forall s$. Therefore, (60) is now:

$$x_1^{CM} = 1 - \frac{\sum_s \Pi(s)\alpha(s)g_s^{1-\sigma} + \alpha_1 - A_1/Y_1}{1 + \sum_s \Pi(s)g_s^{1-\sigma}}. \tag{61}$$

The sign of $x_1^{IM} - x_1^{CM}$ is determined by:

$$\begin{aligned}&\sum_s \Pi(s)(1-D(s))\alpha(s)g_s^{1-\sigma} + [\sum_s \Pi(s)(D(s)-1)g_s^{1-\sigma}](\alpha_1 - \frac{A_1}{Y_1}) + \\ &\quad (\sum_s \Pi(s)D(s)g_s^{1-\sigma})(\sum_s \Pi(s)\alpha(s)g_s^{1-\sigma}) - (\sum_s \Pi(s)g_s^{1-\sigma})(\sum_s \Pi(s)D(s)\alpha(s)g_s^{1-\sigma}).\end{aligned} \tag{62}$$

The first term is negative since $D(s) \geq 1\ \forall s$. The second term is negative as well because $\alpha_1 \leq \frac{A_1}{Y_1}$ and $D(s) \geq 1\ \forall s$. Note that third and fourth terms can be rewritten as

$$\mathbb{E}_s[D(s)g_s^{1-\sigma}]\mathbb{E}_s[\alpha(s)g_s^{1-\sigma}] - \mathbb{E}_s[g_s^{1-\sigma}]\mathbb{E}_s[D(s)\alpha(s)g_s^{1-\sigma}].$$

Note that since both $\alpha(s)$ and g_s are exogenous, they are independent, which makes $\mathbb{E}_s[\alpha(s)g_s^{1-\sigma}] = \mathbb{E}_s[\alpha(s)]\mathbb{E}_s[g_s^{1-\sigma}]$. With this, we can rearrange the above expression to get

$$\mathbb{E}_s[g_s^{1-\sigma}](\mathbb{E}_s[D(s)g_s^{1-\sigma}]\mathbb{E}_s[\alpha(s)] - \mathbb{E}_s[D(s)\alpha(s)g_s^{1-\sigma}]).$$

The term inside the parentheses is $-Cov(\alpha(s), D(s)g_s^{1-\sigma})$ which is negative by the definition of $D(s)$, which makes the second line in (62) negative. Therefore, $x_1^{IM} - x^{CM} \leq 0$ which is the desired result, as we wanted to show. It is important to note that $\alpha_1 \leq \frac{A_1}{Y_1}$ is not a necessary but sufficient condition.

$\square$

B.5 Capital Share in the CES Production Function

The firms maximizes $\pi(s,i) = \left[\alpha\left(g_i g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - \omega L$, which implies $L^d(s,i) = \alpha^{\frac{\rho}{\rho-1}}\left[\left(\frac{\omega}{1-\alpha}\right)^{\rho-1} - (1-\alpha)\right]^{\frac{\rho}{1-\rho}} g_i g_s k$. From the labor market clearing condition $1 = L^s = L^d(s) = \mathbb{E}(L^d(s,i))$ we obtain the following wage:

$$\omega(s) = (1-\alpha)\left[\alpha\left(g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)\right]^{\frac{1}{\rho-1}}$$

Moreover recall that:

$$\alpha(s,i) = \frac{\partial y(s,i)}{\partial k}\frac{k}{y(s,i)} = \frac{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)(L)^{\frac{\rho-1}{\rho}}}$$

so $\alpha(s) = \mathbb{E}_i(\alpha(s,i))$ is given by:

$$\alpha(s) = \frac{\alpha(g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

and, given that $Y(s) = \mathbb{E}(y(s,i))$, in the same way, the labor share is:

$$(1-\alpha(s)) = \frac{\partial Y}{\partial L}\frac{L}{Y} = \frac{(1-\alpha)}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

Then $\omega(s) = (1-\alpha(s))Y(s)$. Given the wage, $L^d(s,i) = g_i$ and therefore $\pi(s,i) = \alpha(s)Y(s)g_i$.

C Numerical Appendix

a. Using $g_s k$, compute once and for all $Y(s)$, $r(s)$ and $w(s)$.

b. Step A: Guess and Solve. Guess the functions $\{\ p(s'|s), \nu(s), \frac{R^{\frac{1}{\gamma}}}{1+Prem(s)}, \Pi(s'|s)\ \}$.

   We start assuming a constant investment, a uniform distribution for Π, and we assume that $p(s'|s) = \beta\Pi(s'|s)\left[\frac{Y(s')}{Y(s)}\right]^{-\gamma}$.

   Using equations (25), (32) and

$$\tilde{\beta}\left(s',s\right) = \frac{W^T(s')/W^T(s)}{\left(\frac{\zeta(s)(1-\zeta(s'))}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\zeta(s)x + \left(\frac{\beta^e}{\beta}\right)^{\frac{(1-\gamma)}{\gamma(1-\sigma)}}\left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\mathbb{R}(s'|s)^{\frac{1}{\gamma}}}$$

   we obtain the savings rates for the consumer, the entrepreneur, and $\tilde{\beta}\left(s',s\right)$. To this end, from equation (110) in the Online Appendix E, the level of wealth consistent with the equilibrium savings ratios and investment choice can be written as

$$W^T(s) = \frac{Y(s)}{(1-\zeta(s))x + [1-\vartheta(s)(1-\nu(s))](1-x)}$$

   Thus, using the last two equations, we simultaneously solve for ϑ and ζ. We do so by appealing to their recursivity. We guess initial function $\vartheta(s) = \zeta(s) = \beta$ and iterate until convergency. We obtain ϕ^w and ϕ^e from (24) and (31).

c. Step B: Update $\{\ p(s'|s), \nu(s), \frac{R(s'|s)^{\frac{1}{\gamma}}}{1+Prem(s)}, \Pi(s'|s)\ \}$. To update $\nu(s)$, we obtain investment k',

$$k'(s) = \sum_{s'|s}\frac{p\left(s' \mid s\right)\left[\alpha\left(s'\right)+(1-\delta)g_{s'}\right]y\left(s'\right)}{(1+\text{Prem}(s))}$$

   and the implied $\nu(s)$ from (30). To obtain $Prem(s)$, we use (41), and to update $\mathbb{R}\left(s'|s\right)$, we use

$$\mathbb{R}\left(s'|s\right) = 1 + \left[\frac{\gamma(1+\gamma)\left(v(s)r\left(s'\right)\right)^2\operatorname{Var}\left(g_i\right)}{2o\left(s',1;\phi',v\right)^2}\right]$$

   The transition matrix $\Pi(s'|s)$ is then given by (39) and (40). We do so by computing the linear interpolation basis of k' in the k grid, which together with the transition probabilities of the exogenous shock generates a stochastic matrix.

Finally, we have $\tilde{\beta}\left(s', s\right)$, and by its definition

$$p(s'|s) = \beta^{\frac{(1-\gamma)}{(1-\sigma)}} \Pi(s'|s) \tilde{\beta}(s', s)^{-\gamma}$$

We use this equation to update the price function.

d. We iterate steps A and B until convergence.

Online Appendix to "The Macroeconomics of Hedging Income Shares"

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D Financial Contract

D.1 Optimal Contract

In this section, we provide the main idea for the optimal contract. Suppose that there is one period, and a risk-neutral principal that can provide insurance to the entrepreneurs. Suppose there are three possible idiosyncratic shocks $g_L < g_M < g_H$, with probability p_i.[30] At the beginning of the period, before knowing the realization of g_i, the firm can enter an insurance contract with the financial intermediary. The firm's profits are $\alpha Y g_i$. In absence of insurance, the entrepreneur's utility is

$$\mathbb{E}_i[u(e_i)] = \mathbb{E}_i[u(\alpha Y g_i)]$$

The principal (financial intermediary) can sign a contract and offer insurance to the entrepreneur.

Full Insurance. In the benchmark when g_i is observable, the contract is simple: the principal "buys" all the proceeds of the production with a lump sum payment of J. Then, after the shock is realized, the entrepreneur hands over the profits to the principal. Because the principal must break even, it must be that $\mathbb{E}_i(\alpha Y g_i) - J = 0$. Thus, the utility of the entrepreneur in this case with full insurance is

$$\mathbb{E}_i[u(e_i^O)] = E_i[u(J)] = u(\mathbb{E}_i[\alpha Y g_i]) > \mathbb{E}_i[u(\alpha Y g_i)] = \mathbb{E}_i[u(e_i)]$$

Moral Hazard. However, the entrepreneur is subject to the moral hazard problem, because g_i *is not observable*. The entrepreneur can report an alternative value of g_i, say $g_{i'}$, and keep the difference for herself. Therefore, any contract adds a constraint so that the entrepreneur reveals the true realization of g_i (incentive compatibility). But, transforming these "stolen" profits into consumption is not free. Each unit of stolen profit transforms

[30] The contract with three shocks can be generalized to any finite numbers of shocks. With only two shocks, the results might not generalize to more states.

into consumption at the rate $0 \leq \psi \leq 1$. Thus, when the entrepreneur steals profits, she obtains an additional consumption of only $\psi\alpha Y(g_i - g_{i'})$. To force truthful revelation, the principal must hand over additional payments d_i contingent on the realization of g_i. Since the entrepreneur will not lie in equilibrium, her consumption is $e_i^C = J + d_i$, while because the principal must break even, the contract must also satisfy $\mathbb{E}_i(\alpha Y g_i - d_i) - J = 0$, where we normalize the outside option of the principal to zero without loss of generality. As a result, the optimal contract solves

$$\max_{\{J,d_i\}} \mathbb{E}_i u(J + d_i)$$

$$\text{st.} \quad \psi\alpha Y(g_i - g_{i'}) + d_{i'} + J \leq J + d_i; \quad \forall i, i'$$

$$\mathbb{E}_i(\alpha Y g_i - d_i) - J = 0$$

The first set of constraints are the *incentive compatibility* (IC), or truth-telling, constraints. Only the adjacent constraints matter. To see this, consider that the entrepreneur would never lie when she observes the low shock. So, only the following can be binding:

$$\begin{aligned}
\psi\alpha Y(g_H - g_M) + d_M &\leq d_H \\
\psi\alpha Y(g_M - g_L) + d_L &\leq d_M \\
\psi\alpha Y(g_H - g_L) + d_L &\leq d_H
\end{aligned}$$

Adding the first two inequalities,

$$\begin{aligned}
\psi\alpha Y(g_H - g_M) + d_M + \psi\alpha Y(g_M - g_L) + d_L \leq &\quad d_H + d_M \\
\psi\alpha Y(g_H - g_L) + d_L \leq &\quad d_H
\end{aligned}$$

Thus, the third constraint is irrelevant. In general, this is a version of the single crossing property; it can be generalized to any arbitrary number of idiosyncratic shocks. Rewriting the problem, we have

$$\max_{\{J,d_i\}} \sum_i p_i u(J + d_i)$$

$$\text{st.} \quad \psi\alpha Y(g_H - g_M) + d_M \leq d_H$$

$$\psi\alpha Y(g_M - g_L) + d_L \leq d_M$$

$$\sum_i p_i(\alpha Y g_i - d_i) - J = 0$$

Let λ be the multiplier in the break-even constraint and μ_i the multiplier in each incentive compatibility. Taking first-order conditions,

$$\begin{aligned}
\sum_i p_i u'(J+d_i) &= \lambda \\
\gamma_L u'(J+d_L) &= \gamma_L \lambda + \mu_M \\
\gamma_M u'(J+d_M) &= \gamma_M \lambda + \mu_H - \mu_M \\
\gamma_H u'(J+d_H) &= \gamma_H \lambda - \mu_H
\end{aligned}$$

Clearly, $\mu_L = \mu_H = 0$ cannot be a solution because it violates the IC constraints. Now, suppose $\mu_M = 0$, while $\mu_H > 0$. Then it must be that $d_L = d_M$. If $d_L > d_M$, the IC constraint implies

$$\psi \alpha Y(g_M - g_L) + d_L - d_M < 0$$

which is a contradiction. If $d_L < d_M$, a small increase in d_L accompanied by a small reduction on d_M, keeping the break-even constraint satisfied, generates a welfare change of

$$\gamma_L d_L[u'(e_L) - u'(e_M)] > 0$$

which is true because $u''(.) < 0$ and $e_L < e_M$, thus increasing welfare. A similar argument can be used to show that $\mu_M > 0$ and $\mu_H = 0$ is not possible either. As a result, because μ_M and μ_H are both strictly positive, we must have

$$\begin{aligned}
\psi \alpha Y(g_H - g_M) &= d_H - d_M \\
\psi \alpha Y(g_M - g_L) &= d_M - d_L
\end{aligned}$$

Clearly, $d_i = \psi \alpha Y g_i$, together with $J = (1-\psi)\mathbb{E}_i(\alpha Y g_i)$, is a solution for all the equations. And since the problem has a unique solution, it must be the solution. This contract can be interpreted as an equity contract. Each entrepreneur sells a share $1-\psi$ of her firm to the intermediary and uses the proceeds to buy an indexed stock market financial instrument. This completely smooths out a proportion $(1-\psi)$ of the idiosyncratic risk. However, to prevent stealing, not all the shares can be sold; the entrepreneur must retain a proportion ψ of her shares, which is her "skin in the game." This is the best insurance possible with only short-term contracts. Here, we assume that there was no aggregate risk. This result would not be affected by it, since it would affect all the IC constraints proportionally. It would only change the pricing of J.

D.2 Constrained Efficiency

In this section, we show that the equilibrium in the two-period model is constrained-efficient. The notion of constrained efficiency follows Stiglitz (1982) and Geanakoplos and Polemarchakis (1986); it provides the planner with the same instruments as the market. In particular, the planner can intervene, redistributing consumption across aggregate states with a lump-sum transfer $T(s)$. Consumption for the worker and the capitalist are given by

$$\begin{aligned} c_2(s) &= T(s) + (1-\alpha(s))Y_2(s) \\ e_2(s,i) &= -T(s) + \alpha(s) g_i Y_2(s). \end{aligned}$$

Without loss of generality, and to follow the notation of this paper, we define

$$T(s) := \frac{\phi(s)}{Y_2(s)}$$

Planning Program. The planner solves

$$\max_{\{e_1, c_1, \phi(s), c_2(s)\}_{s \in S}} \frac{e_1^{1-\gamma}}{1-\gamma} + \mathbb{E}_{i,s} \frac{e_2(s,i)^{1-\gamma}}{1-\gamma}$$

$$c_1 + e_1 = Y_1 \tag{63}$$

$$c_2(s) + e_2(s) = Y_2(s) \tag{64}$$

$$c_2(s) = \phi(s) Y_2(s) + (1-\alpha(s))Y_2(s) \tag{65}$$

$$e_2(s,i) = -\phi(s) Y_2(s) + \alpha(s) g_i Y_2(s) \tag{66}$$

$$e_2(s) = \mathbb{E}_i e_2(s,i) \tag{67}$$

$$\frac{c_1^{1-\gamma}}{1-\gamma} + \mathbb{E}_s \frac{c_2(s)^{1-\gamma}}{1-\gamma} \geq \underline{u} \tag{68}$$

for all (s,i) . Equations (63) and (64) are the resource constraints for periods 1 and 2. Equations (65) and (66) pin down consumption for the consumer and the entrepreneur in period 2. The last constraint maps the Pareto frontier. Lets rewrite the program in terms of consumption of the entrepreneur, as follows:

$$\max_{\{e_1, \phi(s)\}_{s \in S}} \frac{e_1^{1-\gamma}}{1-\gamma} + \mathbb{E}_{i,s} \frac{\left(-\phi(s) Y_2(s) + \alpha(s) g_i Y_2(s)\right)^{1-\gamma}}{1-\gamma}$$

$$\frac{(Y_1 - e_1)^{1-\gamma}}{1-\gamma} + \mathbb{E}_s \frac{(\phi(s)Y_2(s) - \alpha(s)Y_2(s))^{1-\gamma}}{1-\gamma} \geq \underline{u}$$

The first-order conditions are

$$\begin{aligned} e_1 &: e_1^{-\gamma} - \lambda (Y_1 - e_1)^{-\gamma} = 0 \\ \phi(s) &: -Y_2(s)\Pi(s)\mathbb{E}_i (-\phi(s)Y_2(s) + \alpha(s)g_i Y_2(s))^{-\gamma} \\ &\quad + \lambda Y_2(s)\Pi(s) (\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\gamma} = 0 \end{aligned}$$

Thus, for every state, the ratio of consumption is equal to

$$\frac{e_1^{-\gamma}}{c_1^{-\gamma}} = \frac{\mathbb{E}_i (-\phi(s)Y_2(s) + \alpha(s)g_i Y_2(s))^{-\gamma}}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\gamma}}$$

This is exactly the same relative allocation of consumption. The level of consumption will depend on the reservation utility $\underline{u}$. Thus, for a particular choice of $\underline{u}$, we can recover the allocation of the competitive equilibrium.

Efficiency. We summarize the discussion in the following proposition. A competitive equilibrium is Pareto-efficient if there exists some $\underline{u}^*$ such that the allocation of the competitive equilibrium for a given initial distribution of wealth (α_1, ϕ_1)[31] is equal to the solution of the planning problem for a level of reservation utility $\underline{u}^*$.

Proposition 4. *The competitive equilibrium with idiosyncratic risk is the solution of the planning problem when*

$$\underline{u} = \frac{\left(Y_1 - e_1^{IM}(\alpha_1, \phi_1)\right)^{1-\gamma}}{1-\gamma} + \mathbb{E}_s \frac{\left(\phi^{IM}(s)(\alpha_1, \phi_1) Y_2(s) - \alpha(s)Y_2(s)\right)^{1-\gamma}}{1-\gamma} \tag{69}$$

Thus, the competitive equilibrium is constrained-efficient.

The proof is immediate. For the level $\underline{u}$ defined in (69), the participation constraint of the consumer holds with equality, and the allocation of incomplete markets meets all of the first-order conditions. When that the planner has more instruments, in particular, it can perfectly control consumption, then the planning problem will be the same one as the complete-markets allocation. Thus, we need to choose which problem to focus on. This result is coming from the fact that in the two-period problem we are not microfounding

[31] This pins down the initial assets of each agent and their initial income (which depends on the capital and labor share).

the amount of idiosyncratic risk that the entrepreneurs face. This is also the case in the infinite horizon.

E General Model With Epstein-Zin Preferences.

In this appendix, we characterize the equilibrium with Epstein-Zin preferences in detail. These equations support the expressions presented in Section 3.

E.1 Worker's Problem

Any representative worker solves

$$V^w(a;s) = \max_{\{c,a(s'|s)\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \left(\mathbb{E}_{s'} V^w \left(a\left(s' \mid s\right)\right)^{\frac{1-\gamma}{1-\sigma}} \right)^{\frac{1-\sigma}{1-\gamma}} \right\}$$

$$c + \sum_{s'} p\left(s' \mid s\right) a\left(s' \mid s\right) \le a(s) + \omega(s)$$

We guess and verify that the solution has the following structure:

$$\begin{aligned}
V^w(a;s) &= \frac{\left(A(s)(a+\omega(s)+h(s))\right)^{1-\sigma}}{1-\sigma}, &(70)\\
c(a;s) &= (1-\zeta(s))(a+\omega(s)+h(s)), &(71)\\
h(s) &= \sum_{s'|s} p\left(s' \mid s\right)\left[\omega\left(s'\right)+h\left(s'\right)\right], &(72)\\
a\left(s' \mid s\right) &= \phi^w\left(s' \mid s\right)\zeta(s)[a+\omega(s)+h(s)] - \omega\left(s'\right) - h\left(s'\right). &(73)
\end{aligned}$$

Recall that we have defined $W^w(s) = a + \omega(s) + h(s)$ as the worker's total wealth. Using (73), we can get the following law of motion for the worker's total wealth

$$W^w(s') = \phi^w(s'|s)\zeta(s)W(s). \tag{74}$$

The unknowns in (70)-(73) are:

$$\boxed{A(s), \phi\left(s' \mid s\right), \zeta(s)}.$$

Once we have these functions we can obtain $\{c, a\left(s' \mid s\right)\}$. We start by getting an expression for $A(s)$ by using the envelope theorem. From the envelope theorem, it holds that:

$$
\begin{aligned}
V_1^w(a;s) =& U'(c(s)), \\
A(s)(A(s)W^w(s))^{-\sigma} =& \left[(1-\zeta(s))W^w(s)\right]^{-\sigma}, \\
A(s)^{1-\sigma} =& (1-\zeta(s))^{-\sigma}.
\end{aligned}
$$

The last equation implies $A(s)^{1-\gamma} = (1-\zeta(s))^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}$.

To derive the expressions for $\phi^w(s'|s)$ and $\zeta(s)$, we first derive a useful relationship between them. We take the first-order condition in the workers problem with respect to $a(s'|s)$, which gives

$$
\begin{aligned}
p\left(s' \mid s\right)(c(s))^{-\sigma} =& \beta\left(\mathbb{E}_{s'}\left[\left(A(s')(a\left(s' \mid s\right) + \omega(s' \mid s) + h(s' \mid s))\right)^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}-1} \times \\
& \Pi(s' \mid s)\left[A(s')(a\left(s' \mid s\right) + \omega(s' \mid s) + h(s' \mid s))\right]^{-\gamma} A(s').
\end{aligned}
$$

From the definition of $W^w(s)$, we have that:

$$
p\left(s' \mid s\right)(c(s))^{-\sigma} = \beta\left(\mathbb{E}_{s'}\left[\left(A(s')W^w(s')\right)^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}-1} \Pi(s' \mid s)\left[A(s')W^w(s')\right]^{-\gamma} A(s'). \tag{75}
$$

Plugging in the guessed decision rules (71)-(74), rearranging terms, (75) becomes

$$
\left(\frac{1-\zeta(s)}{\zeta(s)}\right)^{-\sigma} p(s'|s) = \beta\Big(\mathbb{E}_{s'}[(A(s')\phi^w(s'|s))^{1-\gamma}]\Big)^{\frac{1-\sigma}{1-\gamma}-1} \Pi(s'|s)A(s')^{1-\gamma}\phi^w(s'|s)^{-\gamma}. \tag{76}
$$

Multiplying both sides with $\phi^w(s'|s)$ and summing over s', we get

$$
\left(\frac{(1-\zeta(s))}{\zeta(s)}\right)^{-\sigma} \sum_{s'} p\left(s' \mid s\right)\phi^w(s' \mid s) = \beta\left(\mathbb{E}_{s'}\left[\left(A(s')\phi^w(s' \mid s)\right)^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}}. \tag{77}
$$

Consider $\sum_{s'} p\left(s' \mid s\right)\phi^w(s' \mid s)$. Multiplying both sides of (73) with $p(s'|s)$ and summing over s' we get

$$
\sum_{s'} p(s'|s)a(s'|s) = \sum_{s'} p(s'|s)\phi^w(s'|s)\zeta(s)W^w(s) - \sum_{s'} p(s'|s)(\omega(s') + h(s')).
$$

The second term in the right-hand side is $h(s)$ by definition. When we sum the last equation with (71), we get

$$
c(s) + \sum_{s'} p(s'|s)a(s'|s) = (1-\zeta(s))W^w(s) + \sum_{s'} p(s'|s)\phi^w(s'|s)\zeta(s)W^w(s) - h(s).
$$

By the budget constraint of the consumer, the left-hand side is equal to $a(s)+\omega(s)$. Plugging this, rearranging terms, and recalling the definition of $W^w(s)$, we have

$$W^w(s) = \Big(1-\zeta(s)+\zeta(s)\sum_{s'} p(s'|s)\phi^w(s'|s)\Big)W^w(s).$$

This holds for all s if and only if, $\sum_{s'} p(s'|s)\phi^w(s'|s) = 1$. Using this in (77), we get a useful relationship between $\phi^w(s'|s)$ and $\zeta(s)$:

$$\left(\frac{(1-\zeta(s))}{\zeta(s)}\right)^{-\sigma} = \beta\left(\mathbb{E}_{s'}\left[\left(A(s')\phi^w(s'\mid s)\right)^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}}. \tag{78}$$

To derive an expression for $\phi^w(s'|s)$, we rewrite (76):

$$\left(\frac{1-\zeta(s)}{\zeta(s)}\right)^{-\sigma} p(s'|s) = \beta^{\frac{1-\gamma}{1-\sigma}}\beta^{\frac{\gamma-\sigma}{1-\sigma}}\left(\mathbb{E}_{s'}[(A(s')\phi^w(s'|s))^{1-\gamma}]\right)^{\frac{1-\sigma}{1-\gamma}-1}\Pi(s'|s)A(s')^{1-\gamma}\phi^w(s'|s)^{-\gamma}. \tag{79}$$

Then, raise both sides of (78) to the power of $\frac{\gamma-\sigma}{1-\sigma}$, and plug to (79), which generates:

$$\phi^w(s'\mid s)^\gamma = \left(\frac{\zeta(s)}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}\tilde{\beta}(s',s)^\gamma(1-\zeta(s'))^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}, \tag{80}$$

$$\phi^w(s'\mid s) = \left[\left(\frac{\zeta(s)}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}\tilde{\beta}(s',s)^\gamma(1-\zeta(s'))^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}\right]^{\frac{1}{\gamma}}, \tag{81}$$

where $\tilde{\beta}(s',s) = \frac{\beta^{\frac{(1-\gamma)}{\gamma(1-\sigma)}}\Pi(s'|s)^{\frac{1}{\gamma}}}{p(s'|s)^{\frac{1}{\gamma}}}$, which is (24) in Section 3.1. For CRRA preferences and $\sigma=\gamma$, $\tilde{\beta}(s',s)$ simplifies to:

$$\tilde{\beta}(s',s) = \hat{\beta}(s',s) = \left(\frac{\beta\Pi(s'\mid s)}{p\,(s'\mid s)}\right)^{\frac{1}{\gamma}}.$$

Lastly, to derive an expression for $\zeta(s)$, rearrange terms in (80) to obtain:

$$\left(\frac{(1-\zeta(s))}{\zeta(s)}\right)^{\frac{1-\gamma}{1-\frac{1}{\sigma}}} = \tilde{\beta}(s',s)^\gamma(1-\zeta(s'))^{\frac{1-\gamma}{1-\frac{1}{\sigma}}}\phi^w(s'\mid s)^{-\gamma}. \tag{82}$$

As the objective is to solve for savings ratios, we would like to write a linear equation. To do so, in (82), we multiply both sides by $\phi^w(s'|s)^\gamma$ to the left-hand side, raise both sides to the power of $\frac{1}{\gamma}$, multiply both sides by $p(s'|s)$, sum over s', and recall that $\sum_{s'} p(s'|s)\phi^w(s'|s) = 1$ to get:

$$\frac{1}{\zeta(s)} = 1 + \left[\sum_{s'} p(s'|s) \tilde{\beta}(s',s)(1-\zeta(s'))^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}} \right]^{\frac{\gamma(1-\frac{1}{\sigma})}{1-\gamma}},$$

which is (25) in Section 3.1.

E.2 Capitalists' Problem

Our aim is to derive the risk-adjustment factor and risk premium terms in equations (33)-(35), together with the equations themselves. To that end, we will, as in the workers' problem, adopt a guess-and-verify approach.

Step 1: Setup of the Problem and Guesses. The problem of the entrepreneur is given by

$$\begin{aligned} V^e(E,k,i;s) = \max_{\{e,E(s'|s),k'\}} & \left\{ \frac{e^{1-\sigma}}{1-\sigma} + \beta \left(\mathbb{E}_{s',i'} V^e \left(E',k',s',i' \right)^{\frac{1-\gamma}{1-\sigma}} \right)^{\frac{1-\sigma}{1-\gamma}} \right\} \\ s.t. \quad & e + k' + \sum_{s'} p(s'|s)E(s'|s) \leq E(s) + g_i R(s) k; \qquad \forall i,i',s,s' \end{aligned}$$

We will make the following guesses for the value function and the decision rules:

$$\begin{aligned} V^e(E,k,i;s) &= \frac{(B(s)W^e(s,i,k))^{1-\sigma}}{1-\sigma}, & (83) \\ e(E,k,i;s) &= (1-\vartheta(s))W^e(s,i,k), & (84) \\ k'(E,k,i;s) &= \nu(s)\vartheta(s)W^e(s,i,k), & (85) \\ E(s' \mid s) &= \phi^e(s' \mid s)E_1(s,i), & (86) \end{aligned}$$

where $B(s)$ is the marginal utility of wealth, $W^e(s,i,k) = E(s) + R(s)g_i k$ is the entrepreneur's total wealth, $\vartheta(s)$ is the entrepreneur's saving rate, $\nu(s)$ is the portion of savings invested in capital, and $E_1(s,i) = \sum_s p(s'|s)E(s'|s)$, total market value of the entrepreneur's savings for the next period in the form of Arrow securities.

With these guesses, the problem becomes

$$\begin{aligned} V^e(E,k,i;s) = \max_{\{e,E(s'|s),k'\}} & \left\{ \frac{(e^{1-\sigma}}{1-\sigma} + \frac{\beta}{1-\sigma} \left[\mathbb{E}_{s',i'} \left\{ \left(B(s')(E(s' \mid s) + R(s' \mid s)g_i' k') \right)^{1-\gamma} \right\} \right]^{\frac{1-\sigma}{1-\gamma}} \right\} \\ s.t. \quad & e = E(s) + R(s)kg_i - k' - \sum_{s'} p\left(s' \mid s\right) E\left(s' \mid s\right) \end{aligned}$$

Step 2: First Order Conditions and Laws of Motion. We will take the first-order conditions with respect to $E(s' \mid s)$ and k'. The former is given by

$$E(s' \mid s) : p(s'|s)e^{-\sigma} = \beta \left[\mathbb{E}_{s',i'}[(B(s')W^e(s',i',k'))^{1-\gamma}]\right]^{\frac{\gamma-\sigma}{1-\gamma}} \times \\ \Pi(s'|s)B(s')^{1-\gamma}W^e(s',i',k')^{-\gamma}; \;\; \forall s' \quad (87)$$

By increasing (decreasing) the amount of Arrow securities, we decrease (increase) consumption today, but we increase (decrease) the payoff in the future in state s'. The right hand side is the marginal change in the continuation utility. In the case of risk neutrality, the correction of idiosyncratic risk is equal to one. The first-order condition with respect to capital, k', is given by

$$k' : \quad e^{-\sigma} = \beta \left[\mathbb{E}_{s',i'}[(B(s')W^e(s',i',k'))^{1-\gamma}]\right]^{\frac{\gamma-\sigma}{1-\gamma}} \times \\ \mathbb{E}_{s',i'}[B(s')^{1-\gamma}W^e(s',i',k')^{-\gamma}R(s'|s)g_{i'}]. \quad (88)$$

Before manipulating these first-order conditions, we will need a law of motion for total wealth of the entrepreneur. To get it, using (84) and (85), we first observe

$$\begin{aligned} E_1(s,i) &\equiv \sum_{s'} p\left(s' \mid s\right) E\left(s' \mid s\right) \\ &= E(s) + R(s)kg_i - k' - e \\ &= W^e(s,i,k) - \nu(s)\vartheta(s)W^e(s,i,k) - (1-\vartheta(s))W^e(s,i,k) \\ &= \vartheta(s)(1-\nu(s))W^e(s,i,k), \end{aligned}$$

which, by (86), gives

$$E(s'|s) = \phi^e(s'|s)\vartheta(s)(1-\nu(s))W^e(s,i,k). \quad (89)$$

Define

$$o(s',i;\phi^e(s'|s),\nu(s)) = \phi^e(s' \mid s)(1-\nu(s)) + R(s'|s)g_i\nu(s). \quad (90)$$

Plugging (90) to (89), rearranging terms, and remembering the definition of $W^e(s,i,k)$, we can get a low of motion for the entrepreneur's total wealth

$$W^e(s',i',k') = \vartheta(s)o(s',i;\phi^e(s'|s),\nu(s))W^e(s,i,k). \quad (91)$$

This still holds even in the case of Epstein-Zin preferences.

Step 3: Defining risk-related terms and deriving (35).

Step 3.a: Defining $\mathbb{R}(s', s)$. To derive $\mathbb{R}(s', s)$, we take the second-order Taylor approximation of $o(s', i; \phi^e(s'|s), \nu(s))$ around $g_i = 1$, and take expectation of both sides of the expansion with respect to i to get

$$\begin{aligned}\mathbb{E}_i\left(\left[o(s', i; \phi^e(s'|s), \nu(s))\right]^{-\gamma}\right) &= \mathbb{E}_i\left(\left[\phi^e(s' \mid s)(1-\nu(s)) + R(s'|s)g_i\nu(s)\right]^{-\gamma}\right) \\ &= \left[\phi^e(s' \mid s)(1-\nu(s)) + R(s'|s)\nu(s)\right]^{-\gamma}\mathbb{R}(s', s) \\ &= o(s', 1; \phi^e(s'|s), \nu(s))^{-\gamma}\mathbb{R}(s', s),\end{aligned}$$

where we define $\mathbb{R}(s', s)$ as

$$\mathbb{R}(s', s) := \left[1 + \frac{\gamma(1+\gamma)\left(\nu(s)R\left(s'|s\right)\right)^2 Var\left(g_i\right)}{2o\left(s', 1; \phi^e(s'|s), \nu(s)\right)^2}\right]. \tag{92}$$

Step 3.b: Deriving (35). Now, to derive , add up over s' equation (87) and equalize to (88) to get

$$\mathbb{E}_{s',i'}\left[B(s')^{1-\gamma}W^e(s', i', k)^{-\gamma}\left(R(s'|s)g_i - \frac{1}{\sum_{s'} p(s'|s)}\right)\right] = 0.$$

Use (91):

$$\mathbb{E}_{s',i}\left[B(s')^{1-\gamma}o(s', i; \phi^e(s'|s), \nu(s))^{-\gamma}\left(R(s'|s)g_i - \frac{1}{\sum_{s'} p(s'|s)}\right)\right] = 0. \tag{93}$$

This equation determines the investment ratio $\nu(s)$. As in the worker's problem, we can use the envelope theorem to obtain $B(s')$:

$$\begin{aligned}V'(W^e(s, i, k)) =& U'(e), \\ B(s)^{1-\gamma} =& (1-\vartheta(s))^{\frac{1-\gamma}{1-1/\sigma}}.\end{aligned} \tag{94}$$

Plugging (94) to (93) we get

$$\mathbb{E}_{s',i}\left[(1-\vartheta(s'))^{\frac{1-\gamma}{1-1/\sigma}}\left[o(s', i; \phi^e(s'|s), \nu(s))\right]^{-\gamma}\left(R(s'|s)g_{i'} - \frac{1}{\sum_{s'} p(s'|s)}\right)\right] = 0, \tag{95}$$

which is (35).

Step 3.c: Deriving $Prem(s)$. To simplify (93), we first take the expectation of both sides with respect to i. Then we use the second-order Taylor approximation of $o(s', i; \phi^e(s'|s), \nu(s))^{-\gamma}g_i$ around $g_i = 1$ which gives, for $o(.)$, we suppress the dependence on $\phi^e(s'|s)$ and $\nu(s)$ for ease of notation,

$$\mathbb{E}_i[o(s',i)^{-\gamma}g_i] = \mathbb{E}_i\left[o(s',1)^{-\gamma}\left(\mathbb{R}(s',s) - \gamma\frac{\nu(s)R\left(s'|s\right)Var\left(g_i\right)}{o\left(s',1\right)}\right)\right].$$

Plugging this back to (95), and expanding the expectation with respect to s', we get

$$\sum_{s'}\Pi(s'|s)B(s')^{1-\gamma}o(s',1)^{-\gamma}R(s' \mid s)\left(\mathbb{R}(s',s) - \gamma\frac{\nu(s)R\left(s'|s\right)Var\left(g_i\right)}{o\left(s',1\right)}\right) = \frac{1}{\sum_{s'}p(s'|s)}\sum_{s'}\Pi(s'|s)B(s')^{1-\gamma}o(s',1)^{-\gamma}\mathbb{R}(s',s).$$

Also, plugging (84) and (91) to (87), taking the expectation of both sides with respect to i', using the very first Taylor expansion, and arranging terms, we have

$$o(s',1)^{-\gamma}B(s')^{1-\gamma} = \left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{-\sigma}\frac{p(s'|s)}{\beta\Pi(s'|s)}\left[\mathbb{E}_{s',i}[(B(s')o(s',i))^{1-\gamma}]\right]^{-\frac{\gamma-\sigma}{1-\gamma}}\mathbb{R}(s',s)^{-1}. \quad (96)$$

Combining the two and simplifying, we obtain

$$\sum_{s'}p(s'|s)R(s'|s)\left(1-\gamma\frac{\nu(s)R\left(s'|s\right)Var\left(g_i\right)}{o\left(s',1\right)\mathbb{R}(s',s)}\right) = 1, \quad (97)$$

which is the equation for the risk premium in the case of Epstein-Zin preferences.

This implies a capitalist's growth rate of wealth because it gives

$$\sum_{s'}p(s'|s)R(s'|s) = 1 + \sum_{s'}p(s'|s)R(s'|s)\gamma\frac{\nu(s)R\left(s'|s\right)\mathbb{V}ar\left(g_i\right)}{o\left(s',1\right)\mathbb{R}(s',s)}.$$

By multiplying both sides by $\nu(s)$, adding 1 to both sides, and rearranging terms, this can also be written as

$$1-\nu(s)+\nu(s)\sum_{s'}p(s'|s)R(s' \mid s) = 1 + Prem(s),$$

where we define $Prem(s)$ as

$$Prem(s) := \sum_{s'}p(s'|s)\left(\gamma\frac{\nu(s)^2R\left(s'|s\right)^2\mathbb{V}ar\left(g_i\right)}{o\left(s',1\right)\mathbb{R}(s',s)}\right). \quad (98)$$

Step 3.d: Further results on $Prem(s)$. In particular, the above relationship implies

$$\sum_{s'} p(s'|s)o(s',1) = \sum_{s'} p(s'|s)\left(\phi^e(s' \mid s)(1-\nu(s)) + R(s'|s)\nu(s)\right). \tag{99}$$

Now, consider $\sum_s p(s'|s)\phi^e(s'|s)$. Multiplying both sides of (89) by $p(s'|s)$, and summing over s', we get

$$\sum_{s'} p(s'|s)E(s'|s) = \vartheta(s)(1-\nu(s))W^e(s,i,k)\sum_{s} p(s'|s)\phi^e(s'|s).$$

At the same time, by the entrepreneur's budget constraint, (84)-(85), and the definition of $W^e(s,i,k)$ we have

$$\sum_{s'} p(s'|s)E(s'|s) = W^e(s,i,k) - (1-\vartheta(s))W^e(s,i,k) - \nu(s)\vartheta(s)W^e(s,i,k),$$

which implies

$$\vartheta(s)(1-\nu(s)) = \vartheta(s)(1-\nu(s))\sum_{s} p(s'|s)\phi^e(s'|s).$$

This holds if and only if $\sum_s p(s'|s)\phi^e(s'|s) = 1$. Using this, (99) can be rewritten as

$$\sum_{s'} p(s'|s)o(s',1) = 1-\nu(s) + \nu(s)\sum_{s'} p(s'|s)R(s' \mid s) = 1 + Prem(s). \tag{100}$$

Step 4: Deriving (33). Having found an equation for the $Prem(s)$, we now want to find the equations for $\vartheta(s), \phi^e(s' \mid s)$, that is (33) and (34) respectively. We start with $\vartheta(s)$. First, reorganize (96) to get

$$\beta\frac{\Pi(s'|s)}{p(s'|s)}B(s')^{1-\gamma}o(s',1)^{-\gamma}\left(\mathbb{E}_{s',i}\left[(B(s')o(s',i))^{1-\gamma}\right]\right)^{\frac{\gamma-\sigma}{1-\gamma}} = \left(\frac{1-\vartheta(s)}{\vartheta(s)}\right)^{-\sigma}\mathbb{R}(s',s)^{-1}. \tag{101}$$

Take a second-order Taylor approximation of the term $o(s',i)^{1-\gamma}$, take expectation of both sides of the expansion with respect to i to get

$$\mathbb{E}_i\left(\left[o(s',i)\right]^{1-\gamma}\right) = \left[\phi^e(s' \mid s)(1-\nu(s)) + R(s'|s)\nu(s)\right]^{1-\gamma}R_1(s',s),$$

where

$$R_1(s',s) = 1 - \gamma(1-\gamma)\frac{(\nu(s)R(s'|s))^2}{2o(s',1)^2}\mathbb{V}ar(g_i).$$

Raising both sides of (101) to the power of $\frac{1-\gamma}{\gamma-\sigma}$, using the previous expansion, then raising both sides to the power of $\frac{\gamma-\sigma}{1-\sigma}$, multiplying both sides by $p(s'|s)o(s',1)R_1(s',s)$, and adding over s′, we obtain

$$\beta\left[\mathbb{E}_{s'}[[B(s')o(s',1)]^{1-\gamma}R_1(s',s)]\right]^{\frac{1-\sigma}{1-\gamma}} = \left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{-\sigma}\sum_{s'}p(s'|s)o(s',1)\frac{R_1(s',s)}{\mathbb{R}(s',s)}. \tag{102}$$

Note that using the definitions of $R_1(s',s)$ and $\mathbb{R}(s',s)$, we can write

$$\frac{R_1(s',s)}{\mathbb{R}(s',s)} = 1 - \gamma\frac{(\nu(s)R(s'|s))^2}{o(s',1)^2}\frac{\mathbb{V}ar(g_i)}{\mathbb{R}(s',s)}.$$

Then, using this and (100), (102) becomes

$$\beta\left[\mathbb{E}_{s'}[[B(s')o(s',1)]^{1-\gamma}R_1(s',s)]\right]^{\frac{1-\sigma}{1-\gamma}} = \left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{-\sigma}\left[1+\widetilde{Prem}(s)\right], \tag{103}$$

where

$$\widetilde{Prem}(s) = Prem(s) - \gamma Var(g_i)\sum_{s'}\frac{p(s'|s)}{\mathbb{R}(s',s)}\frac{(\nu(s)R(s'|s))^2}{o(s',1)}.$$

However, since $Prem(s)$ is exactly the last term, we have $\widetilde{Prem}(s) = 0$.

Raise both sides of (103) to the power of $\frac{\gamma-\sigma}{1-\sigma}$ to have

$$\beta^{\frac{\gamma-\sigma}{1-\sigma}}\mathbb{E}_{s'}\left[[B(s')o(s',1)]^{1-\gamma}R_1(s',s)\right]^{\frac{\gamma-\sigma}{1-\sigma}} = \left(\frac{1-\vartheta(s)}{\vartheta(s)}\right)^{-\sigma\frac{(\gamma-\sigma)}{1-\sigma}},$$

and plug this and (94) to (101) to get

$$(1-\vartheta(s'))^{\frac{1-\gamma}{1-1/\sigma}}\frac{\beta^{\frac{(1-\gamma)}{(1-\sigma)}}\Pi(s'|s)}{p(s'|s)}\mathbb{R}(s',s) = o(s',1)^{\gamma}\left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{\frac{1-\gamma}{1-1/\sigma}}. \tag{104}$$

Multiplying both sides by $p(s'|s)$, using (100) and the definition of $\tilde{\beta}(s',s)$, adding up over

s', we get

$$\sum_{s'} p(s'|s)\tilde{\beta}(s',s)(1-\vartheta(s'))^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\mathbb{R}(s'|s)^{\frac{1}{\gamma}} = (1+Prem(s))\left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}.$$

We can rewrite this as

$$\left(\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}} = \sum_{s'} p(s'|s)\tilde{\beta}(s',s)(1-\vartheta(s'))^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\frac{\mathbb{R}(s'|s)^{\frac{1}{\gamma}}}{1+Prem(s)} \tag{105}$$

which has the same form as with CRRA preferences; it is the same as in the draft when $\gamma = \sigma$. We can rewrite the latter equation as

$$\frac{1}{\vartheta(s)} = 1 + \left[\sum_{s'} p(s'|s)\tilde{\beta}(s',s)(1-\vartheta(s'))^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\frac{\mathbb{R}(s'|s)^{\frac{1}{\gamma}}}{1+Prem(s)}\right]^{\frac{\gamma(1-1/\sigma)}{1-\gamma}},$$

which is (33).

Step 5: Deriving (34). To recover $\phi^e(s'|s)$, hence to derive (34), we use (104) writing it as

$$(1-\nu(s))\phi^e(s'|s) = \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\tilde{\beta}(s',s)\mathbb{R}(s'|s)^{\frac{1}{\gamma}} - \nu(s)R(s'|s). \tag{106}$$

Plugging this to (90), we get (34) when $g_{i'} = 1$:

$$o(s',1) = \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\tilde{\beta}(s',s)\mathbb{R}(s'|s)^{\frac{1}{\gamma}}. \tag{107}$$

E.3 Equilibrium and Prices

Remember that we have defined, $W^T(s) = \int_i W^e(s,i,k)dH(i) + W^w(s)$, as the total wealth in the economy, with $G(.)$ being marginal cumulative distribution function of g_i, and $x = \frac{W^w(s)}{W^T(s)}$. We will use these, together with market clearing conditions, in order to derive the equilibrium prices and the laws of motion that help characterize the transition probabilities, $\Pi(s'|s)$.

Step 1: Deriving $p(s'|s)$. **Asset-market** clearing condition is

$$a(s'|s) + E(s'|s) = 0 \quad \forall s, s'. \tag{108}$$

Plugging in the decision rules derived above and using the definition of x, it reads

$$\phi^w(s'|s)\zeta(s)x + \phi^e(s'|s)[\vartheta(s)(1-\nu(s))](1-x) = \frac{\omega(s') + h(s')}{W^T(s)};\ \forall s, s' \tag{109}$$

We also have the **goods-market clearing** to check

$$\begin{aligned} c(s) + e(s) + k'(s) = &\quad y(s);\ \ \forall s \\ (1-\zeta(s))W^w(s) + (1-\vartheta(s))W^e(s) + \vartheta(s)\nu(s)W^e(s) = &\quad y(s);\ \ \forall s \end{aligned}$$

$$(1-\zeta(s))x + [1-\vartheta(s)(1-\nu(s))](1-x) = \tfrac{y(s)}{W^T(s)};\ \forall s \tag{110}$$

We use the asset-market clearing to find the prices. Recall that the equation for $\phi^e(s'|s)$ is

$$\phi^e(s'|s) = \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}} \tilde{\beta}(s', s)\frac{\mathbb{R}(s', s)^{\frac{1}{\gamma}}}{(1-\nu(s))} - \frac{\nu(s)R(s'|s)}{(1-\nu(s))}.$$

The equivalent expression for the worker is given by

$$\phi^w(s' \mid s) = \left(\frac{\zeta(s)(1-\zeta(s'))}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}} \tilde{\beta}(s', s).$$

Using both in (109), we get an equation for the prices.

$$\frac{p(s'|s)}{\beta^{\frac{(1-\gamma)}{(1-\sigma)}}\Pi(s'|s)} = \left\{ \frac{\left(\frac{\zeta(s)(1-\zeta(s'))}{(1-\zeta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\zeta(s)x + \left(\frac{\vartheta(s)(1-\vartheta(s'))}{(1-\vartheta(s))}\right)^{\frac{1-\gamma}{\gamma(1-1/\sigma)}}\mathbb{R}(s'|s)^{\frac{1}{\gamma}}\vartheta(s)(1-x)}{R(s'|s)\nu(s)\vartheta(s)(1-x) + \frac{\omega(s')+h(s')}{W^T(s)}} \right\}^{\gamma}. \tag{111}$$

Now consider the denominator. Multiplying it by $W^T(s)$, and adding $a(s'|s) + E(s'|s)$; which, by (108) is 0, we get

$$\underbrace{R(s'|s)\nu(s)\vartheta(s)(1-x)W^T(s) + E(s'|s)}_{=\int_{i'} W^e(s', i', k')dH(i') = W^e(s')} + \underbrace{a(s'|s) + \omega(s') + h(s')}_{=W^w(s')} = W^T(s'). \tag{112}$$

Therefore,

$$R(s'|s)\nu(s)\vartheta(s)(1-x) + \frac{\omega(s') + h(s')}{W^T(s)} = \frac{W^T(s')}{W^T(s)}.$$

Plugging to (111), we get

$$\frac{p(s'|s)}{\beta^{\frac{1-\gamma}{1-\sigma}}\Pi(s'|s)} =$$
$$\left[\left(\frac{\zeta(s)(1-\zeta(s'))}{1-\zeta(s)}\right)^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}}\zeta(s)x+\left(\frac{\vartheta(s)(1-\vartheta(s'))}{1-\vartheta(s)}\right)^{\frac{1-\gamma}{\gamma(1-\frac{1}{\sigma})}}\vartheta(s)\mathbb{R}(s',s)^{\frac{1}{\gamma}}(1-x)\right]^{\gamma}\left(\frac{W^T(s)}{W^T(s')}\right)^{\gamma}, \tag{113}$$

which is (38) in Section 3.3.

Step 2: Deriving (39)-(40). Since it is only the entrepreneurs who invest in capital, next period's aggregate capital is determined only by the entrepreneurs' investments in the current period. Namely,

$$\begin{aligned} K'(s) &= \int_i k'(s,i)dH(i); && (114)\\ &= \int_i \nu(s)\vartheta(s)W^e(s,i,k)dH(i); && (115)\\ &= \nu(s)\vartheta(s)(1-x)W^T(s). && (116) \end{aligned}$$

Since $x(s'|s) = \frac{W^w(s')}{W^T(s')}$, plugging in the law of motion for the wealth of the worker derived in the workers' problem, we get

$$x(s'|s) = \phi^w(s'|s)\zeta(s)\frac{W^T(s)}{W^T(s')}x; \tag{117}$$

which, together with (116), gives (39)-(40), laws of motion that describe the transition probabilities $\Pi(s'|s)$.

F Correlation Between Risk-Premium and GDP

In this section, we compute the correlations, including transition paths, when the economy starts from a very low wealth ratio x. To be precise, we start with $x_0 = 0.37$. Table 5 shows that the average moments are similar to the main calibration in Section 4.2. Thus, the slight differences with that section are due to the inclusion of the transition path in the computation of the moments. For this computation, we simulate for only $1,500$ periods. This relatively "short" sample allows us to approximate the average moments in Table 2, while simultaneously maximizing the impact of the transition path.

Table 6 shows the observed correlations. Notice that correlation between $\alpha(s)$ and the

Table 5: Averages, Including Transition Paths

δ	α_k	$\mathbb{E}(\alpha(s))$	$\mathbb{V}ar(\alpha(s))$	$\frac{K}{Y}$	$\mathbb{E}(x)$	A	B	σ	γ	Premium
0.07	0.265	0.374	0.0067	2.812	0.815	0.776	-1.607	2.0	5.0	0.0527

risk premium is reduced from 0.92 in Table 3 to 0.02 in this exercise. Since a perfect positive correlation exists between α and GDP (by construction), this also implies a reduced correlation between the risk premium and GDP. This exercise is maximizing the impact of the wealth effects described in Section 3.5. Because the starting x is significantly low and distant from the stationary one ($x_0 = 0.37$ vs. $E(x_t) = 0.83$), during the transition x_t is mostly increasing. The speed of convergence is notably slow. If we were to reduce the sample to 100 periods, the correlation would definitively be negative, but then the averages would be off.

Table 6: Implied Correlations With Low x_0

α_t	B_t	A_t	R_t	Risk Premium
1.00	-0.99	-0.22	-0.88	0.02
-0.99	1.00	0.15	0.90	-0.08
-0.22	0.15	1.00	-0.28	0.97
-0.88	0.90	-0.28	1.00	-0.50
0.02	-0.08	0.97	-0.50	1.00

G Proof of Proposition 3

G.1 Proof of Part (a)

When capitalists can fully insure their idiosyncratic risk, $m(s) = \frac{\mathbb{R}(s',s)^{\frac{1}{\sigma}}}{1+Prem(s)} = 1$ for all s. Then from equations (25) and (33), it follows that $\zeta(s) = \vartheta(s)$. The wealth ratio's law of motion is given by $x(s'|s) = W^w(s'|s)/W^T(s'|s)$. Using the laws of motion of each agent's wealth, $x(s'|s)$ can be written as

$$x(s'|s) = \frac{\phi^w(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i;\phi^e)\vartheta(s)(1-x) + \phi^w(s'|s)\zeta(s)x},$$

where from equation (34), $\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}$ satisfies

$$\left(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}\right)^{-\frac{1}{\sigma}} = \tilde{\beta}(s', s)\frac{(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))}; \ \forall s, s'.$$

When all idiosyncratic risk is insured, $\mathbb{E}_i o(s', i; \phi^e)^{-\sigma} = [\mathbb{E}_i o(s', i; \phi^e)]^{-\sigma}$, thus using this and $\zeta(s) = \vartheta(s)$ from the last equation together with (24), we obtain

$$\mathbb{E}_i o(s', i; \phi^e) = \frac{\tilde{\beta}(s', s)(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))} = \frac{\tilde{\beta}(s', s)(1-\zeta(s))}{\zeta(s)(1-\zeta(s'))} = \phi^w(s'|s); \ \forall s, s'.$$

Using the last equation in the first delivers the result.

G.2 Proof of Part (b)

Assume that $\delta = 1$ and guess an equilibrium with constant x. To do this, assume that capitalists have a different discount rate β^e. We will pick its value to make sure that x is constant. Since we are assuming $\gamma = \sigma$, we use a guess-and-verify strategy, guessing that prices and the risk adjustment factor satisfy

$$p(s'|s) = A_0 \beta \Pi(s'|s)\tilde{g}(s'|s)^{-\sigma} \qquad \textit{and} \qquad m(s) = m; \qquad \forall s, s' \tag{118}$$

for some constants $A_0 > 0$, $m \geq 1$; $\quad \forall x$, and $\tilde{g}(s'|s) = \frac{Y(s')}{Y(s)}$. Later we verify this guess. We prove this proposition in a series of steps showing that (1) the savings rates are independent of the state, (2) holdings of contingent assets are proportional to growth, (3) the investment rate and portfolio allocations are constant, and (4) the wealth growth rates are independent of the state. In the final steps, we verify the guesses in (118).

Step 1: Savings rates are independent of aggregate shock. Using the guessed prices in the definition of $\tilde{\beta}(s'|s)$, we obtain

$$\tilde{\beta}(s'|s) = \left[\frac{\beta \Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} = \left[\frac{\beta \Pi(s'|s)}{A_0 \beta \Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}}\right]^{1/\sigma} = \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}},$$

and

$$\tilde{\beta}^e(s'|s) = \left[\frac{\beta^e \Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} = \left[\frac{\beta^e \Pi(s'|s)}{A_0 \beta \Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}}\right]^{1/\sigma} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}}.$$

Guess that the savings rates are constant. Plugging the latter expression in equation (25), together with $\gamma = \sigma$, and the guessed price imply that the solution for the worker's saving rate is

$$\zeta(s') = \zeta(s) = \zeta = \beta \frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}; \qquad \forall s, s'.$$

Doing similar calculations with (33), we obtain

$$\vartheta(s) = \vartheta = \beta \frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{m A_0^{\frac{1-\sigma}{\sigma}}} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}; \qquad \forall s, s'. \tag{119}$$

Step 2: AD securities are proportional to $\tilde{g}(s'|s)$. From equation (24) with $\gamma = \sigma$ and the computed value of $\tilde{\beta}(s'|s)$, we obtain

$$\phi^w(s'|s) = \frac{\tilde{g}(s'|s)}{\zeta A_0^{1/\sigma}} = \frac{\tilde{g}(s'|s)}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}.$$

The equivalent condition for the capitalist (see equation (106) in the Online Appendix) generates

$$\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \frac{\tilde{g}(s'|s)}{\vartheta A_0^{1/\sigma}} = \frac{\tilde{g}(s'|s) m}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}} \qquad \forall s, s'. \tag{120}$$

This implies that $\phi^w(s'|s) = \frac{\tilde{\beta}(s'|s)}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}$ and $\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = \frac{\tilde{\beta}(s'|s) m}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}$.

The capitalist's growth rate of wealth is $o(s', i; \phi^e) \equiv [(1 - \nu(s))\phi^e(s'|s) + \nu(s) r(s') g_i]$. Define the portfolio allocation as

$$\frac{(1 - \nu(s))\phi^e(s'|s)}{\nu(s) r(s')} = \frac{1}{D},$$

also guessing that D is constant. Then, arranging terms in (34), plugging it to (33), arranging terms in (33), and given that $\sum_{s'} p(s'|s)\phi^e(s'|s) = 1$, we have

$$m(s) = \sum_{s'|s} p(s'|s) \left(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}\right)^{-1/\sigma} = (1 - \nu(s)) \left[\mathbb{E}_i (1 + D g_i)^{-\sigma}\right]^{-1/\sigma}. \tag{121}$$

Also rearranging terms in the definition of D, we get

$$\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = (1 - \nu(s))\phi^e(s'|s) \left[\mathbb{E}_i (1 + D g_i)^{-\sigma}\right]^{-1/\sigma} = \frac{\tilde{\beta}(s'|s) m(s)}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}.$$

Because of (121), the latter equation implies $\phi^e(s'|s) = \phi^w(s'|s), \quad \forall s, s'$.

Step 3: Investment rates are constant. Using portfolio choice D in equation (35),

$$\mathbb{E}_{s',i|s}\left[\left[(1-\nu(s))\phi^e(s'|s)(1+Dg_i)\right]^{-\sigma}\left(\frac{(1-\nu(s))\phi^e(s'|s)}{\nu(s)}Dg-\frac{1}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{-\sigma}}\right)\right]=0.$$

Since i is independent of s, we can write

$$\mathbb{E}_{s'|s}\left[\frac{\left[(1-\nu(s))\phi^e(s'|s)\right]^{1-\sigma}}{\nu(s)}\right]\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right)= \frac{\mathbb{E}_{s'|s}\left[(1-\nu(s))\phi^e(s'|s)\right]^{-\sigma}\mathbb{E}_i(1+Dg_i)^{-\sigma}}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{-\sigma}};$$

$$\frac{(1-\nu(s))}{\nu(s)}\mathbb{E}_{s'|s}[\phi^e(s'|s)]^{1-\sigma}\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right)=\frac{\mathbb{E}_{s'|s}\left[\phi^e(s'|s)\right]^{-\sigma}\mathbb{E}_i(1+Dg_i)^{-\sigma}}{\beta A_0\mathbb{E}_{s'}\tilde{g}(s'|s)^{-\sigma}}.$$

Given that we have $\phi^e(s'|s)=\phi^w(s'|s)$, plugging in the expression we have found for $\phi^w(s'|s)$ above, we obtain

$$\frac{\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right)}{\mathbb{E}_i(1+Dg_i)^{-\sigma}}=\frac{\nu}{(1-\nu)}. \tag{122}$$

So ν is constant whenever D is constant. Also, with D and ν we can compute the value of $m(s)$ given by equation (121), which confirms that m is constant. To solve for D, use the portfolio equation to get

$$\frac{(1-\nu)}{\nu}D=\frac{r(s')}{\phi^e(s'|s)}=\frac{r(s')}{\phi^w(s'|s)}=\frac{r(s')}{\tilde{g}(s'|s)}\beta A_0\mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}.$$

With an AK model, $r(s')$ is exogenous, so the above equation pins down D, which will not depend on x. In a more general setting, $r(s')$ would depend on aggregate capital. With constant shares and CRS technology, $r(s')=\alpha\frac{y(s')}{K'}$, which can be written as $r(s')=\alpha\tilde{g}(s'|s)\frac{y(s)}{K'}$. But since the capital law of motion is $K'=\vartheta\nu(1-x)W^T(s)$, we can write the previous equation as

$$r(s')=\frac{\tilde{g}(s'|s)}{\beta A_0\mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}\frac{(1-\nu)}{\nu}D,$$

$$\alpha\tilde{g}(s'|s)\frac{y(s)}{\vartheta\nu(1-x)W^T(s)}=\frac{\tilde{g}(s'|s)}{\beta A_0\mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}\frac{(1-\nu)}{\nu}D,$$

$$\frac{\alpha}{\vartheta(1-x)}\frac{y(s)}{W^T(s)}=\frac{(1-\nu)D}{\beta A_0\mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}.$$

Now replacing ϑ from the previously found value,

$$\frac{\alpha m A_0^{\frac{1}{\sigma}}}{(1-x)}\frac{y(s)}{W^T(s)} = (1-\nu)D\left(\frac{\beta^e}{\beta}\right)^{1/\sigma},$$

$$\frac{\alpha\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{-1/\sigma} A_0^{\frac{1}{\sigma}}}{(1-x)}\frac{y(s)}{W^T(s)} = D\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}, \tag{123}$$

where in the last step we have replaced m from equation (121). This equation solves for D, then (122) delivers ν, and then with (121) we obtain m. All these variables are constant if (1) α is constant and (2) the ratio $\frac{y(s)}{W^T(s)}$ is constant. What we need in general is that this solution is independent of the aggregate shock; it could depend on x or k, as long as it does it in a deterministic way. Here, we are considering the case of x constant to simplify the calculations. We consider these cases in the following extensions of this proposition.

Step 4: The GDP-to-wealth ratio is constant. The human capital $h(s)$ can be written per unit of output. Defining $\tilde{h}(s) = \frac{h(s)}{Y(s)}$, it follows that

$$\tilde{h}(s) = \sum_{s'|s} p(s'|s)\left([1-\alpha(s')+\tilde{h}(s')]\tilde{g}(s'|s)\right).$$

Using the latter equation, we can write

$$\frac{\omega(s')+h(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{[1-\alpha(s')+\tilde{h}(s')]}{R(s)(K/Y)+(1-\alpha(s))+\tilde{h}(s)}. \tag{124}$$

(118) implies that $\sum_{s'|s} p(s'|s) = \beta A_0 \mathbb{E}\tilde{g}(s'|s)^{-\sigma}$. Using equation (124) with constant shares, we obtain

$$\frac{\omega(s')+h(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{[1-\alpha(s')+\tilde{h}(s')]}{1+\tilde{h}(s)} = \tilde{g}(s'|s)\frac{(1-\alpha)[1+G(s')]}{1+(1-\alpha)G(s)},$$

where

$$G(s) = \sum_{s'|s} p(s'|s)\left[(1+G(s'))\tilde{g}(s'|s)\right] \tag{125}$$

is the present value of a constant dividend unit with growth factor $\tilde{g}(s'|s)$. The last equality holds because with constant shares $\tilde{h}(s) = (1-\alpha)G(s)$. Similarly:

$$\frac{W^T(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{1+(1-\alpha)G(s')}{1+(1-\alpha)G(s)}.$$

Because the distributions of growth rates are independent of the state and using the pricing function, (125) has a fixed point, that is $G(s') = G(s) = G,\ \forall s, s'$. Then it follows that

$$\frac{W^T(s')}{W^T(s)} = \tilde{g}(s'|s); \qquad \frac{\omega(s') + h(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1+(1-\alpha)G}.$$

Also,

$$\frac{y(s)}{W^T(s)} = \frac{y(s)}{y(s)+h(s)} = \frac{1}{1+\tilde{h}(s)} = \frac{1}{1+(1-\alpha)G}. \tag{126}$$

Thus the ratio $\frac{y(s)}{W^T(s)}$ is constant and therefore also ν, D, and m are as well.

Step 5: Use feasibility to find A_0. We can use feasibility constraint in (110), which can be written as

$$\zeta(s)x + \vartheta(s)(1-\nu(s))(1-x) = 1 - \frac{y(s)}{W^T(s)}; \ \forall s \tag{127}$$

to dig further into the solutions. Firstly, replacing (121) in (119) generates

$$\vartheta(1-\nu) = \beta\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}.$$

Plugging this, the expression for ζ, and (126) in (127), we get:

$$\beta\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}\left[x + (1-x)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right] = \frac{(1-\alpha)G}{1+(1-\alpha)G}; \ \forall s. \tag{128}$$

Secondly, using the guessed function for $p(s'|s)$, we obtain the fixed point of (125):

$$G = \frac{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{1-\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}. \tag{129}$$

Hence, equation (128) generates the value of A_0.

Step 6: Verify guessed prices. For the AD prices, we compute a price equation akin to that in the Online Appendix E (see equation (111) when $\gamma = \sigma$). To make the proof self-contained, we replicate some calculations adapted to this environment. Recall that the AD securities must satisfy

$$\phi^w(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma}\frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)}; \quad \left[\mathbb{E}_i o(s',i,\phi^e)^{-\sigma}\right]^{-1/\sigma} = \left[\frac{\beta^e\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma}\frac{(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}, \forall s, s'.$$

Using the definition of D, the second equality can be written as

$$(1-\nu)\phi^e(s'|s)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{-1/\sigma} = \left[\frac{\beta^e\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}, \forall s, s'.$$

Plugging the last two in (108), we obtain

$$\left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma}\left[x+(1-x)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right] = \tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1+(1-\alpha)G}; \quad \forall s, s'.$$

Using equation (128), the above becomes

$$\left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma}\left[\frac{(1-\alpha)G}{1+(1-\alpha)G}A_0^{\frac{1-\sigma}{\sigma}}\right] = \beta\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1+(1-\alpha)G}; \quad \forall s, s'.$$

Therefore,

$$p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}\left(\beta\frac{(1+G)}{A_0^{\frac{1-\sigma}{\sigma}}G}\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\right)^{-\sigma}.$$

Using the solution for G from (129), we obtain the initially guessed price function $p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}A_0$.

Step 7: Verify that x is constant with the appropriate choice of β^e. To solve for the evolution of x, recall that

$$x(s'|s) = \frac{\phi^w(s'|s)\zeta(s)x}{\mathbb{E}_i o(s', i, s)\vartheta(s)(1-x) + \phi^w(s'|s)\zeta(s)x}.$$

Using definition of $o(s', i; \phi^e)$ and because we already showed that $\phi^e(s'|s) = \phi^w(s'|s)$, we can write the last as

$$x' = \frac{\zeta x}{[(1-\nu)(1+D)]\vartheta(1-x) + \zeta x}.$$

Now, the savings rates satisfy

$$\frac{\vartheta}{\zeta} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\frac{1}{m} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\frac{\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}}{1-\nu},$$

where in the last step we have used (121). Plugging the last expression above in the law of motion of x generates

$$x' = \frac{x}{\left(\frac{\beta^e}{\beta}\right)^{1/\sigma} (1+D) \left[\mathbb{E}_i (1+Dg_i)^{-\sigma}\right]^{1/\sigma} (1-x) + x},$$

which implies that for $x' = x$ to be true, β^e must satisfy

$$\beta^e = \beta \frac{(1+D)^{-\sigma}}{\mathbb{E}_i (1+Dg_i)^{-\sigma}}. \tag{130}$$

Because of the Jensen's inequality and the convexity of the marginal utility, so that $\mathbb{E}_i (1+Dg_i)^{-\sigma} \geq (1+D)^{-\sigma}$, it is clear that $\beta^e < \beta$. Capitalists must have a smaller discount factor, otherwise x would converge to zero. The correction in the discount factor corrects the upward drift in the capitalist's savings needs. When there is no exposure to idiosyncratic risk, there is no need for the correction.

Step 8: Check solution is correct. Alternatively, the x's law of motion is characterized by

$$x(s'|s) = \phi^w(s'|s) \zeta(s) \frac{W^T(s)}{W^T(s')} x.$$

Replacing the relationships for $\phi^w(s'|s)$, $\zeta(s)$ and the growth rate of wealth implies

$$x(s'|s) = \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}} \frac{1}{\tilde{g}(s'|s)} x.$$

Hence, if $x' = x$, it must be that $A_0 = 1$. We will check whether this holds. The value of A_0 is determined by equation (128), which, using (126), can be written as

$$\beta A_0 \frac{\mathbb{E} \tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}} \left[x + (1-x) \left[\mathbb{E}_i (1+Dg_i)^{-\sigma}\right]^{1/\sigma} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \right] = 1 - \frac{y(s)}{W^T(s)}.$$

To show that indeed $A_0 = 1$ first notice that equation (130) implies

$$\left[\mathbb{E}_i (1+Dg_i)^{-\sigma}\right]^{1/\sigma} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} = 1 - D \left[\mathbb{E}_i (1+Dg_i)^{-\sigma}\right]^{1/\sigma} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}.$$

Therefore, replacing the latter in the former,

$$\beta A_0 \frac{\mathbb{E} \tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}} \left[1 - (1-x) D \left[\mathbb{E}_i (1+Dg_i)^{-\sigma}\right]^{1/\sigma} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \right] = 1 - \frac{y(s)}{W^T(s)}.$$

Now, collecting the term $(1-x)D\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}$ in equation (123) and replacing it in the above equation, we obtain

$$\beta A_0 \frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}}\left[1-\alpha A_0^{\frac{1}{\sigma}}\frac{y(s)}{W^T(s)}\right]=1-\frac{y(s)}{W^T(s)}.$$

Therefore we have

$$\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\left[\frac{1}{A_0^{\frac{1}{\sigma}}}+\frac{(1-\alpha\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma})}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}\frac{y(s)}{W^T(s)}\right]=1.$$

Meanwhile, plugging (129) to (126) we have

$$\frac{y(s)}{W^T(s)}=\frac{1-\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{1-\alpha\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}.$$

Replacing the latter in the former,

$$\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\left[\frac{1}{A_0^{\frac{1}{\sigma}}}+\frac{1-\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}\right]=1,$$

which can only be true if $A_0=1$.