Illiquidity in Sovereign Debt Markets*

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Abstract

We study debt and default policy for sovereign countries when credit and liquidity risk are jointly determined. To account for both types of risks we focus on an economy with incomplete markets, limited commitment, and search frictions in the secondary market for sovereign bonds. We quantify the effect of liquidity on sovereign spreads, debt capacity, and welfare by performing quantitative exercises when our model is calibrated to match key features of the Argentinean default. We find that the liquidity premium is a substantial component of spreads and increases during bad times and reductions in secondary market frictions improve welfare.

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1 Introduction

Sovereign countries borrow to smooth shocks to income and tax revenue. One friction that prevents the smoothing of expenses over time and states of nature is governments' inability to commit to future debt and default policies. This lack of commitment implies that the government will repay its debts only if it is convenient to do so and will dilute debt holders whenever it sees fit. To compensate investors for bearing these risks the sovereign pays a credit risk premium that reduces the available resources for domestic consumption and can substantially increase borrowing costs during bad times. The sovereign debt literature has helped us to understand how a lack of commitment shapes the outcomes of sovereign countries from a positive point of view and what policies are desirable from a normative point of view.[1]

The recent European debt crisis, however, has underscored that decentralized markets impose additional frictions that prevent smoothing by sovereign countries. Their bonds are traded mostly in over-the-counter markets, where trading is infrequent; thus, if an investor holds a large position in a sovereign bond, it might take time to find a counterparty willing to trade at a fair price. For this reason, investors need to be compensated not only for the risk of default or dilution but also due to illiquidity, which introduces, in addition, a liquidity risk premium. This liquidity premium further reduces available resources and constrains sovereign policies. So far, the literature has been silent about this feature of sovereign borrowing. Our objective in this paper is to fill this gap by answering the following questions: How do credit and liquidity premium interact? What portions of total spreads can be explained by credit and by liquidity? What would be the welfare gains of reducing frictions in the secondary market?

Our paper contributes to the literature on sovereign borrowing in two ways. First, we propose a tractable model of sovereign borrowing in which credit and liquidity premia jointly determine borrowing and default decisions. Second, in a quantitative exploration focusing on one of the most studied cases of sovereign default, Argentina's default in 2001, we show that the liquidity premium is a substantial component of total spreads. In particular, through the lens of our model, the welfare gains from the elimination of liquidity frictions are, quantitatively, of the same order of magnitude as the gains from eliminating business cycle fluctuations.

We begin our paper by constructing a model of sovereign debt where debt and default policy take into account credit and liquidity risk. We focus on a small open economy that

[1] See for example Arellano (2008), Aguiar and Gopinath (2006), and Aguiar and Amador (2013) for a review.

borrows from international investors to smooth income shocks following the quantitative literature on sovereign debt that builds on Eaton and Gersovitz (1981). A benevolent government designs debt and default policies to maximize the utility of the households by issuing non-contingent debt. The government cannot commit to future debt and default policies and might default in some states of nature. The distinctive feature of our model, in comparison to the previous literature, is the introduction of frictions in the secondary market for sovereign bonds, following the literature on over-the-counter markets, such as Duffie et al. (2005). In our model, investors buy bonds in the primary market and can receive idiosyncratic liquidity shocks. If a shock occurs, they will bear a cost for holding the asset and therefore become natural sellers of it. Due to search frictions in the secondary market, it will take time for them to find a counterparty with whom to transact.

One of the main features of the model we propose is that default and liquidity risk will be jointly determined. On the one hand, the presence of search frictions in the secondary market introduces a liquidity premium that affects prices in the primary market, thereby affecting debt and default policies, which in turn affect the credit risk premium. On the other hand, as the credit risk premium increases, the probability of default also increases, and because investors foresee worse liquidity conditions in the future, liquidity conditions will also deteriorate. Therefore, in our model, the default and liquidity risk premia are jointly determined. This joint determination is important because it will enable us to decompose total spreads into liquidity and credit components and to study the effects on welfare of reducing liquidity frictions in the secondary market.

After building a model of sovereign borrowing in which both credit and liquidity premia constrain the choices of the government, we perform quantitative exercises to assess how much of total spreads are due liquidity frictions and what would be the welfare gains of eliminating these frictions. To do so, we calibrate the model to match key features of Argentina's default in 2001. In particular, we match debt levels, the mean and volatility of spreads, bid-ask spreads, and turnover, to counterparts in the data.

Our first quantitative finding is that the liquidity premium is a substantial component of total spreads. In particular, we find that during good times, characterized by low debt and high output, approximately 30 percent of total spreads can be explained by liquidity frictions in the case of Argentina's 2001 default. During bad times, characterized by high debt and low output, when the credit risk premium is high, the percentage of total spreads is lower. The intuition for these results is as follows. Bid-ask spreads measure an investor's loss conditional on receiving a liquidity shock. Thus, the compensation paid to investors due to liquidity risk will be a function of bid-ask spreads. During good times, investors demand a low credit risk premium, because the event of default is unlikely, but

the liquidity risk is bounded away from zero. During bad times, as the probability of default increases, investors demand a higher credit risk premium. This higher probability of default will also increase the liquidity risk premium because the liquidity of defaulted bonds is lower. However, the credit risk premium will increase faster than the liquidity risk premium, which is why during bad times the liquidity premium will decrease as a fraction of total spreads.

Our second quantitative finding is that the welfare gains from eliminating liquidity risk are substantial. To explore the size of these welfare gains, we perform an exercise following Lucas (2003). We find that the welfare loss induced by secondary market frictions is 0.17 percent in consumption equivalent terms. To put this number in perspective, given the volatility of consumption for Argentina in the period of study, a representative agent would pay 0.40 percent in consumption equivalent terms to eliminate fluctuations.

We believe that the distinction between credit and liquidity risk is important for the design of debt policies for three reasons. First, in the long run, the policies to mitigate lack of commitment differ from those to mitigate frictions in the secondary market. For example, Hatchondo and Martinez (2015) and Chatterjee and Eyigungor (2015) show that fiscal rules and covenants on debt improve welfare in models when the government lacks commitment. However, policies that would decrease the liquidity premium in the long run include the development of a centralized exchange for sovereign bond trading or increasing transparency in the secondary market, as reported in Edwards et al. (2004). Second, the policies implemented during a short-term crisis might also be different. For example, a government could use resources to repay debt or to bail-out financial institutions that hold government debt and are in distress. An alternative policy, focusing on the secondary market, would be to provide liquidity to intermediaries. Finally, we believe that that the liquidity premium has implications for debt management. By issuing debt in different currencies or with different maturities, the government caters to investors and completes the market. However, this increase in the number of assets might imply low liquidity for each one of these bonds, which in turn increases the cost of debt for the government.

Our paper connects with different strands of the literature. First, to the quantitative literature of sovereign default that follows Eaton and Gersovitz (1981). In particular, our model builds on the setting developed by Aguiar and Gopinath (2006) and Arellano (2008) to study business cycles in economies with a risk of default. These early quantitative implementations study economies with short-term debt and no recovery on default. Long-term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). In a model of long-term debt Chatterjee and Eyigungor

(2012) introduces randomization to obtain continuous policy functions. Aided with this randomization the authors prove the existence of a fixed point in the pricing operator. Endogenous recovery of defaulted debt was introduced by Yue (2010).[2] In our paper the sovereign borrows by issuing long-term bonds and after a default investors recover a fraction of their investment. The first ingredient is crucial to obtain positive liquidity premia and the second is necessary to guarantee that bonds have a positive value during default. We model long-term debt following Chatterjee and Eyigungor (2012). To keep the model numerically tractable we introduce a reduced form recovery after default and we abstract from the bargaining process; it is worth noting that our reduced form recovery after default resembles the endogenous bargaining outcome in a setup as in Yue (2010). The main difference between the present paper and the previous literature is that we introduce liquidity frictions in the market for sovereign bonds to study the positive and normative implications of liquidity risk.

Second, our paper relates to the literature on over-the-counter markets. To model secondary market frictions we build on the setting of over-the-counter markets first studied by Duffie et al. (2005). In that paper, investors hold one unit of an asset and can receive non-diversifiable liquidity shocks. Once they receive a shock, they search for a counterparty, and meet them randomly. This framework was extended by Lagos and Rocheteau (2009) to allow for arbitrary asset holdings for investors. [3] Our paper structures the debt market as in Duffie et al. (2005) but to keep the model numerically tractable we follow He and Milbradt (2013) and we do not keep track of the asset holdings of high- and low-valuation investors.

Third, our paper is closely related to models of corporate borrowing. In particular, we

[2]Recent contributions to the quantitative literature of sovereign debt are Bianchi et al. (2017), Bocola and Dovis (2016), Dovis et al. (2017), Chatterjee and Eyigungor (2015), Hatchondo et al. (2016), Roch and Uhlig (2018), Pouzo and Presno (2016), Arellano and Bai (2014), Arellano and Bai (2017), Ottonello and Perez (2018), Aguiar et al. (2017), Sanchez et al. (2018). See Aguiar and Amador (2013) for a review.

[3]There has been an extensive literature that studies search in asset markets following the seminal contribution of Duffie et al. (2005). Some recent examples are Lagos et al. (2011) that studies crises in over-the-counter markets; Afonso and Lagos (2012) that studies high frequency trading in the market for federal funds; Atkenson et al. (2013), that studies the decisions of financial intermediaries to enter and exit an over-the-counter market. A recent literature studies the consequences of search frictions in asset markets for macroeconomic outcomes, such as output, employment and asset prices. For example, Cui and Radde (2016) studies an economy with financial frictions in which private debt is illiquid and traded by intermediaries in an over-the-counter market. Kozlowski (2017) develops a joint theory of corporate investment and maturity choice. Both decisions are affected by the liquidity premia of corporate bonds issued to finance the investment and these premia are generated by frictions in the secondary market. Gutkowski (2017) studies, through the lens of a New Keynesian Model, how an exogenous inability to sell sovereign bonds used as collateral impacts output and employment. On the empirical side, using a VAR identification, Gazzani and Vicondoa (2016) finds that shocks to liquidity, proxied by shocks to bid-ask spreads, affect macroeconomic outcomes, for example unemployment, and indicators of confidence.

build on the framework of He and Milbradt (2013), which extends the models of corporate default as in Leland and Toft (1996) by introducing an over-the-counter market as in Duffie et al. (2005). That paper uncovers a joint determination of liquidity and credit risk. Building on this framework Chen et al. (Forthcoming) decompose spreads into a liquidity and a credit component over the business cycle. Our paper differs from He and Milbradt (2013) and Chen et al. (Forthcoming) in two dimensions. First, there is a crucial qualitative difference between the sovereign and corporate settings. The value of default in our model is endogenously determined, whereas in He and Milbradt (2013) and Chen et al. (Forthcoming), this value is fixed, does not depend on future liquidity conditions, and is zero in equilibrium. Second, in our setup the total debt of the sovereign country is changing over the cycle. In both He and Milbradt (2013) and Chen et al. (Forthcoming), the capital structure is fixed. In our paper, the fact that total debt varies over the cycle implies that the response of debt and default policies to liquidity frictions is to decrease debt levels and to default in more states of nature. The former case cannot occur with a fixed capital structure.

Finally, our paper relates to a growing empirical literature that documents that liquidity is an important factor in explaining sovereign spreads.[4] Pelizzon et al. (2013) study market micro-structure using tick-by-tick data and document the strong non-linear relationship between changes in Italian sovereign risk and liquidity in the secondary bond market. Bai et al. (2012) finds that liquidity risk explains most of the spread variations before the European sovereign debt crisis and credit risk explains most of these variations at the onset of the crisis. Ashcraft and Duffie (2007) find evidence of trading frictions in the pricing of overnight loans in the federal funds market. Fleming (2002) finds evidence of liquidity effects in treasury markets. Using data on Italian bonds Pelizzon et al. (2016) finds that, in the context of the European sovereign debt crisis, credit risk drives liquidity premia. Our paper builds on these empirical findings, which document sizable liquidity frictions in the market for sovereign bonds, and complements these empirical studies by quantifying the size of the liquidity premium over the business cycle and assessing the welfare losses due to liquidity frictions.

[4]The evidence showing that liquidity is a factor explaining the spread of corporate bonds is more established. Longstaff et al. (2005) use data on credit default swaps to measure the size of the default and non-default components of credit spreads. They find that most of the spread is due to default risk and that the nondefault component is explained mostly by measures of bond illiquidity. Bao et al. (2011) show that there is a tight link between illiquidity and bond prices. Edwards et al. (2007) study transaction costs in over-the-counter markets and find that transaction costs decrease significantly with transparency, trade size, and bond rating, and increase with maturity. Friewald et al. (2012) find that liquidity effects account for approximately 14 percent of the explained market-wide corporate yield spread changes. Chen et al. (2007) also find that liquidity is priced into corporate debt for a wide range of liquidity measures after controlling for common bond-specific, firm-specific, and macroeconomic variables.

2 Model

In this section, we present a model of sovereign default with trading frictions in the secondary market for sovereign bonds. We first describe the setting: section 2.1 describes the macroeconomic environment, section 2.2 describes the secondary bond market, and 2.3 describes the the timing of the model. We then proceed to characterize the equilibrium: section 2.4 characterizes the decisions of the government given prices, sections 2.5 and 2.6 define bond prices and valuations, and section 2.7 defines the equilibrium . Finally, section 2.8 discusses why each of the ingredients in the model are needed to quantify the credit and liquidity component of spreads and to study the welfare implications of frictions in the secondary market.

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. The small open economy receives a stochastic stream of income denoted by y_t. Income follows a first-order Markov process $\mathbb{P}\left(y_{t+1} = y' \mid y_t = y\right)$. The government is benevolent and wants to maximize the utility of the household. To do this, it trades bonds in the international bond market to smooth the household's consumption. The household evaluates consumption streams, c_t, according to:

$$(1-\beta)\,\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

with time-preference $\beta \in (0, 1)$ and utility function $u(\cdot)$.

The sovereign issues long-term debt when it is not in default. As in Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012), each unit of debt matures with probability m each period. Non-maturing bonds pay coupon z. This memoryless formulation of the debt maturity structure means that the face value of outstanding debt is the only relevant state variable for the obligations of the government. The government can issue bonds at a price q_t in the primary bond market. In equilibrium, the price of debt depends on current income, y_t, and the next period's bond position, b_{t+1} (our convention is that $b_{t+1} > 0$ denotes debt). The budget constraint for the economy is given by:

$$c_t = y_t - \left[m + (1-m)\,z\right] b_t + q_t\left[b_{t+1} - (1-m)b_t\right], \tag{2.1}$$

where mb_t is the repayment of principal for maturing debt, $(1-m)zb_t$ is the total coupon payment for non-maturing debt, and $q_t\left[b_{t+1} - (1-m)b_t\right]$ represents the proceeds from

newly issued debt.

There is limited enforcement of debt; thus, the government can default at its convenience. There are two consequences of default. First, the government loses access to the international credit market and goes into autarky. Second, output is lower during default and is given by $y_t - \phi(y_t)$. That is, there is also a direct output cost of default, $\phi(y_t)$, which is a standard assumption in the literature. The government can regain access to the international credit market, which occurs with probability θ each period. A fraction of defaulted debt is written off when the government regains access to credit markets. In particular, the new face value of outstanding debt is $\mathcal{R}(b_t) = \min\left\{\bar{b}, b_t\right\}$, where b_t is the amount of defaulted debt and $\bar{b}$ is a maximum recovery value.[5] That is, the fraction of recovered debt is $\mathcal{R}(b_t)/b_t = \min\left\{\bar{b}/b_t, 1\right\}$. Note that this fraction converges to zero as the amount of defaulted debt goes to infinity.

2.2 Primary and Secondary Bond Markets

There are two bond markets: the primary market and the secondary market. The government issues debt in the primary bond market. International investors can initially purchase newly issued debt in the primary market. Subsequent trading of bonds occurs in the secondary market. As in Duffie et al. (2005) and He and Milbradt (2013), trading in the secondary bond market is subject to search frictions.

Each international investor is assumed to be small and can hold at most a single unit of government debt. These investors are risk-neutral and can be either constrained or unconstrained. Unconstrained investors price bonds by discounting future payoffs at the rate risk-free rate, r. Unconstrained investors can become constrained if they receive a liquidity shock. Liquidity shocks are idiosyncratic in nature and have a per period probability ζ of occurrence. Constrained investors also discount payoffs at the risk free rate, but are additionally subject to per period holding costs $h_c > 0$. As a result, unconstrained investors have a high bond valuation, q^H, while constrained investors have a low bond valuation, q^L (the exact expressions for q^H and q^L are given given in sections 2.5 and 2.6). Therefore, unconstrained investors are the natural buyers of bonds in both the primary and the secondary markets, while constrained investors are the natural sellers of bonds

[5] Models of sovereign debt renegotiation such as Yue (2010) and Bai and Zhang (2012) homogenize recovery as the equilibrium outcome of a Nash bargaining game between the government and its creditors. These models produce recovery values of the form $\mathcal{R}(\bar{b}(y_t), b_t) = \min\left\{\bar{b}(y_t), b_t\right\}$ with state-dependent maximum recovery values, $\bar{b}(y_t)$. In our simple specification, the maximum recovery value does not vary across states.

in secondary markets.[6]

Unconstrained investors try to offload their bond positions in the secondary market. As in He and Milbradt (2013), secondary market trading is intermediated. The per period contact probability between constrained investors and intermediaries is λ.[7] Constrained investors and intermediaries bargain upon making contact. The total surplus is:

$$S = A - q^L,$$

where q^L is the low valuation of the constrained investor, and A denotes the ask price at which the intermediary can then offload the bond. Following He and Milbradt (2013), we assume that there is a large mass of competitive unconstrained investors waiting on the sidelines who intermediaries could contact with immediate effect. This simplifying assumption means that we do not have to keep track of the dealer's inventory as intermediaries could instantaneously offload bonds to high-valuation investors. Since the large mass of high-valuation investors on the sidelines act competitively, intermediaries can offload their positions at high valuations, and thus:

$$A = q^H.$$

The total surplus, $S = q^H - q^L$, is then divided according to a Nash bargaining rule with the bargaining power of the constrained low valuation investors being $\alpha \in [0, 1]$. This implies that the price at which constrained investors sell to intermediaries upon contact is:

$$q^S = q^L + \alpha(q^H - q^L).$$

The dollar bid-ask spread, per unit of principal, is the difference between intermediaries' selling price, q^H, and buying price, q^S,

$$ba_d = (1 - \alpha)(q^H - q^L).$$

[6]We interpret liquidity events as investor-specific events that prompt an immediate need to sell (e.g. to meet an expenditure). Holding costs represent the utility loss associated with delayed transactions. Valuations of debt are non-negative due to free disposal of the asset. When solving the model numerically, we additionally assume that bonds can be freely disposed. This is necessary so that the presence of holding costs do not imply negative bond prices for some off-the-equilibrium path values of the state space. Note that bond prices are always positive along the equilibrium path.

[7]In equilibrium, there is no need for unconstrained investors to contact intermediaries: unconstrained investors already have high valuations, and thus, there are no gains from trade.

The proportional bid-ask spread:

$$ba = \frac{(1-\alpha)(q^H - q^L)}{\frac{1}{2}(q^H + q^L) + \frac{\alpha}{2}(q^H - q^L)},$$

is simply the dollar bid-ask spread normalized by the mid price $\frac{1}{2}\left(q^H + q^S\right)$.

We let that the bargaining power of constrained low-valuation investors depend on whether or not the government is in default, with α_D (α_{ND}) denoting investors' bargaining power when trading bonds that are (not) in default. We assume:

$$\alpha_D < \alpha_{ND}$$

and thus that investors' have lower bargaining power when trading sovereign bonds that are in default. This implies that (dollar) bid-ask spreads are higher during default episodes. As we discuss in subsection 2.8, there is substantial evidence that bid-ask spreads are higher during default for US corporate Bonds (see, e.g., Edwards et al. (2007) and He and Milbradt (2013)).[8]

2.3 Timing

The timing for the government is as follows. First, consider the case in which the government has credit access (i.e., is not in default) and begins period t with an amount b_t of outstanding debt. Income, y_t, is then realized. The government then decides whether or not to default $d_t \in \{0, 1\}$. If the government chooses not to default ($d_t = 0$), principal payments for maturing debt, mb_t, and coupon payments for non-maturing debt, $(1-m)\,zb_t$, are made. The government can then issue new debt in the primary market. An issuance with face value $b_{t+1} - (1-m)b_t$ leads to outstanding debt with face value b_{t+1} at the start of the next period. As previously mentioned, unconstrained investors are the natural buyers of new bond issues, and thus, bonds are always issued at the high valuation, q_t^H. Finally, consumption takes place and is given by $c_t = y_t - [m + (1-m)\,z]\,b_t + q_t^H\,[b_{t+1} - (1-m)b_t]$.

Next, consider the case in which the government is already in default or chooses to default in the current period ($d_t = 1$). In this case, b_t is the amount of debt that is in default. The government is in autarky and does not take any actions. Consumption is simply equal to income adjusted for the costs of default: $c_t = y_t - \phi\,(y_t)$. Nature

[8] To the best of our knowledge, we are unaware of cross-country studies of bid-ask spreads for sovereign bonds that are in default.

determines whether the government regains credit access between the end of period t and the start of the next period, $t+1$. The probability of regaining credit access is θ, and in that event, the government re-accesses the debt market with an outstanding debt of $\mathcal{R}(b_t) = \min\left\{\bar{b}, b_{t+1}\right\}$ at the start of the next period. Otherwise, the government remains in autarky.

The timing for investors is as follows. Investors are assessed holding costs, h_c, for the period if they begin period t constrained. Secondary market trading for outstanding bonds occurs once per period, immediately before the government issues new bonds in the primary market. As mentioned in section 2.2, only constrained bond holders attempt to sell in the secondary market. With probability λ, a constrained investor meets an intermediary and offloads his bond position. The transaction price is $q^S_{ND}(y_t, b_{t+1})$ when the government is not in default, and $q^S_D(y_t, b_t)$ when the government is in default. Constrained investors who fail to contact intermediaries remain constrained going into the next period $t+1$.

An investor who is unconstrained at the beginning of period t is not subject to holding costs for the period. However, such an investor can become constrained for the start of the next period $t+1$ if he receives a liquidity shock during period t. Liquidity shocks occur with probability ζ and take place after the conclusion of trading in the secondary market. This means that a newly constrained investor in period t is unable to immediately offload his position in the same period. In addition, liquidity shocks occur prior to new bond issuances in the primary market. This implies that the unconstrained investors who purchased newly issued bonds during period t will still be unconstrained at the beginning of period $t+1$. Figure 2.3 summarizes the timing within each period.

2.4 The Government's Decision Problem

The government maximizes household welfare while taking bond prices as given. This infinite-horizon decision problem can be cast as a recursive dynamic programming problem. We focus on a Markov equilibrium with income, y, as the exogenous state variable and debt, b, as the endogenous state variable. The value for a government with an option to default, V^{ND}, is the larger of the value of defaulting, V^D, and the value of repayment, V^C,

$$V^{ND}(y,b) = \max_{d\in\{0,1\}} dV^D(y,b) + (1-d)\,V^C(y,b)\,.$$

The solution to this problem yields the government's default policy:

$$d = \boldsymbol{D}(y,b) = 1_{\left\{V^D(y,b) > V^C(y,b)\right\}}. \tag{2.2}$$

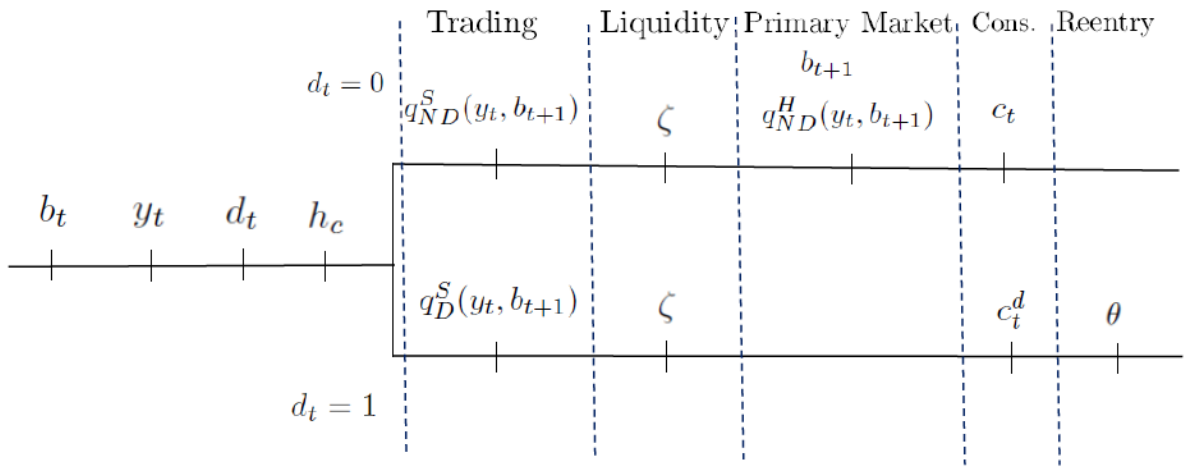

Figure 2.1: This figure summarizes the timing before and after default in period t. The government enters the period with bonds b_t. Then, income, y_t, is realized, and the government chooses whether or not to default, d_t. Constrained investors are subject to holding costs, h_c. The upper branch depicts the sequence of events in the absence of default: secondary market trading of outstanding bonds occurs, and unconstrained investors receive liquidity shocks. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. Then, the liquidity shock is realized. Then, the government issues a face value of debt $b_{t+1} - (1-m)b_t$, facing a price $q^H_{ND}(y_t, b_{t+1})$. Finally, consumption is realized. The lower branch depicts what happens in the case in which the government defaults. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Note that the primary market is closed while the government is in autarky. Then, the government will re-access the debt market in the next period with probability θ. Finally, consumption is equal to $c^d(y_t) = y_t - \phi(y_t)$.

That is, the government defaults whenever the value of defaulting is higher than the value of repayment.

The value of defaulting is:

$$V^D(y,b) = (1-\beta)\, u(y - \phi(y)) + \beta \mathbb{E}_{y'|y} \left[\theta V^{ND}\left(y', \mathcal{R}(b)\right) + (1-\theta) V^D\left(y', b\right) \right],$$

where the flow utility is determined by household consumption in default, $y - \phi(y)$, while the continuation value takes into account the possibility of regaining credit market access with debt level $\mathcal{R}(b)$.

The value of repaying:

$$V^C(y,b) = \max_{b'} \left\{ (1-\beta) u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}\left(y', b'\right) \right] \right\} \tag{2.3}$$

is subject to two constraints. The first constraint is the standard budget constraint:

$$c = y - [m + (1-m)z]\, b + q^H_{ND}\left(y, b'\right)\left[b' - (1-m)b\right],$$

where $q^H_{ND}\left(y, b'\right)$ is the bond issuance price schedule corresponding to the high valuation of unconstrained investors during periods in which the government is not in default. In addition, as in Chatterjee and Eyigungor (2015), the government faces an upper bound in the ex ante one-period-ahead expected default probability:

$$\delta\left(y, b'\right) \equiv \mathbb{E}_{y'|y}\left[d\left(y', b'\right)\right] \leq \bar{\delta} \tag{2.4}$$

whenever there is positive debt issuance, $b' - (1-m)b > 0$. As explained in Chatterjee and Eyigungor (2015), in long-term debt models with positive recovery, the government has incentives to dilute existing bond holders by issuing large amounts of debt just prior to default. Since the liability of the government upon regaining credit access is at most $\bar{b}$, the government will then issue an infinite amount of debt just prior to default to fully dilute existing bond holders. Constraint (2.4) rules out such counterfactual behavior.[9] The solution to the repayment problem yields the debt policy of the government:

$$b' = \boldsymbol{B}\left(y, b\right). \tag{2.5}$$

2.5 Debt Valuations Before Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is not in default. Let y be current income, and suppose that $y'$is the post-issuance face value of outstanding debt. The value of one unit of debt for an unconstrained investor with a high valuation is:

$$q^H_{ND}\left(y, b'\right) = \mathbb{E}_{y'|y}\left\{\left[(1 - d\left(y', b'\right)\right]\frac{m + (1-m)\left[z + \zeta q^L_{ND}\left(y', b''\right) + (1-\zeta)q^H_{ND}\left(y', b''\right)\right]}{1+r}\right.$$

$$\left. + d\left(y', b'\right)\frac{\zeta q^L_D\left(y', b'\right) + (1-\zeta)q^H_D\left(y', b'\right)}{1+r}\right\}, \tag{2.6}$$

which reflects the state-contingent payoffs of the bond. An investor receives principal m and coupon $(1-m)\,z$ in the absence of default during the next period, $d\left(y', b'\right) = 0$. In

[9] As noted in Chatterjee and Eyigungor (2015), sovereign bonds issued in financial centers (e.g. New York) have to be underwritten by some investment bank. Reputational concerns may prevent them from issuing bonds with very high probabilities of immediate default.

this case, the continuation value of the $1-m$ non-maturing fraction of the bond depends on next period's optimal debt policy, $b'' = \boldsymbol{B}(y', b')$, and the realization of the idiosyncratic liquidity shock. A liquidity shock arrives with probability ζ, in which case the investor obtains a low continuation value, $q^L_{ND}(y', b'')$. Otherwise, the investor remains unconstrained and assigns a high continuation value, $q^H_{ND}(y', b'')$, to the bond. An investor does not receive any cashflow in the event of a default, $d(y', b') = 0$. In this case, the government defaults on b' units of debt, and the per unit price of defaulted debt is $q^H_D(y', b')$ and $q^L_D(y', b')$ for investors who do not receive and receive liquidity shocks, respectively. The value of defaulted bonds will be described, in the next section 2.6.

The price of debt for a constrained investor with a low valuation is:

$$q^L_{ND}(y, b') = \mathbb{E}_{y'|y}\left\{\left[(1-d(y', b')\right]\frac{-h_c + m + (1-m)\left[z + (1-\lambda)q^L_{ND}(y', b'') + \lambda q^S_{ND}(y', b'')\right]}{1+r}\right.$$
$$\left. + d(y', b')\frac{-h_c + (1-\lambda)q^L_D(y', b') + \lambda q^S_D(y', b')}{1+r}\right\}. \tag{2.7}$$

The valuation of a constrained investor is similar to that of an unconstrained investor, but reflects the following differences. First, a constrained investor is assessed holding costs h_c, that lower the effective value of a bond. Second, the continuation value for a constrained investor depends on trading outcomes in the secondary market. As described in section 2.2, a constrained investor sells with probability λ. The selling price (in the next period) is:

$$q^S_{ND}(y', b'') = \alpha_{ND} q^H_{ND}(y', b'') + (1-\alpha_{ND}) q^L_{ND}(y', b'')$$

in the absence of default and $q^S_D(y', b')$ in the event of default.

2.6 Debt Valuations After Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is in default. Let the current income be y, and let b be the amount of debt in default. In this case, the value of one unit of debt for unconstrained high-valuation investors is:

$$q^H_D(y, b) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q^H_D(y', b) + (1-\zeta) q^L_D(y', b)\right] + \theta\frac{\mathcal{R}(b)}{b} q^H_{ND}(y, \mathcal{R}(b)). \tag{2.8}$$

The government regains credit access in the next period with probability θ, in which case $\mathcal{R}(b)/b$ is the fraction recovered for each unit of defaulted debt. The value of recovered bonds, $q^H_{ND}(y, \mathcal{R}(b))$, is given by (2.6) and reflects the new value of total outstanding

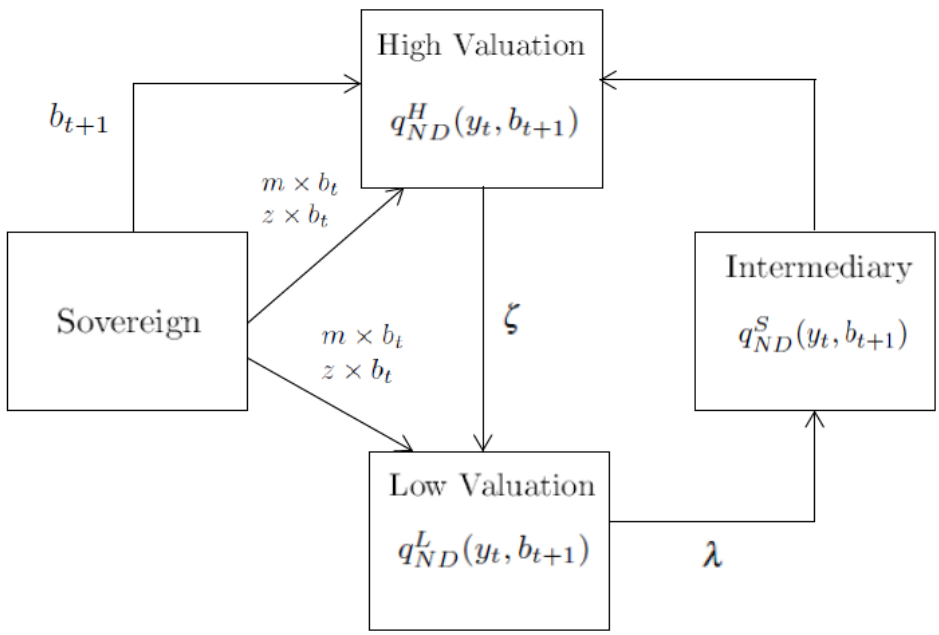

Figure 2.2: This figure details the bond market if the sovereign is not in default and does not default in period t. The sovereign begins by issuing debt b_{t+1}. The high-valuation investors buy this debt in the primary market. After that, with probability λ, the low-valuation investors will meet an intermediary. They will sell their bonds at the price $q^S_{ND}(y_t, b_{t+1})$. After selling their bonds, they exit the market. The low-valuation investors who do not meet an intermediary will attempt to sell their bonds in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond in the next period in the secondary market. Both the high- and low-valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

debt, $\mathcal{R}(b)$. Otherwise, investors receive no payments if default is not resolved and the continuation value reflects the probability ζ of receiving a liquidity shock.

Similarly, the per unit value of debt for constrained low-valuation investors is:

$$q^L_D(y,b) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c + \lambda q^S_D\left(y',b\right) + (1-\lambda)q^L_D\left(y',b\right)\right] + \theta\frac{\mathcal{R}(b)}{b}q^L_{ND}\left(y,\mathcal{R}\left(b\right)\right). \tag{2.9}$$

This valuation is analogous to that of unconstrained investors (2.8), but accounts for holding costs and trading frictions in the secondary market. Constrained investors sell during the next period with probability λ. The selling price for a defaulted bond is:

$$q^S_D(y,b) = \alpha_D q^H_D(y,b) + (1-\alpha_D)q^L_D(y,b),$$

and reflects the lower investor bargaining power during default: $\alpha_D < \alpha_{ND}$.

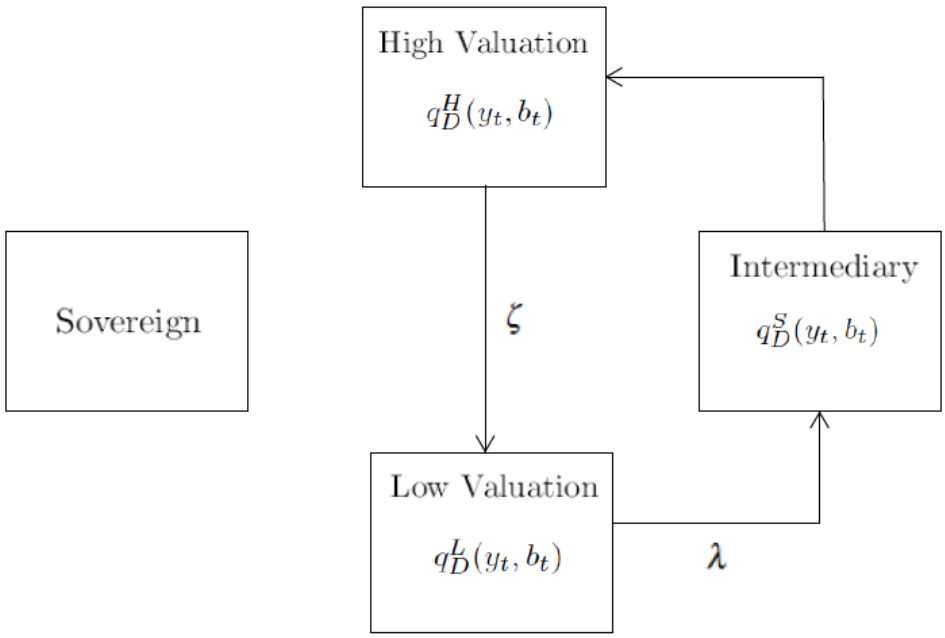

Figure 2.3: This figure details the bond market if the government is in default or defaults in period t. There is no debt issue or debt service. The sovereign has an outstanding balance of debt b_t. The low-valuation investors will meet an intermediary with probability λ. They will sell their bonds at the price $q_D^S(y_t, b_t)$. After they sell the bonds, they exit the market. The low-valuation-investors who do not meet an intermediary will attempt to sell in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell in the next period in the secondary market. Finally, with probability θ, the government resolves the default and re-accesses the credit market in the next period with a total outstanding debt of $b_t \times \mathcal{R}(b_t)$.

2.7 Equilibrium

We focus on a Markov equilibrium with state variables y and b. An equilibrium consists of a set of policy functions for consumption $\boldsymbol{C}(y, b)$, default $\boldsymbol{D}(y, b)$, and debt $\boldsymbol{B}(y, b)$, as well as bond valuations $q_{ND}^H(y, b')$, $q_{ND}^L(y, b')$, $q_D^H(y, b)$ and $q_D^L(y, b)$ such that: (1) the policies solve the goverment's problem (2.3) taking bond valuations as given, and (2) the bond valuations satisfy equations (2.6) (2.7), (2.8) and (2.9).

2.8 Discussion

We now discuss our modeling assumptions. Our goal is to provide a parsimonious framework to study debt and default policy in a setting where default and liquidity risk are jointly determined. We generate bid-ask spreads by introducing search frictions in the

secondary market as in Duffie et al. (2005). Indeed, decentralized bond market trading is an important feature of reality. Having long-term debt is then necessary in order to generate realistic levels of bid-ask spreads. In the extreme case that outstanding debt matures every period bid ask spreads are equal to zero, there is no need to trade as investors can simply wait and receive principal payouts from the government. The combination of long-term debt and secondary market search frictions leads to a difference in valuation between liquidity-constrained and unconstrained investors prior to default, $q^H_{ND} - q^L_{ND}$. Both features of our setup are reasonable. First, there is substantial evidence of trading frictions in the secondary market for sovereign bonds (see, e.g., Pelizzon et al. (2013) for recent evidence). Second, long-term debt is now a standard feature of models of sovereign debt following the contributions of Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012).

We also introduce a positive debt recovery, $\mathcal{R}(b) > 0$, after a default. Positive recoveries are an important feature of the data (see, e.g., Cruces and Trebesch (2013)) and, in our model, is necessary in order to generate realistic behavior of bid-ask spreads. This is because the absence of recovery, $\mathcal{R}(b) = 0$, implies no future cash-flows and therefore zero valuations for both constrained and unconstrained investors during default, $q^H_D = q^L_D = 0$. In turn, this implies that the (dollar) bid-ask spread is zero during default. More importantly, it would also imply that bid-ask spreads approach zero as default probabilities approach one, which is at odds with a widening of bid-ask spreads during sovereign crises documented in Pelizzon et al. (2013) and the evidence for corporate bonds Edwards et al. (2007).

We assume a decrease in the bargaining power of investors after a sovereign default, $\alpha_D < \alpha_{ND}$, to capture the positive comovement between (dollar) bid-ask spreads and sovereign default probabilities. Under this assumption, bid-ask spreads are higher during default. As a result, bid-ask spreads will also increase leading up to a default as is consistent with evidence from Pelizzon et al. (2013). There is substantial evidence of higher bid-ask spreads for defaulted corporate bonds (see, e.g., Edwards et al. (2007) and He and Milbradt (2013)). We are unaware of similar studies that report bid-ask spreads for a large sample of defaulted sovereign bonds.[10]

In Appendix B we discuss our modeling choices using a simple jump-to-default model in which debt policies are fixed and there is an exogenous default probability.

[10]Using Bloomberg data, we frequently observe stale prices for defaulted Argentine bonds that can be constant for weeks. This is indicative of lower trading.

3 Results

We numerically solve a discretized version of the model. As discussed in Chatterjee and Eyigungor (2012) grid-based methods have poor convergence properties when there is long-term debt. To overcome this problem, we follow their prescription and compute a "slightly perturbed" version of the model described in this section. To ensure numerical accuracy, we choose a dense grid with 200 points for the persistent component of output and 450 points for debt. We implement the model in CUDA and numerically compute the model on a Tesla K80 GPU. Appendix A discusses the details.

3.1 Calibration

We calibrate the model developed in section 2 to account for the main features of Argentina's default in 2001. We focus on Argentina over the period from 1993:I to 2001:IV for three reasons. First, using this period facilitates comparison to prior studies in the sovereign debt literature (see, for example, Hatchondo and Martinez (2009), Arellano (2008) and Chatterjee and Eyigungor (2012)), and in this way, we can be transparent about the contribution of our paper. Second, this sample satisfies our model's main assumptions: (1) our model is real and Argentina had a fixed exchange rate vis-a-vis the dollar during this period, and (2) Argentina's bonds were traded in an illiquid secondary market during this period. We calibrate and simulate the model at a monthly frequency because liquidity is inherently a short-run phenomenon. However, we report results at a quarterly frequency to facilitate comparison to previous studies.[11]

Functional Forms and Stochastic Processes. As is standard in the literature, we specify the household utility to be CRRA $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We set the endowment process to be:

$$y_t = e^{z_t} + \epsilon_t,$$
$$z_t = \rho_z z_{t-1} + \sigma_z u_t,$$

where z_t is a discretized AR(1) process with persistence ρ_z, volatility σ_z, and normally distributed innovations $u_t \sim N(0,1)$. We also add a small amount of noise $\epsilon_t \sim trunc\, N(0, \sigma_\epsilon^2)$ that is continuously distributed. As shown in Chatterjee and Eyigungor (2012), this is nec-

[11] To be precise about this conversion, the quarterly debt to output ratio in our paper is the stock of debt at the end of the quarter divided by the sum of monthly output within the quarter. This implies that the average debt to quarterly output will be one-third of average debt to monthly output.

essary to achieve numerical convergence. We set the output loss during default to:

$$\phi(y) = \max\left\{0, d_y y + d_{yy} y^2\right\}.$$

This loss function is proposed by Chatterjee and Eyigungor (2012) and nests several cases in the literature. When $d_y < 0$ and $d_{yy} > 0$, the cost is zero for the range $0 \leq y \leq -\frac{d_y}{d_{yy}}$ and rises more than proportionally with output for $y > -\frac{d_y}{d_{yy}}$. Alternatively, when $d_y > 0$ and $d_{yy} = 0$ the cost is a linear function of output.[12] As is explained in Chatterjee and Eyigungor (2012), the convexity of output costs is necessary to match the volatility of sovereign spreads.

A priori set parameters. We set risk aversion to $\gamma = 2$, which is standard in the sovereign debt literature. The parameters for output are estimated from (linearly) detrended and seasonally adjusted data for Argentina for the quarterly sample from 1980:I to 2001:IV available from Neumeyer and Perri (2005). After estimating an AR (1) model for output at a quarterly frequency, we obtain monthly values $\rho_z = 0.983$, and $\sigma_z = 0.0151$, and we fix $\sigma_\epsilon = 0.004$.[13] We set the risk free rate to $r = 0.0033$ per month such that the quarterly risk free rate is 1 percent, which is standard. We set $m = 1/60$ to match an average debt maturity of 5 years based on values reported in Chatterjee and Eyigungor (2012) and Broner et al. (2013). We set the coupon rate to $z = 0.01$, such that the annualized coupon rate is 12 percent, which is close to the 11 percent value-weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012). We fix the reentry probability at $1 - \theta = 0.0128$, following Chatterjee and Eyigungor (2012). This implies an average exclusion period of 6.5 years.[14] We set the maximum one-month-ahead default probability to $\overline{\delta} = 0.75$. Finally, we fix $\lambda = 0.8647$ such that the average time required for a constrained investor to offload his position to an intermediary is two weeks, as in Chen et al. (Forthcoming).

[12] The case studied in Arellano (2008) features consumption in default that is given by the mean output if the output is over the mean and equal to output if the output is less than the mean. This implies a cost function $\phi^A(y) = \max\{y - \mathbb{E}(y), 0\}$, which closely resembles the case of $d_y > 0$ and $d_{yy} = 0$.

[13] The conversion is as follows. Total monthly volatility is given by $\sqrt{0.004^2 + 0.0151^2} = 0.0156$. Then, total monthly volatility and the monthly autocorrelation are converted to a quarterly frequency. This implies an autocorrelation of $0.983^3 = 0.95$ and an output volatility of $0.0156 \times \sqrt{3} = 0.027$. The latter values are those recovered from the data.

[14] Beim and Calomiris (2001), report that for the 1982 default episode, Argentina remained in a default state until 1993. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in the default state from 2001 until 2005, when it settled with most of its bondholders.

Parameter	Description	Value
β	Sovereign's discount rate	0.9841
γ	Sovereign's risk aversion	2
ρ_z	Persistence of output	0.983
σ_y	Volatility of output	0.0156
m	Rate at which debt matures	0.0167
z	Coupon rate	0.01
θ	Probability of reentry	0.0128
d_y	Output costs for default	-0.264
d_{yy}	Output costs for default	0.337
r	Discount rate of international investors	0.0033
h_c	Holding costs for constrained investors	0.0014
ζ	Probability of getting a liquidity shock	0.139
λ	Probability of meeting a market maker	0.865
α_{ND}, α_D	Bargaining power of the low valuations investor	0.125, 0
$\overline{b}$	Maximum recovery rate for sovereign bonds	0.83
$\overline{\delta}$	Max. Default Probability	0.75

Table 1: Baseline

Calibrated parameters. We choose time preference $\beta = 0.9841$ to target an average debt to quarterly output ratio of 100 percent based on the mean debt to quarterly output ratio for Argentina for the period between 1993:I and 2001:IV.[15] As our model features recovery, we set the maximal debt recovery to be $\overline{b} = 0.83$ to capture a mean recovery of $\mathbb{E}\left[\frac{\min\{\overline{b}, b_{def}\}}{b_{def}}\right] = 0.3$ in the model. This is based on a realized recovery rate of 30 percent for the 2001 Argentine default. In addition, we choose the default cost parameters $d_y = -0.264$ and $d_{yy} = 0.337$ to match the first two moments for the quarterly behavior of (annualized) sovereign spreads. Following Chatterjee and Eyigungor (2012), this involves targeting mean spreads of 0.0815 and a quarterly volatility of 0.0443.[16] We calibrate parameters relating to secondary market frictions as follows. We first normalize the bargaining power of market makers during periods of default to $(1 - \alpha_D) = 1$. We then choose a holding cost of $h_c = 0.0014$ to target a mean proportional bid-ask spread of 500 basis points during default. Due to data limitations, we base this target on the bid-ask spreads of defaulted US bonds which are documented to range between 200 basis points during normal times Edwards et al. (2007) and 620 basis points during recessions Chen et al. (Forthcoming). We then set the bargaining power of market makers outside of default to be $(1 - \alpha_{ND}) = 0.875$ to target a mean proportional bid-ask spread of 50 basis points outside of default. For European bonds, Pelizzon et al. (2013) find a median bid-

[15] Chatterjee and Eyigungor (2012) target a debt to quarterly output ratio of 70 percent because their model does not feature recovery. We target the full 100 percent because our model features recovery.

[16] The annualized sovereign spread is given by $cs(y, b') = (1 + r^H(y, b'))^{12} - (1 + r)^{12}$ where the yield to maturity is given by $r^H(y, b') = [m + (1 - m)z] / q^H(y, b') - m$.

Moment	Data	Model	CE (2012), Table 3
Mean Debt to GDP	1.0	1.0	0.7
Expected Recovery	0.30	0.297	0
Mean Sovereign Spread	0.0815	0.0815	0.0815
Vol. Sovereign Spread	0.0443	0.0437	0.0443
Mean Bid-Ask Spread, ND	0.0050	0.0049	-
Mean Bid-Ask Spread, D	0.0500	0.0503	-
Mean Turnover	0.12	0.12	-

Table 2: **Model moments.**

ask spread of 43 basis points in their sample, which increased to 125 basis points during the period June 2011 to November 2012. For non-investment grade US corporate bonds, Chen et al. (Forthcoming) report bid-ask spreads of 50 basis points during normal times and 218 during bad times. Finally, we set the probability of receiving a liquidity shock each period to $\zeta = 0.139$ to match an average turnover of 12 percent per month based on the average turnover rate for US corporate bonds (see Bao et al. (2011)).[17] This value implies that, on average, an unconstrained investor becomes constrained every 7 months.

Calibration Results. The final parameter values and the results from our baseline calibration are reported in Tables 1 and 2 respectively. The last column of Table 2 lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model closely matches all targeted moments. It generates a mean debt to GDP ratio of 100 percent and an average recovery rate of 29.7 percent. The average sovereign spread is 0.0815 and the volatility is 0.0437. The average bid-ask spread is 50 and 503 basis points before and after default, respectively. Finally, the mean turnover rate is 12 percent.

Coming Next. In the next sections, we use our calibrated model to perform a series of quantitative exercises. In subsection 3.2, we report the pricing functions in the primary market and the bid-ask spreads. In subsections 3.3 and 3.4, we provide a structural decomposition of total spreads into liquidity and default components. In subsection 3.5 we quantify the welfare implications of liquidity frictions. In subsection 3.6 we use our calibrated model to impute the the liquidity component of Argentine spreads in the lead up to Argentina's 2001 default. Finally, subsection 3.7 reports business cycle statistics of the model.

[17] To the best of our knowledge, turnover data for Argentine bonds are unavailable for our period of interest. Turnover, or the fraction of outstanding bonds being traded each period, is $\lambda\zeta / (\lambda + \zeta)$ on average in the model.

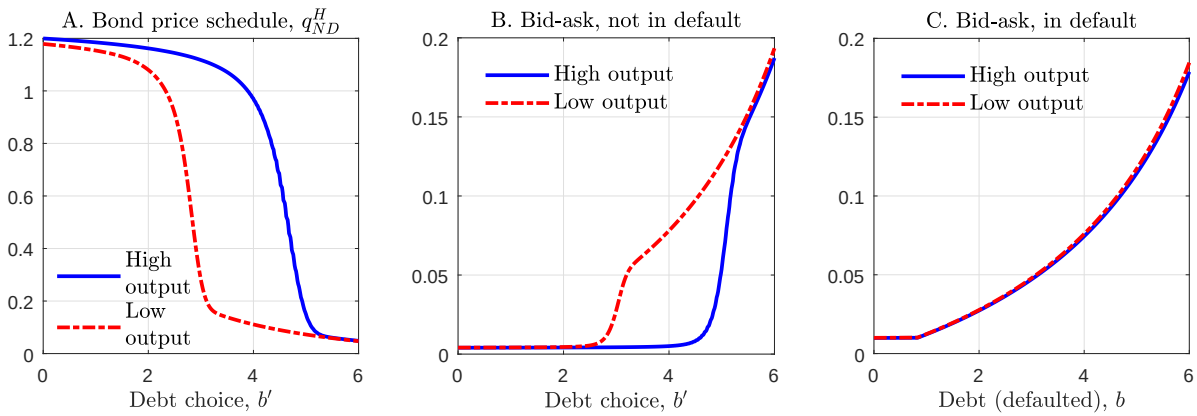

Figure 3.1: **Bond prices and bid-ask spreads.** This figure features bond prices at issue, q^H, and bid-ask spreads which are defined as $\frac{q^H - q^S}{\frac{1}{2}\left(q^H + q^S\right)}$. All prices are a function of (y, b'). Panel A plots prices in the primary market for high-valuation investors. Panels B and C plot bid-ask spreads during credit access and during autarky. In all panels, we plot price schedules and bid-ask spreads for two values of y. The values of z are chosen to be ± 2 standard deviations of the unconditional distribution for z. The actual values are 0.851 and 1.176. Given our monthly calibration, a 3 on the horizontal axis of debt choice corresponds to an average debt to quarterly output ratio of 1. For panels B and C, a 0.05 in the vertical axis corresponds to 500 basis points.

3.2 Bond Prices, Bid-Ask Spreads, and Feedback

Figure 3.1 plots the model-implied bond prices and bid-ask spreads as a function of debt choice b'. The plots are for high and low values of output y, which correspond to values at plus and minus one standard deviation of the unconditional distribution for output, respectively. Panel A plots bond prices in the primary market $q^H(y, b')$, during the credit access regime. Bond prices are always strictly positive as a consequence of having a positive recovery following default. Standard comparative statics apply: bond prices are increasing in output and decreasing in debt choice, as is usually the case in quantitative models of sovereign debt. Note that since the model is calibrated at a monthly frequency, the debt to quarterly output ratio is approximately $b'/3$.

Panel B plots proportional bid-ask spreads, $\left(q^H - q^S\right) / \left[\frac{1}{2}\left(q^H + q^S\right)\right]$, during periods in which the government is not in default. The bid-ask spread is approximately 50 basis points for low levels of debt choice (e.g., $b' \leq 2.5$) regardless of income levels. Bid-ask spreads increase as the likelihood of default increases. This occurs when output is low or debt choice is high. For example, the bid-ask spread reaches 500 basis points when debt choice is $b' = 3$ and output is low, and subsequently continues to increase for larger choices of debt. The logic is as follows. First, recall that bid-ask spreads are larger during

periods of default due to a decrease in investor bargaining power when trading defaulted bonds ($\alpha_D < \alpha_{ND}$). This can be seen in Panel C which plots proportional bid-ask spreads during periods of default as a function of the amount of debt in default. Since prices are forward looking, the increase in bid-ask spreads as the probability of default increases reflects the increased likelihood of encountering the worse liquidity conditions associated with trading defaulted bonds.

Panel C also illustrates that the proportional bid-ask spread of defaulted bonds is also increasing in the amount of debt in default. The reasoning is as follows. To a first-order approximation the dollar bid-ask spread during default, $q_D^H - q_D^S$, is proportional to holding costs. Since holding costs are constant, the proportional bid-ask spread increases as the value of bonds decrease. The latter is true if the amount of debt in default is high and the fractional recovery per unit of defaulted debt, $\mathcal{R}(b)/b$, is low.

In addition, Panel C also shows that, to a first-order approximation there is no difference in the proportional bid-ask spreads of defaulted bonds across different current output states. From equations (2.8) and (2.9), we see that the value of defaulted bonds depends on the current output state only through its influence on the value of recovered debt upon the government reentering international credit markets. In our calibration, periods of autarky last 6.5 years on average. Over such a horizon, the influence of current output on the eventual recovered debt value is weak. As a result, the proportional bid-ask spread for defaulted bonds depends mainly on the amount of debt in default.

In combination, Panels B and C of 3.1 demonstrate a liquidity-default feedback loop first highlighted by He and Milbradt (2013) in the context of corporate bonds. To see this feedback loop, consider the net proceeds from debt rollover, the difference in the value of newly issued bonds and the repayment of the principal of maturing debt, $q_{ND,t}^H [b_{t+1} - (1-m) b_t] - mb_t$. A larger debt issuance is necessary to break even when rolling over debt if the probability of default is high and the price of newly issued bonds, $q_{ND,t}^H$, is low. This leads to higher levels of indebtedness and further increases the government's incentives to default in future periods. This rollover risk channel is already understood (see, .e.g, Chatterjee and Eyigungor (2012)). The presence of liquidity frictions further amplifies this rollover risk channel. Higher debt levels further increase bid-ask spreads in the secondary market (Panel B), which results in an even lower issuance price in the primary bond market. In turn, this further exacerbates the government's default incentives and results in a liquidity-default feedback loop.

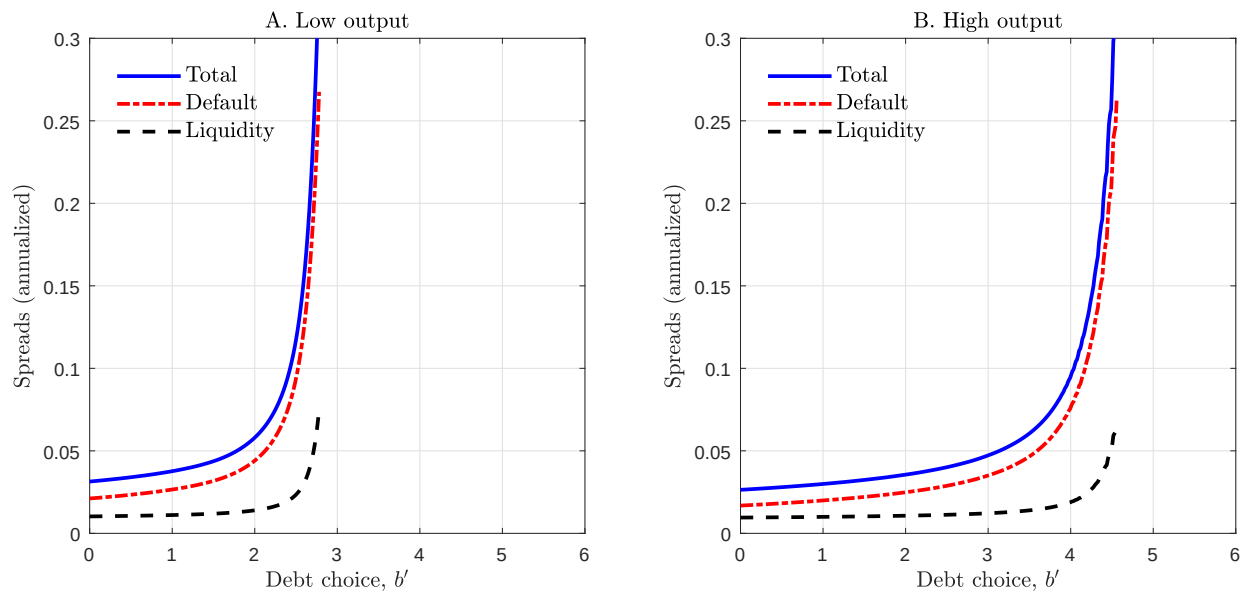

Figure 3.2: **Sovereign spread decomposition.** This figure decomposes total sovereign spreads, CS, into a default component, CS_{DEF}, and a liquidity component, CS_{LIQ}, for two values of z are chosen to be ± 2 standard deviations of the unconditional distribution for z. The actual values are 0.851 and 1.176. The default component is the one defined in equation (3.2), and the liquidity component is the one defined in (3.3).

3.3 Sovereign Spread Decomposition

In this subsection, we present a decomposition of total spreads into a liquidity and a credit component. Our main result is that liquidity premia can be a substantial component of total spreads. As a first step toward a decomposition, we start by defining total spreads, $cs\left(y, b'\right)$, as a function of government policies. Consider a government with debt and default policies given by $b' = \widetilde{\boldsymbol{B}}\left(y, b\right)$ and $d = \widetilde{\boldsymbol{D}}\left(y, b\right)$, respectively. Let $q_{ND}^{H}\left(y, b'\right)\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)}$ be the corresponding value of this government's debt to an unconstrained investor who receives liquidity shocks with probability ζ when the government follows policies $\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}$. That is, $q_{ND}^{H}\left(y, b'\right)\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)}$ solves equation (2.6) under debt policy $\widetilde{\boldsymbol{B}}$ and default policy $\widetilde{\boldsymbol{D}}$. Since our calibration is monthly, the corresponding annualized sovereign spread is then given by:

$$cs\left(y, b'\right)\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)} \equiv \left(1 + r^{H}(y, b')\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)}\right)^{12} - (1 + r)^{12}, \tag{3.1}$$

where $r^{H}(y, b')\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)}$ is given by $r^{H}(y, b')\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)} = [m + (1 - m)z] \,/\, \left(q_{ND}^{H}\left(y, b'\right)\Big|_{\left(\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{D}}, \zeta\right)}\right) - m$.

Aided by the definition of spreads as a function of policies, given by equation (3.1), we will now introduce a decomposition of total spreads as the sum of a credit and a liquidity

component.[18] Denote by $\boldsymbol{B}, \boldsymbol{D}$ the default and debt policies of the baseline calibration (i.e. the parameters listed in Table 1). The default component of the sovereign spread is defined as:

$$cs_{DEF}\left(y, b'\right) \equiv cs\left(y, b'\right)\Big|_{(\boldsymbol{B},\boldsymbol{D},0)}. \tag{3.2}$$

Thus, the default component of total spreads, $cs_{DEF}(y, b')$, is computed with a price that takes into account the default and debt policies of the baseline but the investor pricing these policies faces a zero probability of a liquidity shock. More precisely, the policies $(\boldsymbol{B}, \boldsymbol{D})$ are the responses of a government that faces prices $q^H_{ND}(y, b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)}$ in the primary market. However, the bond price associated with $cs_{DEF}(y, b')$ is given by $q^H_{ND}(y, b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)}$.[19] The liquidity component is then defined as the residual:

$$cs_{LIQ}(y, b') \equiv cs(y, b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y, b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} \tag{3.3}$$

and accounts for the portion of the total sovereign spread that is not explained by the default component. These two definitions amount to decomposing sovereign spreads as:

$$cs\left(y, b'\right) = cs_{DEF}\left(y, b'\right) + cs_{LIQ}\left(y, b'\right). \tag{3.4}$$

Now we delve into the numerical results. What portion of total annualized spreads is explained by each type of risk? The above decomposition is depicted in Figure 3.2. Panels A and B plot the total spreads, credit risk premium, and liquidity premium for two levels of output; the values of z are chosen to be ± 2 standard deviations of the unconditional distribution of z. There are two features of this decomposition that are worth noting. First, Panels A and B show that the liquidity component is increasing as the debt choice increases. This increase reflects a higher default probability combined with higher bid-ask spreads during default. Second, note also that as a percentage of total spreads, the liquidity component is sizable. In particular, when default risk is low (i.e. when output is high and/or debt levels are low) the liquidity component is predominant; however, as the overall default risk increases, the liquidity component as a fraction of total spreads becomes smaller, but remains a first order factor. For example, in Panel B, we observe that the fraction of the total sovereign spreads attributable to liquidity is approximately

[18] The decomposition is analogous to the decomposition provided in He and Milbradt (2013) in the context of corporate bond spreads. As it will be made clear in the next subsection, one important difference is that our decomposition takes into account the endogenous response of debt policy, while the decomposition in He and Milbradt (2013) is for a fixed debt policy.

[19] The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account when choosing debt and default policies), individual investors are heterogeneous and in particular there may be some investors without liquidity concerns who discount at the risk-free rate.

45 percent for a debt choice of 1 and a high level of output. However, close to default, for example when output is low and debt choice is around 2, liquidity is responsible for approximately 25 percent of total spreads. These magnitudes are in line with the CDS-basis based calculations in Longstaff et al. (2005) and structural decompositions in He and Milbradt (2013) for Corporate Bonds.

How does liquidity premia depend on default risk? To see this, consider a "jump-to-default" model similar to the model developed in section 2 except that debt is fixed and the default probability is exogenous and given by p_d (we present the jump-to-default model in detail in Appendix B). In this case, the valuation of the unconstrained investor (2.6) becomes

$$q_{ND}^H = \frac{1}{1+r+\ell_{ND}^H}\left[(1-p_d)\left(m+(1-m)\left(z+q_{ND}^H\right)\right)+p_d q_D^H\right]$$

where

$$\ell_{ND} = (1-p_d)(1-m)\zeta\frac{q_{ND}^H - q_{ND}^L}{q_{ND}^H} + p_d\zeta\frac{q_D^H - q_D^L}{q_{ND}^H}$$

is the liquidity premium that is needed to equate the market price q_{ND}^H to the valuation of an investor who is not subject to liquidity concerns. Two observations are in order here. First, the value of the liquidity premium is pinned down by the bid-ask spreads. Thus, as long as the calibration generates bid-ask spreads that are in line with the data, the liquidity premia that we obtain are disciplined by the friction we observe empirically. Second, there will be a positive co-movement between liquidity and credit premia as long as bid-ask spreads are larger in default. When this holds, liquidity premia increase during bad times when default risk (p_d) is high.

How does liquidity risk drive default risk? Worsening liquidity conditions exacerbate default incentives by increasing debt rollover costs. To see this, consider a stationary debt structure $b_{t+1} = b_t$. In this case, the budget constraint (2.1) becomes $c_t = y_t - [m+(1-m)z]\,b_t + mb_t q_{ND,t}^H$. In the jump-to-default setting, the debt issuance price can also be written as

$$q_{ND}^H = \frac{(1-p_d)\left[m+(1-m)\left(z+q_{ND}^H\right)\right]+p_d q_D^H}{1+r} - EL,$$

where

$$EL \equiv \zeta\frac{(1-p_d)(1-m)\left(q_{ND}^H - q_{ND}^L\right)+p_d\left(q_D^H - q_D^L\right)}{1+r}$$

is price discount that compensates investors for expected losses due to secondary market illiquidity. Worse liquidity conditions lead to more substantial price discounts (EL) and

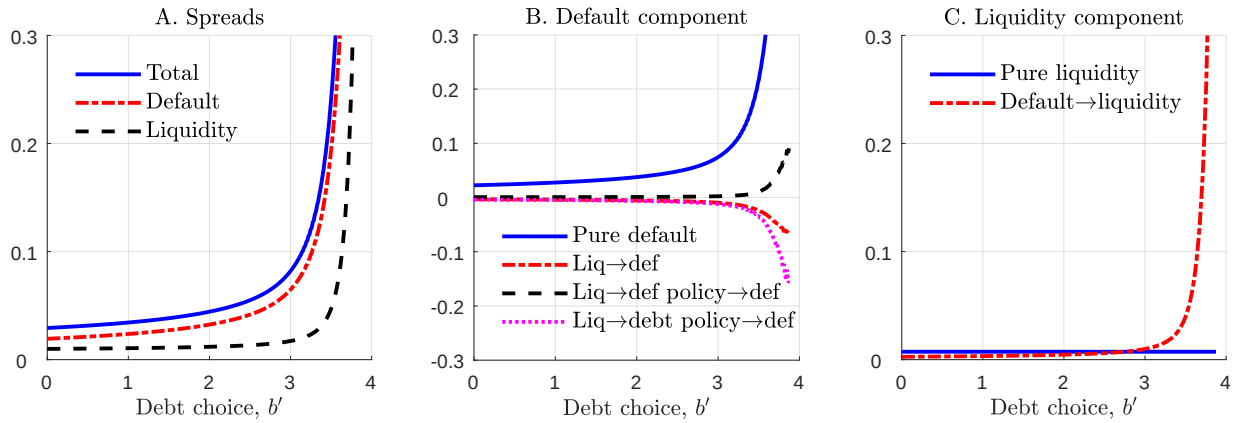

Figure 3.3: **Sovereign spread decomposition.** This figure plots the different components of spreads for a fixed value of y equal to 1. Panel A plots total spreads, the default component defined in equation (3.2), and the liquidity component defined in (3.3). Panel B plots the different components of credit spreads. Panel C plots the two components of the liquidity spreads defined in equation (3.11) in equation (3.12).

lower consumption through increased debt rollover costs. In turn, this generates more significant sovereign default incentives.

3.4 Liquidity, Policies, and Spreads

We now more closely examine the two components of spreads: default and liquidity. We would like to quantitatively answer the following two questions. First, what are the determinants of the credit and liquidity components of sovereign spreads? Second, how much of the default premium is explained by liquidity frictions? How much of the liquidity premium is explained by default risk?

A Closer Examination of the Default Risk Premium. We begin by analyzing the default component. To do so, we further decompose cs_{DEF} into two components:

$$cs_{DEF}(y, b') = cs_{DEF,DEF}(y, b') + cs_{LIQ \to DEF}(y, b'). \tag{3.5}$$

Denote by $\boldsymbol{B}_0, \boldsymbol{D}_0$ the debt and default policies for the baseline calibration when the probability of receiving a liquidity shock, ζ, is equal to zero.[20] These two terms are defined

[20]More precisely, we solve for the optimal policies when the parameters are those in Table 3.1, with $\zeta = 0$ instead of the baseline of $\zeta = 0.139$.

as:

$$cs_{DEF,DEF}(y,b') \equiv cs(y,b') \mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)}; \tag{3.6}$$

$$cs_{LIQ\to DEF}(y,b') \equiv cs(y,b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} - cs(y,b') \mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)} . \tag{3.7}$$

The first term, equation (3.6), is the pure default component. This is the spread that the sovereign would pay if there were no liquidity frictions. That is, the investors do not receive liquidity shocks, and the policies are those in the baseline calibration with no liquidity shocks. The second term, equation (3.7), is the liquidity-induced component of default spreads. This is the spread in addition to the pure default component that is caused by a change in policies due to liquidity frictions. However, the change in spreads comes only through a change in policies. Thus, we fix the probability of a liquidity shock at zero, i.e. $\zeta = 0$.

Depending on the response of debt and default policies when liquidity shocks are switched on, from $(\boldsymbol{B}_0, \boldsymbol{D}_0)$ to $(\boldsymbol{B}, \boldsymbol{D})$, the component $cs_{LIQ\to DEF}(y,b')$ might be positive or negative. For example, holding default policies fixed, if the government takes on less debt due to liquidity frictions, $cs_{LIQ\to DEF}(y,b')$ will be negative. Alternatively, fixing debt policies, if due to liquidity frictions the government defaults in more states of nature, then $cs_{LIQ\to DEF}(y,b')$ will be positive.[21] To quantify the response of debt and default policies, it will be useful to to further decompose $cs_{LIQ\to DEF}(y,b')$, into two terms:

$$cs_{LIQ\to DEF}(y,b') \equiv cs_{LIQ\to DEF,Debt} + cs_{LIQ\to DEF,Def}.$$

These two terms are defined as:

$$cs_{LIQ\to DEF,Debt} \equiv cs \mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)} - cs \mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)} \tag{3.8}$$

$$cs_{LIQ\to DEF,Def} \equiv cs \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} - cs \mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)} . \tag{3.9}$$

The first term, equation (3.8), is the change in spreads due to changes in default policies, while holding default policy fixed. This term measures the response of spreads to changes in the frictions in the secondary market that operate through debt policy. The second

[21] The intuition is as follows. Suppose that the default policy remains fixed when the friction ζ changes. On the one hand, an increase in the liquidity frictions might increase interest rates in the primary market, driving debt issuance down, and this might, in turn, imply a decrease in total spreads. On the other hand, this increase in the liquidity friction might induce the country to borrow even more, to sustain consumption, which in turn will exacerbate the increase in spreads caused by the liquidity friction. The same can occur with default policies: fixing debt policies, a change in the liquidity friction might induce the sovereign to default in a higher (lower) number of states of nature, which in turn implies higher (lower) spreads.

term, equation (3.9), is the change in spreads due to changes in default policies, holding debt policy fixed, and measures the response in spreads due to default policy.

Do liquidity frictions increase or decrease default risk? The overall response and the one of each of the components $cs_{DEF,DEF}, cs_{LIQ\to DEF}, cs_{LIQ\to DEF,Debt}, cs_{LIQ\to DEF,Def}$ are depicted in Figure 3.3 Panel B. Note that as we mentioned, in our calibration the component $cs_{LIQ\to DEF,Debt}$ is negative. This is because worse liquidity conditions, given default policies, will imply a more precautionary debt policy. Furthermore, note that the component $cs_{LIQ\to DEF,Def}$ is always positive.

A Closer Examination of the Liquidity Risk Premium. We can further decompose the liquidity premium into two terms:

$$cs_{LIQ}(y,b') = cs_{DEF\to LIQ}(y,b') + cs_{LIQ,LIQ}(y,b'). \tag{3.10}$$

These two terms are defined as follows:

$$cs_{LIQ,LIQ}(y,b') \equiv cs(y,b') \mid_{(\boldsymbol{B}_0,\boldsymbol{D}=0,\zeta)} \tag{3.11}$$

$$cs_{DEF\to LIQ}(y,b') \equiv cs(y,b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y,b') \mid_{(\boldsymbol{B},\boldsymbol{D},0)} - cs(y,b') \mid_{(\boldsymbol{B}_0,\boldsymbol{D}=0,\zeta)} . \tag{3.12}$$

The first component, equation (3.11), measures the spread when investors can receive liquidity shocks but the government never defaults on debt. The second term, equation (3.12), measures how much in addition to the spread due solely to liquidity shocks the government needs to pay to compensate for default risk. This term is a residual that measures the portion of the liquidity component that is not explained by the pure liquidity component, and thus, by default risk. Panel C of Figure 3.3 illustrates these two components. As one would imagine, the pure liquidity component does not vary with debt levels because this component is not sensitive to the default probabilities, which are affected by the output realizations. Note, however, that the default-induced liquidity component is increasing as debt increases, and a default is more likely. This increase in the liquidity premia, as a consequence of higher default risk, simply illustrates what was clear from the jump-to-default model: that default probabilities drive the variations in the liquidity premia.

Relationship to the Literature. The previous two decompositions highlight one of the key differences of the sovereign setting with respect to the decompositions for the corporate setting in He and Milbradt (2013) and Chen et al. (Forthcoming). Note that from (3.5)

Moment	Data	$h_c = 0$	$h_c = 0.0005$	$h_c = 0.001$	$h_c = 0.00145$
Mean Debt to GDP	1.00	1.017	1.012	1.007	1.002
Mean Spread	0.0815	0.0767	0.0785	0.0802	0.0815
Vol. of Spread	0.0443	0.0474	0.0466	0.0450	0.0436
Mean Bid-Ask Spread	0.0050	0	0.0017	0.0034	0.0049
Welfare	-	1.0164	1.0158	1.0152	1.0147

Table 3: Comparative Statistics.

and (3.10), we can define total spreads as:

$$cs(y, b') \equiv cs_{DEF,DEF}(y, b') + cs_{LIQ \to DEF}(y, b') + cs_{DEF \to LIQ}(y, b') + cs_{LIQ,LIQ}(y, b'). \tag{3.13}$$

This is precisely the decomposition of sovereign spreads studied in He and Milbradt (2013) and Chen et al. (Forthcoming). The main difference is that these papers feature a fixed debt policy and the outside option of default does not depend on future policies.[22] Thus, in that setup, one can show that all four terms in (3.13) are positive. The reason is that the response of spreads to liquidity frictions that comes through a change in debt policies, $cs_{LIQ \to DEF,Debt}$, is equal to zero. In addition, in both of these papers, the response to liquidity spreads that comes through default, $cs_{LIQ \to DEF,Def}$, is an increase in spreads, because the corporation defaults in more states of nature, unambiguously, in response to higher liquidity shocks.

3.5 Secondary Market Frictions and Welfare

In this section, we examine the implications of secondary market illiquidity for household welfare. We define welfare as the certainty equivalent consumption:

$$c\left(h_c\right) \equiv u^{-1}\left(\mathbb{E}\left[V^C\left(y, b = 0; h_c\right)\right]\right)$$

obtained by a sovereign with no initial debt ($b = 0$) operating in an economy in which constrained international investors are subject to holding costs h_c. We vary the illiquidity of secondary bond markets, as measured by bid-ask spreads, by varying the severity of liquidity shocks for international investors and consider the resulting welfare impli-

[22] In a corporate setting these assumption are standard. First, the default for equity-holders corresponds to bankruptcy, the value of which is usually exogenously fixed at zero (they optimally liquidate the firm when it has no value for them). Second, the rationale for a fixed debt policy is that the bond issue might have a covenant that restricts further issues, and this covenant is enforceable in court. This assumption of a constant debt structure is thus, usually, for simplicity.

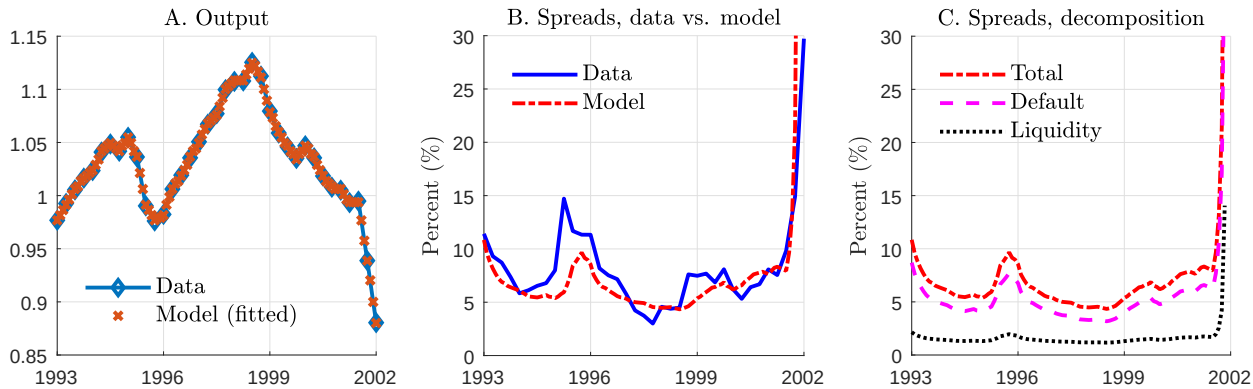

Figure 3.4: **Event Study: Argentina's Default, 2001:IV.** The first figure plots output, Argentinean spreads in the data, and Argentinean spreads generated by the model when the output realizations for Argentina are fed in. The second figure plots total spreads and the credit and liquidity components as defined in 3.3.

cations for the sovereign. In particular, we vary international investors' holding costs conditional on receiving a liquidity shock, h_c, starting with an economy in which international investors never become constrained (i.e. $h_c = 0$), and we then slowly worsen the severity of liquidity shocks by increasing holding costs, h_c.

The results of this exercise are summarized in Table 3. We see that the average bid-ask spread is increasing in holding costs. Importantly, household welfare decreases as the secondary bond market becomes more illiquid. In particular, decreasing bid-ask spreads from 50 basis points to zero results in a 0.17 percent increase in household welfare. To put this number in perspective, the welfare gain from eliminating the business cycle in a representative agent setting with CRRA preferences is 0.40 percent of consumption equivalent. Thus, the welfare gains from removing secondary market frictions are substantial. In particular, through the lens of our model, in that period, secondary market frictions can account for 42.5 percent of the cost of the business cycle for Argentina. [23]

3.6 Case Study: Argentina's Default in 2001

In this subsection, we conduct an event study of Argentina's default in December of 2001. For the policy functions of the baseline calibration, depicted in Table 1, we we input the output shocks that Argentina received in the 1990s and the initial level of debt. Panel A plots the level of GDP in logs. Panel B plots the total spreads generated by our model in

[23] Appendix C details the calculations of the welfare cost of the business cycles for Argentina following Lucas (2003). Note also that the welfare cost of business cycles is substantially higher once deviations from the the CRRA framework in Lucas (2003) are introduced. See, for example Krusell et al. (2009), Barlevy (2004) and Alvarez and Jermann (2004). In that case, the welfare losses from, in relative terms, liquidity frictions will be smaller.

Variable	Data	Model	CE (2012), Table 4
$\sigma(c)/\sigma(y)$	1.09	1.15	1.11
$\sigma\left(\frac{NX}{y}\right)/\sigma(y)$	0.17	0.09	0.20
$corr(c,y)$	0.98	0.98	0.99
$corr\left(\frac{NX}{y},y\right)$	-0.88	-0.497	-0.44
$corr(r-r^f,y)$	-0.79	-0.65	-0.65
Debt service	0.053	0.079	0.055
Default frequency	0.125	0.066	0.068

Table 4: **Business cycle properties.**

comparison with the data. Finally, Panel C plots total spreads and the liquidity and credit components computed using the decomposition in (3.5).

First, note that our model can replicate the most salient features of the series of spreads. The first spike is the Tequila crisis in Mexico in 1996, where there is a sharp increase in the spreads coming from a recession. In addition, we can see that for 2001 the model can correctly account for the spike in spreads and default. Note that although the recession started in 1998:III, spreads continued to be below 800 basis points until the beginning of 2001.

Second, in Panel C, we observe that that the liquidity premium is a larger portion of total spreads during good times, when total spreads are low. However, when the spreads began to spike in 1999, the liquidity component started to be a smaller portion of total spreads. In fact, in our calibration, it can explain 20 percent of the total spreads during the period of the crisis.

3.7 Business Cycle Properties

The model's business cycle properties are summarized in Table 4. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model performs well. As in the data, consumption is as volatile as output, and nearly perfectly correlated with it. The volatility of the current account relative to output volatility is 0.09 in the model, which is close to its empirical counterpart of 0.17. The model performs well at capturing counter-cyclical sovereign credit risk, with a correlation of -0.65 between the sovereign spread and output. In addition, there is a negative correlation, -0.50, between the current account and output. Finally, debt service (as a fraction of output) is 7.9 percent and the default frequency is 6.6 percent. Overall, the business cycle properties generated by the calibrated model are similar to those generated by the model in Chatterjee and Eyigungor (2012).

4 Conclusion

The quantitative literature of sovereign-debt has helped us to understand debt capacity, spreads, and welfare when the central friction is lack of commitment to repayment. The recent Sovereign Debt Crisis in Europe, however, has highlighted that there is substantial liquidity risk associated with sovereign lending. Motivated by these facts, in this paper we studied debt and default policy when the government lacks commitment, and there is search friction in the secondary market for sovereign bonds.

After proposing a tractable framework in which credit and liquidity risk constrain the choices of the government we proceeded to study the quantitative importance of liquidity in sovereign spreads and welfare. To do so, we calibrated our model to match the main features of the Argentinean default in the 1990s, one of the most widely studied cases for sovereign default. Our first result is that liquidity risk is a substantial component of sovereign spreads. In particular, in a model based decomposition, we found that around 30 percent of total sovereign spreads. Our second result is that liquidity also matters for welfare. In particular, through the lens of our model, a representative agent would be willing to pay around 42 percent of what she would be willing to pay to shut down business cycles to shut down liquidity frictions.

Why should we distinguish between credit and liquidity risk? Why should we incorporate liquidity risk in models of sovereign borrowing? An important reason to distinguish between credit and liquidity risk are the different normative implications of each friction. As we mentioned before, there are at least three reasons why the introduction of liquidity would matter for debt management policy. First, policies in the long run to fight spreads caused by lack of commitment might be different to those to combat spreads due to the lack of liquidity. Second, policies in the short run might also be different; for example, one strategy during bad times could be to capitalize financial intermediaries as opposed to decreasing government spending or repaying debt. Third, maturity and currency management policies might also differ. The benefit of spreading debt across currencies and maturities is to complete the market. However, the unintended consequence would be lower liquidity in the market for these bonds, which could undo any benefits due to higher costs. All of these are topics for further research.

References

Afonso, Gara and Ricardo Lagos, "Trade dynamics in the market for federal funds," Technical Report, Staff Report, Federal Reserve Bank of New York 2012.

Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 2006, *69* (1), 64 – 83.

_ **and Manuel Amador**, "Sovereign Debt: A Review," Working Paper 19388, National Bureau of Economic Research August 2013.

_ **, Satyajit Chatterjee, Harold Cole, and Zachary Stangebye**, "Self-Fulfilling Debt Crises, Revisited: The Art of the Desperate Deal," Technical Report, National Bureau of Economic Research 2017.

Alvarez, Fernando and Urban J Jermann, "Using asset prices to measure the cost of business cycles," *Journal of Political economy*, 2004, *112* (6), 1223–1256.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 2008, *98* (3), 690–712.

_ **and Ananth Ramanarayanan**, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 2012, *120* (2), pp. 187–232.

_ **and Yan Bai**, "Renegotiation policies in sovereign defaults," *American Economic Review*, 2014, *104* (5), 94–100.

_ **and** _ , "Fiscal austerity during debt crises," *Economic Theory*, 2017, *64* (4), 657–673.

Ashcraft, Adam B. and Darrell Duffie, "Systemic Illiquidity in the Federal Funds Market," *American Economic Review*, 2007, *97* (2), 221–225.

Atkenson, Andrew, Andrea Eisfeldt, and Rierre Olivier Weill, "The Market for OTC Derivatives," *Working Paper*, 2013.

Bai, Jennie, Christian Julliard, and Kathy Yuan, "Eurozone Sovereign Bond Crisis: Liquidity or Fundamental Contagion," *Working Paper*, 2012.

Bai, Yan and Jing Zhang, "Duration of sovereign debt renegotiation," *Journal of International Economics*, 2012, *86* (2), 252 – 268.

Bao, Jack, Jun Pan, and Jiang Wang, "The Illiquidity of Corporate Bonds," *The Journal of Finance*, 2011, *66* (3), 911–946.

Barlevy, Gadi, "The cost of business cycles under endogenous growth," *American Economic Review*, 2004, *94* (4), 964–990.

Beim, DO. and CO Calomiris, "Emerging Financial Markets," *McGraw-Hill*, 2001.

Benjamin, D and M Wright, "Recovery Before Redemption: A Theory of Sovereign Debt Renegotiation," *Working Paper*, 2009.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez, "International reserves and rollover risk," Technical Report, American Economic Review, Forthcoming 2017.

Bocola, Luigi and Alessandro Dovis, "Self-fulfilling debt crises: A quantitative analysis," Technical Report, National Bureau of Economic Research 2016.

Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler, "WHY DO EMERGING ECONOMIES BORROW SHORT TERM?," *Journal of the European Economic Association*, 2013, *11*, 67–100.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, pp. 2674–2699.

_ **and** _ , "A seniority arrangement for sovereign debt," *The American Economic Review*, 2015, *105* (12), 3740–3765.

Chen, Hui, Rui Cui, Zhiguo He, and Konstantin Milbradt, "Quantifying liquidity and default risks of corporate bonds over the business cycle," *The Review of Financial Studies*, Forthcoming.

Chen, Long, David Lesmond, and Jason Wei, "Corporate Yield Spreads and Bond Liquidity," *The Journal of Finance*, 2007, *62* (1), 119–149.

Cruces, Juan J. and Christoph Trebesch, "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 85–117.

Cui, Wei and Sören Radde, "Search-based endogenous asset liquidity and the macroeconomy," 2016.

Dovis, Alessandro et al., "Efficient sovereign default," *The Review of Economic Studies, forthcoming*, 2017.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, "Over-the-Counter Markets," *Econometrica*, 2005, *73* (6), 1815–1847.

Eaton, Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 1981, *48* (2), pp. 289–309.

Edwards, Amy, Lawrence Harris, and Michael Piwowar, "Corporate bond market transparency and transaction costs," *University of Pennsylvania-Financial Economists Roundtable*, 2004.

_ , _ , and _ , "Corporate Bond Market Transaction Costs and Transparency," *The Journal of Finance*, 2007, *62* (3), 1421–1451.

Fleming, Michael J., "Are Larger Treasury Issues More Liquid? Evidence from Bill Reopenings," *Journal of Money, Credit and Banking*, 2002, *34* (3), pp. 707–735.

Friewald, Nils, Rainer Jankowitsch, and Marti G. Subrahmanyam, "Illiquidity or credit deterioration: A study of liquidity in the {US} corporate bond market during financial crises," *Journal of Financial Economics*, 2012, *105* (1), 18 – 36.

Gazzani, Andrea and Alejandro Vicondoa, "The Real Effects of Liquidity Shocks in Sovereign Debt Markets: Evidence from Italy," Technical Report, working paper, European University Institute 2016.

Gutkowski, Violeta, "Sovereign Illiquidity and Recessions," 2017.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79* (1), 117 – 125.

_ and _ , "Debt dilution and sovereign default risk," *Available at SSRN*, 2015.

_ , _ , and Cesar Sosa-Padilla, "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 2016, *124* (5), 1383–1422.

He, Zhiguo and Konstantin Milbradt, "Endogenous Liquidity and Defaultable Bonds," *Econometrica*, 2013, *forthcoming.*

Kozlowski, Julian, "Long-Term Finance and Investment with Frictional Asset Markets," 2017.

Krusell, Per, Toshihiko Mukoyama, Ayşegül Şahin, and Anthony A Smith Jr, "Revisiting the welfare effects of eliminating business cycles," *Review of Economic Dynamics*, 2009, *12* (3), 393–404.

Lagos, Ricardo and Guillaume Rocheteau, "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 2009, *77* (2), 403–426.

_ , _ , and Pierre-Olivier Weill, "Crises and liquidity in over-the-counter markets," *Journal of Economic Theory*, 2011, *146* (6), 2169 – 2205.

Leland, Hayne and Klaus Toft, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *The Journal of Finance*, 1996, *51* (3), 987–1019.

Longstaff, Francis., Sanjay Mithal, and Eric Neis, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *The Journal of Finance*, 2005, *60* (5), 2213–2253.

Lucas, Robert E, "Macroeconomic priorities," *The American Economic Review*, 2003, *93* (1), 1.

Neumeyer, Pablo A. and Fabrizio Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 2005, *52* (2), 345 – 380.

Ottonello, Pablo and Diego Perez, "The currency composition of sovereign debt," *Mimeo*, 2018.

Pelizzon, Loriana, Marti G Subrahmanyam, Davide Tomio, and Jun Uno, "Sovereign credit risk, liquidity, and European Central Bank intervention: Deus ex machina?," *Journal of Financial Economics*, 2016, *122* (1), 86–115.

_ , Marti Subrahmanyam, Davide Tomio, and Jun Uno, "The Microstructure of the European Sovereign Bond Market: A Study of Euro-zone Crisis," *Working Paper*, 2013.

Pouzo, Demian and Ignacio Presno, "Sovereign default risk and uncertainty premia," *American Economic Journal: Macroeconomics*, 2016, *8* (3), 230–66.

Roch, Mr Francisco and Harald Uhlig, *The dynamics of sovereign debt crises and bailouts* 2018.

Sanchez, Juan, Horacio Sapriza, Emircan Yurdagul et al., "Sovereign default and the choice of maturity," *Journal of Monetary Economics, Forthcoming*, 2018.

Tauchen, George, "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 1986, *20* (2), 177–181.

Yue, Vivian Z., "Sovereign default and debt renegotiation," *Journal of International Economics*, 2010, *80* (2), 176 – 187.

Appendix

A Numerical Method

It is well-known that numerical convergence is often a problem in discrete-time sovereign debt models with long-term debt. To circumvent this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). This involves altering total output to be: $y_t + \epsilon_t$, where $\epsilon_t \sim trunc\, N(0, \sigma_\epsilon^2)$ is continuously distributed. As shown in Chatterjee and Eyigungor (2012), the noise component ϵ_t guarantees the existence of a solution of the pricing function equation. The government's repayment problem (2.3) is altered as follows:

$$V^C(y, b, \varepsilon) = \max_{b'} \left\{ (1-\beta)\, u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}(y', b') \right] \right\},$$

where the budget constraint is now given by:

$$c = y + \varepsilon - b\,[m + (1-m)\, z] + q_{ND}^H(y, b')\,[b' - (1-m)\, b]$$

which contains the randomization component ε. Debt choice is denoted as $b'(b, y, \varepsilon)$. We impose that $\varepsilon_t \equiv 0$ during the autarky regimes, meaning that the expression for the value to default remains the same; i.e.,

$$V^D(y, b) = (1-\beta)\, u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta V^{ND}(y', \mathcal{R}(b)) + (1-\theta)\, V^D(y, b) \right].$$

The default decision is given by:

$$d(y, b, \varepsilon) = 1_{\left\{ V^C(y,b,\varepsilon) \geq V^D(y,b) \right\}},$$

and depends on the randomization component. The continuation value is adjusted as follows:

$$V^{ND}(y, b) = \mathbb{E}_\varepsilon \left[\max \left\{ V^D(y, b), V^C(y, b, \varepsilon) \right\} \right],$$

to take into account the randomization component. Finally, bond prices are also adjusted to take into account the additional randomization variable:

$$q_{ND}^{H}\left(y,b'\right)=\mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c}\frac{1-d(y',b',\varepsilon')}{1+r}\left[m+(1-m)\left[\begin{array}{c}z+\zeta q_{ND}^{L}\left(y',b'\left(b',y',\varepsilon'\right)\right)\\+(1-\zeta)\,q_{ND}^{H}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right]\\+\frac{d(b',y',\varepsilon')}{1+r}\left[\zeta q_{D}^{L}\left(b',y'\right)+(1-\zeta)\,q_{D}^{H}\left(y',b'\right)\right]\end{array}\right\}$$

$$q_{ND}^{L}\left(b',y\right)=\mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c}\frac{1-d(y',b',\varepsilon')}{1+r}\left[-h_c+m+(1-m)\left[\begin{array}{c}z+(1-\lambda)\,q_{ND}^{L}\left(y',b'\left(b',y',\varepsilon'\right)\right)\\+\lambda q_{ND}^{S}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right]\\+\frac{d(y',b',\varepsilon')}{1+r}\left[-h_c+(1-\lambda)\,q_{D}^{L}\left(y',b'\right)+\lambda q_{D}^{S}\left(y',b'\right)\right]\end{array}\right\}$$

$$q_{D}^{H}\left(y,b\right)=\frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q_{D}^{H}\left(y',b\right)+(1-\zeta)q_{D}^{L}\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q_{ND}^{H}\left(y,\mathcal{R}(b)\right)$$

$$q_{D}^{L}\left(y,b\right)=\frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c+\lambda q_{D}^{S}\left(y',b\right)+(1-\lambda)q_{D}^{L}\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q_{ND}^{L}\left(y,\mathcal{R}(b)\right)$$

$$q_{ND}^{S}\left(y,b\right)=(1-\alpha_{ND})q_{ND}^{L}\left(y,b\right)+\alpha_{ND}q_{ND}^{H}\left(y,b\right)$$

$$q_{D}^{S}\left(y,b\right)=(1-\alpha_{D})q_{D}^{L}\left(y,b\right)+\alpha_{D}q_{D}^{H}\left(y,b\right)$$

The rest of the numerical scheme is standard and follows the routine outlined in Chatterjee and Eyigungor (2012). We summarize the scheme in 4 steps:

a. Start by discretizing the state space. This involves choosing grids $\{y_i\}_{i=1}^{N_y}$ and $\{b_j\}_{j=1}^{N_b}$ for output and debt. The grid points and transition probabilities for output are chosen in accordance with the Tauchen (1986) method and encompass ± 3 standard deviations of the unconditional distribution of output. In the baseline model the number of states for output is chosen to be $N_y = 200$. The grid points for debt values are uniformly distributed over the range $[0, b_{max}]$, with the upper limit, b_{max}, being chosen large enough so that this (implicit) constraint never binds in simulations. The baseline calibration has $b_{max} = 6.0$ and $N_b = 450$.

b. Next perform value function iteration. Given bond prices, update value functions V^C and V^D. The debt and default policies, $b'(\cdot)$ and $d(\cdot)$, are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, linear interpolation is used to obtain terms involving $\mathcal{R}(b)$.

c. Given the debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.

B Jump to Default

In this appendix, we describe a particular case of our model in which default probabilities are exogenous, and debt is fixed. The idea is to introduce a clear definition of the liquidity

premium, show how bid-ask spreads map onto the liquidity premium as a function of default risk, and to clarify the role of the features of the model in the results. Assume an unconditional constant default probability p^{LR} in each period. Then the pricing equations yield a system of 6 equations and 6 unknowns $\bar{q}^H_{ND}, \bar{q}^L_{ND}, \bar{q}^H_D, \bar{q}^L_D, \bar{q}^S_{ND}, \bar{q}^S_D$. The system is given by:[24]

$$
\begin{aligned}
\bar{q}^H_{ND} &= \frac{1}{1+r}\left[\left(1-p^{LR}\right)\left(m+(1-m)\left(z+\zeta\bar{q}^L_{ND}+(1-\zeta)\,\bar{q}^H_{ND}\right)\right)\right. \\
&\quad \left.+p^{LR}\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)\right], \qquad \text{(B.1)} \\
\bar{q}^L_{ND} &= \frac{1}{1+r}\left[\left(1-p^{LR}\right)\left(-h_c+m+(1-m)\left(z+\lambda\bar{q}^S_{ND}+(1-\lambda)\,\bar{q}^L_{ND}\right)\right)\right. \\
&\quad \left.+p^{LR}\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)\right], \\
\bar{q}^H_D &= \frac{1-\theta}{1+r}\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)+\theta\mathcal{R}\bar{q}^H_{ND}, \\
\bar{q}^L_D &= \frac{1-\theta}{1+r}\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)+\theta\mathcal{R}\bar{q}^L_{ND}, \\
\bar{q}^S_{ND} &= \bar{q}^L_{ND}+\alpha_{ND}\left(\bar{q}^H_{ND}-\bar{q}^L_{ND}\right), \\
\bar{q}^S_D &= \bar{q}^L_D+\alpha_D\left(\bar{q}^H_{ND}-\bar{q}^L_{ND}\right),
\end{aligned}
$$

where $\mathcal{R} \in [0,1]$ denotes the fraction recovered after a default. The solution to this system yields four value functions that depend on the unconditional default probability, p^{LR}, and the parameters of the model. Fix the unconditional default probability, p^{LR}, and suppose there is a short-run departure. In particular, the current default probability is p_d. After a default, the default probability will remain fixed at the long run probability of default,

[24] For ease of exposition, the equations do not include the free asset disposal conditions that guarantee non-negative prices. Bond prices in the presence of free asset disposal can be characterized as the solution to a linear complementarity problem.

p^{LR}.[25] Then, current prices are given by:

$$
\begin{aligned}
q_{ND}^{H} &= \frac{1}{1+r}\left[(1-p_d)\left(m+(1-m)\left(z+\zeta q_{ND}^{L}+(1-\zeta)\,q_{ND}^{H}\right)\right)\right. \\
&\quad \left.+p_d\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)\right], \\
q_{ND}^{L} &= \frac{1}{1+r}\left[(1-p_d)\left(-h_c+m+(1-m)\left(z+\lambda q_{ND}^{S}+(1-\lambda)\,q_{ND}^{L}\right)\right)\right. \\
&\quad \left.+p_d\left(-h_c+\lambda q_{D}^{S}+(1-\lambda)\,q_{D}^{L}\right)\right], \\
q_{D}^{H} &= \frac{1-\theta}{1+r}\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)+\theta\mathcal{R}\bar{q}_{ND}^{H}(p^{LR}), \\
q_{D}^{L} &= \frac{1-\theta}{1+r}\left(-h_c+\lambda q_{D}^{S}+(1-\lambda)\,q_{D}^{L}\right)+\theta\mathcal{R}\bar{q}_{ND}^{L}(p^{LR}), \\
q_{ND}^{S} &= q_{ND}^{L}+\alpha_{ND}\left(q_{ND}^{H}-q_{ND}^{L}\right), \\
q_{D}^{S} &= q_{D}^{L}+\alpha_{D}\left(q_{D}^{H}-q_{D}^{L}\right).
\end{aligned}
\tag{B.2}
$$

Note that difference between the first system of four equations and the second system is that in the first one $\bar{q}_{ND}^{H}(p^{LR})$,$\bar{q}_{ND}^{L}(p^{LR})$ are taken as given. This will permit us to take limits on p_d over $[0,1]$ and still have a well-defined system of equations. For both systems of equations, that define valuations (B.1) and (B.2), free disposal of the asset implies that all the valuations are non-negative.

Bid-Ask Spreads and Liquidity Premia. How do the observable frictions in the secondary market, the bid-ask spreads, map onto liquidity premia? To see the connection, rewrite the bond price (B.2) as:

$$
q_{ND}^{H}=\frac{(1-p_d)\left(m+(1-m)\left(z+q_{ND}^{H}\right)\right)+p_d q_{D}^{H}}{1+r+\ell_{ND}},
$$

where the endogenous liquidity component is:

$$
\ell_{ND}=(1-p_d)\,(1-m)\,\zeta\frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}}+p_d\zeta\frac{q_{D}^{H}-q_{D}^{L}}{q_{D}^{H}}. \tag{B.3}
$$

That is, ℓ_{ND} is the additional liquidity spread that is needed to equate the market price q_{ND}^{H} to the valuation of an investor who is not subject to liquidity concerns. Two obser-

[25] Note that this process captures the idea of mean reversion in the hazard rates, which is common in the literature on credit risk modeling (see, for example Longstaff et al. (2005)). Formally, we can consider an irreducible Markov chain with 2 states and transition matrix $\mathbf{P}$. The p^{LR} will be defined by the invariant distribution Π.

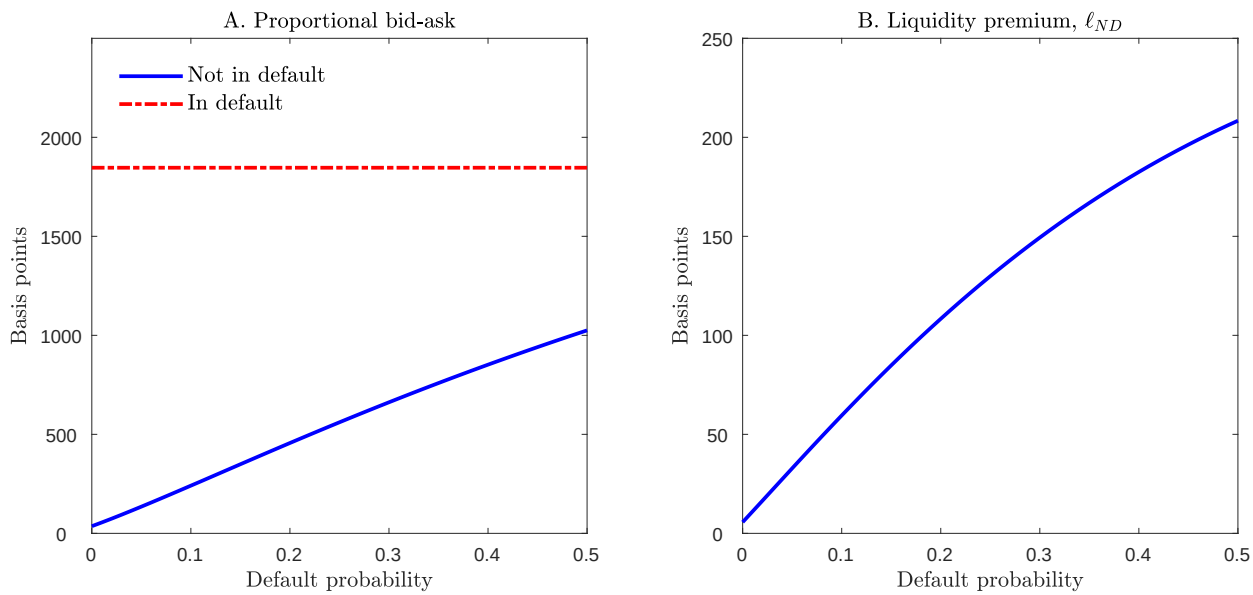

Figure B.1: **Liquidity Premium**. Panel A plots proportional bid-ask spreads $\left(q^H - q^S\right)/q^H$ in and out of default. Panel B plots the corresponding liquidity premium (B.3).

vations about this liquidity component are in order. First, it is the expected valuation loss after taking into account the loss conditional on having to sell and the probability of incurring a liquidity event and having to sell. Second, it depends on distance to default (or p_d); this is the case because the loss conditional on selling is higher during a default episode. Figure B.1 illustrates the relationship between proportional bid-ask spreads in and out of default, default probabilities, and the liquidity premium (B.3).[26]

Liquidity and Default. In our baseline model with endogenous default, worsening liquidity conditions exacerbate default incentives by increasing debt rollover costs. To see this, consider the stationary setting in which debt is fixed $b_t = b$. In this case, the budget constraint (2.1) becomes

$$c = y - [m + (1 - m)\, z]\, b + mbq^H_{ND}.$$

From this, we see that worsening liquidity conditions will increase default incentives if it leads to lower debt issuance prices q^H_{ND}. The link between liquidity conditions and issuance prices can be illustrated using our simple jump-to-default model by rewriting

[26]The numerical examples in this section use the following parameters: $z = 0.03$, $\theta = 0.0128$, $r = 0.0033$, $\zeta = 0.139$, $\alpha_{ND} = 0.125$, $\alpha_D = 0$, $\mathcal{R} = 0.3$, and $\bar{p}^{LR} = 0.03$.

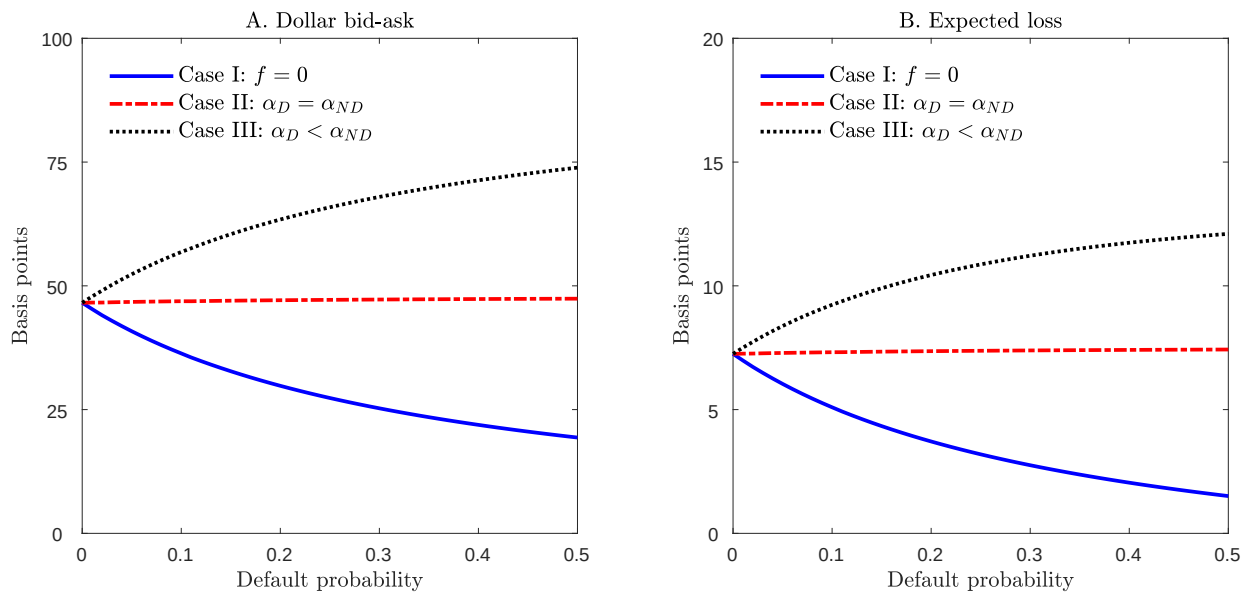

Figure B.2: **Liquidity Premium**. Panel A plots dollar bid-ask spreads $q^H_{ND} - q^S_{ND}$. Panel B plots expected loss due to secondary market illiquidity (B.4).

the bond price (B.2) as

$$q^H_{ND} = \underbrace{\frac{(1-p_d)\left[m + (1-m)\left(z + q^H_{ND}\right)\right] + p_d q^H_D}{1+r}}_{\text{Valuation for } \zeta=0} - EL,$$

where

$$EL \equiv \zeta \frac{(1-p_d)(1-m)\left(q^H_{ND} - q^L_{ND}\right) + p_d\left(q^H_D - q^L_D\right)}{1+r} \tag{B.4}$$

is the compensation for expected losses due to secondary market illiquidity.

A liquidity-default spiral will be present whenever increases in the likelihood of default (p_d) increase expected liquidity losses (EL). For this to happen, each of the model's ingredients are needed: long-term debt and frictions on the secondary market; positive recovery ($\mathcal{R}(b) > 0$); and, worse liquidity conditions during a default ($\alpha_D < \alpha_{ND}$). First, Long-term debt and frictions in the secondary market are needed to generate a bid-ask spread. That is, the following conditions need to hold: $\lambda \in (0,1)$ and $m < 1$. Second, recovery is needed for the bonds to have a positive price during default. If this were not to be the case, bid-ask spreads would not defined during the default state. Third, worser liquidity conditions during default are needed to generate higher bid-ask spreads during default.

Figure B.2 illustrates the role of the model's ingredients by plotting the relationship between liquidity premia and default probabilities for three different scenarios. The solid

lines plot the case in which there is long-term debt and frictions in the secondary market, but no recovery ($f = 0$). In this case, we see in Panels A and B that both dollar bid-ask spreads and expected losses decrease as default probabilities increase. Next, the dash-dot lines illustrate the case for which there is a positive recovery value for bonds ($f > 0$), but bondholders' bargaining does not decrease in default ($\alpha_D = \alpha_{ND}$). In this case, both dollar bid-ask spreads and expected losses remain constant as default probabilities increase. Finally, the dotted lines show the case for which both positive recovery and a post-default decrease in bondholders' bargaining power (i.e. $\alpha_D < \alpha_{ND}$) are present. Here, we see that liquidity premia increas as default becomes more likely. This implies the presence of the default-liquidity feedback channel: in the baseline model with endogenous default rates, increased liquidity premia increase the cost of rolling over debt which, in turn, further increases default incentives.

C Welfare Cost of Business Cycles

In order to put the welfare cost of liquidity frictions in perspective, we compute how much a representative agent in Argentina would pay in order to avoid fluctuations in consumption. The exercise follows Lucas (2003). In this Appendix we detail our calculations. As in the main body of the paper, the de-trended output process is given by:

$$y_t = e^{z_t} + \epsilon_t$$
$$z_t = \rho_z z_{t-1} + \sigma_z u_t.$$

We assume that the de-trended consumption process is AR(1) and that, in particular, follows:

$$c_t = y_t^{\kappa} \tag{C.1}$$

We choose $\kappa = 1.1$ so that $\frac{\sigma(c)}{\sigma(y)} = 1.1$; as in the data for Argentina. The value function of the stream of consumption (C.1) solves:

$$V(c) = (1-\beta)u(c) + \beta \mathbb{E}_{c'|c}\left[V\left(c'\right)\right].$$

Note that the expectation is taken with respect to the conditional distribution of consumption, where consumption is defined in (C.1), and that this distribution is derived from the distribution of output. The unconditional value function is then given by:

$$V \equiv \mathbb{E}_{c'}\left[V\left(c'\right)\right]$$

where the expectation is taken with respect to the unconditional distribution of consumption. What is the welfare of a constant stream of consumption c^E? This is the measure of welfare in terms of consumption equivalent. It is given by:

$$V\left(c^E\right) = (1-\beta)u(c^E) + \beta V\left(c^E\right).$$

This implies that the certainty equivalent of stream of consumption with value $\tilde{V}$ is given by:

$$c^E = u^{-1}\left(\tilde{V}\right).$$

For the CRRA case, this is given by:

$$c^E = \left[(1-\gamma)\,\tilde{V}\right]^{\frac{1}{1-\gamma}}.$$

How much consumption is the agent willing to resign in order to avoid shocks? In the absence of shocks, $u_t = 0$ for all t, then $\bar{c} = 1$. Therefore, the cost of business cycle: $1 - c^E$. The implementation of the calculation is as follows. First, $V(c)$ is computed numerically using our calibrated income process for Argentina (the smoothing component is ignored). Second, given $V(c)$ we obtain $V = \mathbb{E}_{c'}\left[V\left(c'\right)\right]$ using the Ergodic distribution of consumption implied by the process $c_t = y_t^{\lambda}$. Third, we obtain the consumption equivalent and therefore cost of the business cycle as $1 - c^E$. This value is 0.0040, or 0.4 percent in terms of consumption equivalent.