Illiquidity in Sovereign Debt Markets*

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January 23, 2018

Abstract

We study debt and default policy for sovereign countries when credit and liquidity risk are jointly determined. To account for both types of risks we focus on an economy with incomplete markets, limited commitment, and search frictions in the secondary market for sovereign bonds. We quantify the role of liquidity on sovereign spreads, debt capacity, and welfare, by performing quantitative exercises when our model is calibrated to match key features of the Argentinean default. We find that liquidity premia is a substantial component of spreads, increases during bad times and reductions in secondary market frictions improve welfare.

*We would like to thank Robert Townsend, Ivan Werning, George-Marios Angeletos, Alp Simsek, Nicolas Caramp, Arnaud Costinot, Claudio Michelacci, Athanasios Orphanides, Dejanir Silva, Robert Ulbricht, seminar participants at MIT, European University Institute, University of Naples, Toulouse School of Economics, Tinbergen Institute, Universidad Di Tella, and conference participants at SED Meetings in Poland, RIDGE, New Faces in Macro Madrid, LACEA Medellin, Central Bank of Uruguay, for helpful comments. We thank Marzio Bassanin and Adriana Grasso for excellent research assistance and the Macro-Financial Modeling Group from the Becker and Friedman Institute for financial support. All errors are our own. First Version: November 2014. This Version: January 2018.

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1 Introduction

Sovereign countries borrow in order to smooth shocks to income and tax revenue. One friction that prevents the smoothing of expenses over time and across states of nature is the inability for governments to commit to future debt and default policies. This inability to commit implies that the government will repay its debts only if it is convenient to do so and will dilute debt holders whenever it sees fit. To compensate investors for bearing these risks, it is necessary for the sovereign to pay a *credit risk premium*. This premium reduces the available resources for domestic consumption and can substantially increase borrowing costs during bad times. The sovereign debt literature has helped us to understand how a lack of commitment shapes the outcomes of sovereign countries from a positive point of view (see, for example, Arellano (2008) and Aguiar and Gopinath (2006)) and what policies are desirable from a normative point of view (see, for example, Aguiar and Amador (2013)).

The recent European debt crisis, however, has underscored that decentralized markets impose additional frictions that prevents smoothing for sovereign countries. Their bonds are traded mostly in over the counter markets, where trading is infrequent, so if an investor holds a large position in a sovereign bond it might take time to find a counterparty willing to trade at a fair price. For this reason investors need to be compensated not only for the risk of default or dilution, but also due to illiquidity, introducing in addition a *liquidity risk premium*. This liquidity premium further reduces available resources and constrains policies of the sovereign. So far, the literature has been silent about this feature of sovereign borrowing and our main objective is to fill this gap by answering the following questions: How does liquidity and credit premium interact? How much of total spreads can be explained by credit and how much by liquidity? What would be the welfare gains of reducing frictions in the secondary market?

Our paper contributes to the literature on sovereign borrowing in two ways. First, we propose a tractable model of sovereign borrowing in which credit and liquidity premia jointly determine borrowing and default decisions. Second, in a quantitative exploration focusing on one of the most studied cases of sovereign default, Argentina's default in 2001, we show that liquidity premia is a substantial component of total spreads. Not only that, we also show that the welfare gains from improving secondary market liquidity are substantial. In particular, within our setting, the welfare gains from such an improvement in liquidity are comparable to the gains from the elimination of business cycle fluctuations.

We start our paper by proposing a model of sovereign debt where debt and default policy take into account credit and liquidity risk. We study a small open economy that borrows from international investors to smooth income shocks following the quantitative literature of sovereign debt that builds on Eaton and Gersovitz (1981). A benevolent government designs debt and default policies in order to maximize the utility of the households by issuing non-contingent debt. The government cannot commit to future debt and default policies and might default in some states of nature. The distinctive feature of our model, in comparison to the previous literature, is the introduction of frictions in the secondary market for sovereign bonds, following the literature on over-the-counter markets as in Duffie et al. (2005). In our model, investors buy bonds in the primary market and can receive idiosyncratic liquidity shocks. If that occurs they will have a positive cost of carry and therefore become natural sellers of the asset. Due to search frictions in the secondary market, it will take time for them to find a counterparty with whom to transact.

In our model default and liquidity risk will be jointly determined. On the one hand, the presence of search frictions in the secondary market introduces a liquidity premia that affects prices in the primary market, thereby affecting debt and default policies, which in turn changes credit premia. On the other hand, as credit premium increases, the probability of default increases, and liquidity conditions will also deteriorate, because investors foresee worse liquidity conditions in the future. Therefore, in our model, default and liquidity premia are jointly determined. This joint determination is important because it will permit us to decompose total spreads into liquidity and credit components.

After building a model of sovereign borrowing where both credit and liquidity premia constrain the choices of the government, we perform quantitative exercises to assess how much of total spreads are due liquidity frictions and what would be the welfare gains of eliminating these frictions. In order to do so, we calibrate the model to match key features of Argentina's default in 2001. In particular, to match debt levels, mean spreads, and bid ask spreads, as well as turnover of debt.

Our first finding is that liquidity premia is a substantial component of total spreads. We find that around 30 percent of total spreads can be explained by liquidity frictions in the case of Argentina's 2001 default, during good times, through the lens of our model. As we explain in a simple version of the model where default probabilities are exogenous, the liquidity component of total spreads maps one to one with bid ask spreads. Intuitively, bid ask spreads measure the loss of an investor conditional on receiving a liquidity shock. The loss upon receiving the shock will in turn depend on whether the shock is received before or after default, because the liquidity conditions in these two states are different; in particular, in our calibration (and also in the data) liquidity conditions are worse during default. So, the liquidity premia will be a weighted combination of bid ask spreads before and after default where the weights are the default probabilities. Therefore, even though bid ask spreads before default might be low, large bid ask spreads after default in combination with a high default probability can translate in a high liquidity premium.

Our second finding is that that the welfare gains from eliminating liquidity risk are substantial. To explore the size of these welfare gains we perform an exercise following Lucas (2003). We find that, with respect to the calibrated benchmark, the welfare loss of secondary market frictions is 0.17 percent in consumption equivalent terms. To put this number in perspective, given the volatility of consumption for Argentina in the period of study, a representative agent would pay 0.24 percent of consumption equivalent to eliminate fluctuations in consumption.

We think that the distinction between credit and liquidity risk is important for the design of debt policies for three reasons. First, in the long run, the policies to fight lack of commitment are different than the policies to fight frictions in the secondary market. For example, fiscal rules and covenants have been associated as welfare improving in Hatchondo and Martinez (2015) and Chatterjee and Eyigungor (2015). However, policies in the long run that decrease liquidity premium would be the introduction of a centralized exchange for sovereign bond trading or increasing transparency in the secondary market as reported in Asquith et al. (2013). Second, the policies during a short term crisis might also be different. For example, a government could use resources to repay debt or to bailout financial institutions that hold government debt and are under distress. An alternative policy, focusing in the secondary market, would be providing liquidity to intermediaries. Finally, we think that liquidity premia has implications for debt management. By issuing debt in different currencies and maturities the government caters to investors and completes the market. However, this increase in the number of assets might imply low liquidity of each one of these bonds, which in turn increases the cost of debt for the government.

Our paper connects with different strands of the literature. Our model builds on the setting of the quantitative models of sovereign debt as in Aguiar and Gopinath (2006) and Arellano (2008); these two papers, extend the Eaton and Gersovitz (1981) framework of endogenous default to study business cycles in economies with risk of default. These early quantitative implementations study economies with short-term debt and no recovery on default. Long-term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). Chatterjee and Eyigungor (2012) introduce randomization to guarantee convergence of the numerical algorithm and show the existence of an equilibrium pricing function. Endogenous recovery of defaulted debt was introduced by Yue (2010). In our setting both long-term debt and recovery are crucial for the joint determination of credit and liquidity risk. Long term debt is modelled as in Chatterjee and Eyigungor (2012) and recovery depends on total debt defaulted. To keep the model numerically tractable, we abstract from a bargaining process because it is not crucial for our model. However, our reduced form recovery after default resembles the endogenous bargaining outcome in a setup as in Yue (2010). Our papers differs from the previous quantitative models of sovereign default by introducing a liquidity risk premia as a component of total spreads.

We build on the setting of over-the-counter markets first studied by Duffie et al. (2005). This framework was extended by Lagos and Rocheteau (2009) to allow for arbitrary asset holdings for investors. Lagos and Rocheteau (2007) studies the entry of dealers into the market.[1] Our paper structures the debt market as in Duffie et al. (2005) but to keep the model numerically tractable we follow He and Milbradt (2013) and we do not keep track of the asset holdings of high and low valuation investors.

Our paper is closely related to He and Milbradt (2013), which extends the models of corporate default as in Leland and Toft (1996) by introducing an over-the-counter market as in Duffie et al. (2005). This paper, uncovers a joint determination of liquidity and credit risk. Building on this framework Chen et al. (2013) and decompose spreads into a liquidity and credit component over the business cycle. Our paper extends the model of sovereign debt with long-term debt instruments as in Chatterjee and Eyigungor (2012) to account for liquidity frictions as in Duffie et al. (2005). Our paper differs from He and Milbradt (2013) and Chen et al. (2013) in two dimensions. First, there is a crucial qualitative difference between the sovereign and corporate settings. The value of default in our model is endogenously determined whereas in the corporate setting this value is fixed (does not depend on future liquidity conditions) and is zero in most cases. Second, in our setup total debt of the sovereign country is changing over the cycle. In the corporate setting the capital structure is usually fixed, and in particular in this two papers. The fact that total debt varies over the cycle could imply that the reponse to liquidity frictions is a decreases welfare via a decrease in debt levels.

Recent studies show that liquidity is a factor explaining sovereign spreads; this paper builds on these empirical findings. Pelizzon et al. (2013) study market micro-structure using tick by tick data and document the strong non-linear relationship between changes

[1] There has been an extensive literature following Duffie et al. (2005). Some examples are Lagos et al. (2011) which studies crises in over-the-counter markets; Afonso and Lagos (2012) which studies high frequency trading in the market for federal funds; Atkenson et al. (2013) which studies the decisions of financial intermediaries to enter and exit an over-the-counter market; Cui and Radde (2016) finds that secondary market frictions will imply endogenous financing constraints for firms.

in Italian sovereign risk and liquidity in the secondary bond market. Bai et al. (2012) find that most of the spread variations before the European sovereign debt crisis were due to liquidity and that most of the spreads were explained by credit risk in the onset of the crisis. Beber et al. (2009), on the contrary, show that for the Euro area, the majority of the spread is explained by credit risk.[2]Our paper complements these studies by quantifying the effects of liquidity frictions, uncovered in these paper, over spreads and welfare.

2 Model

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. The small open economy receives a stochastic stream of income denoted by y_t. Income follows a first order Markov process $\mathbb{P}(y_{t+1} = y' \mid y_t = y) = F(y', y) > 0$. The government is benevolent and wants to maximize the utility of the household. To do this it trades bonds in the international bond market smoothing the households consumption. The household evaluates consumption streams according to

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right].$$

The sovereign issues long-term debt. To simplify the maturity structure of government debt, we follow Chatterjee and Eyigungor (2012). Each unit of outstanding debt will mature with probability m. If the unit does not mature, it pays a coupon z.[3] The advantage of this formulation of debt is that it is memory-less; whether debt was issued 1 or n periods before, the probability that this debt will mature next period will be m.

[2] Ashcraft and Duffie (2007) find evidence of trading frictions in the pricing of overnight loans in the federal funds market. Fleming (2002) finds evidence of liquidity effects in treasury markets. The evidence showing that liquidity is a factor explaining the spread of corporate bonds is more established. Longstaff et al. (2005) use data of credit default swaps to measure the size of the default and non-default component of credit spreads. They find that most of the spread is due to default risk and that the non default component is explained mostly by measures of bond illiquidity. Bao et al. (2011) show that there is a strong link between illiquidity and bond prices. Edwards et al. (2007) study transaction costs in over-the-counter (OTC) markets and find that transaction costs decrease significantly with transparency, trade size, and bond rating, and increase with maturity. Friewald et al. (2012) liquidity effects account for approximately 14 per cent of the explained market-wide corporate yield spread changes. Chen et al. (2007) also find that liquidity is priced into corporate debt for a wide range of liquidity measures after controlling for common bond-specific, firm-specific, and macroeconomic variables.

[3] In the sovereign setting long-term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012); these two papers model long-term debt as consols, without a coupon; the approach is analogous to the one we adopt in this paper. The probabilistic maturity structure was introduced in the corporate setting by Leland and Toft (1996).

Therefore, the relevant state variable to measure the obligations of the government due in next period is the face value of debt.

There is limited enforcement of debt. Therefore, the government will repay debts only if it is more convenient to do so. There are two consequences of default. First, the government loses access to the international credit market so it is effectively in autarky. It regains access next period with probability θ. Once the government regains access the face value of debt will be $\mathcal{R}(b_t) = \min\left\{\bar{b}, b_t\right\}$, where b_t is the amount of defaulted debt and $\bar{b}$ is a maximum recovery value.[4] Second, during default output is lower and given by $y_t - \phi(y_t)$. So, in addition to market exclusion there is a direct output cost of default, a standard assumption in the literature.

There are two markets for debt. In the primary market, the government can sell bonds at a price q_t. The price of debt will depend on next periods bond position and current income. Our convention is that $b_{t+1} > 0$ denotes debt and $b_{t+1} < 0$ denotes savings. In the case of borrowing, after paying debt that matured this period mb_t, and the coupon on outstanding debt $(1-m)zb_t$, the country increases its debt position to b_{t+1}. The capital inflow that the country receives today is given by $q_t[b_{t+1} - (1-m)b_t]$. The budget constraint for the economy is then

$$c_t = y_t - [m + (1-m)\,z]\,b_t + q_t[b_{t+1} - (1-m)b_t].$$

In the secondary market, government debt can be resold.

2.2 Investors

There are two types of investors (high valuation and low valuation) and two markets (primary and secondary). High valuation investors are risk neutral and discount payoffs at the rate r. They are the only type of investors in the primary market buying debt from the government. An investor with high valuation receives an idiosyncratic liquidity shock that is uninsurable; with probability ζ the investor will become liquidity constrained and will bear a holding cost h_c. Once a high valuation investor receives a liquidity shock

[4] In models with endogenous recovery, as in Yue (2010), the maximum amount that international investors can recover after a default is bounded by the net present value of the country's endowment. Depending on the relative outside options, recovering zero for investors or consuming forever the endowment for the sovereign, an endogenous recovery rate is determined via Nash bargaining. To keep the model simple, we abstract from the bargaining process and adopt a reduced form upper bound $\bar{b}$ on the amount of resources to be recovered. The fraction recovered will be $\min\left\{\bar{b}, b_t\right\}/b_t$ that converges to zero as the amount of debt converges to infinity, to discourage infinite debt dilution by the sovereign just before default.

she becomes a low valuation investor (liquidity constrained) and is a natural seller of the asset; she values the asset less than the high valuation investors. The low valuation investor will sell the bond in the secondary market. As in Duffie et al. (2005), there is a search friction: a low valuation investor will meet a counterparty with probability λ. Once this investor meets an intermediary and sells, she exits the market. We will denote the valuations of the investors by q^H_{ND}, q^L_{ND} for debt before default and q^H_D, q^L_D for debt in default for the high and low valuation investors respectively.

2.3 Intermediaries

Our formulation of the secondary market follows He and Milbradt (2013). There is a continuum of intermediaries (broker dealers) in perfect Bertrand competition holding no stock as in Duffie et al. (2007). The intermediary buys from high valuation investors (H) and resells immediately to low valuation investors (L). The intermediaries contact low valuation investors with probability λ. We will assume that there is a big mass of high valuation investors ready to buy in the primary market or in the secondary market.[5] There is Nash bargaining between the intermediary and the investors. We assume that the bargaining power of the high and low valuation investors is zero and α_D, α_{ND} during default, and before default, respectively.

Bid-Ask Spreads. The surplus for an intermediary that is trading with the high valuation investors is given by

$$S_H = A - M$$

where A is the asking price at which they are buying from high valuation investors and M is the price at which the intermediary buys in the inter-dealer market. This surplus is zero because of Bertrand competition, the assumption that there is a high mass of high valuation investors, and the zero inventory restriction. Therefore, $S_H = 0$ and this implies $A = M$. The surplus of the high valuation investors is $\left(q^H_i - A\right) - q^H_{i,0}$, where $q^H_{i,0}$ denotes the valuation of the high valuation investor that has no bonds (where $i \in \{D, ND\}$). Because they have no bargaining power, they have a surplus of zero. Also, $q^H_{i,0} = 0$, since the value of not having the asset is the claim on any future surplus; because this surplus is zero, the price is zero. Then

$$A = M = q^H_i. \tag{2.1}$$

[5]The assumption that the number of type H investors is much higher than the number of type L investors is for tractability since it allows us to avoid keeping track of the distribution of asset holdings. These investors are ready to jump in and buy the bonds and will have no bargaining power.

Trading between the intermediary and the low valuation investor determines the selling price. The surplus for an intermediary trading with the low valuation investors is given by

$$S_L = M - B = q_i^H - B.$$

The surplus of the low valuation investors is given by $\left(B - q_{i,0}^L\right) - q_i^L$. Because the low valuation investors exit the market once they sell, $q_{i,0}^L = 0$. The total surplus (investors plus intermediary) is then $q_i^H - q_i^L$. The bid price is such that the intermediary gets α_i (from Nash bargaining) of the total surplus and is given by

$$B = q_i^L + \alpha_i(q_i^H - q_i^L). \tag{2.2}$$

From (2.1) and (2.2) the bid ask spread will be

$$A - B = (1 - \alpha_i)(q_i^H - q_i^L).$$

Finally, note that the bargaining power of the intermediaries will depend on the state of the economy.

2.4 Timing

In period t, if the government has credit access (is not in default), it starts the period with b_t bonds outstanding. For these bonds the government will have to pay a coupon and pay principal as they mature. The total amount due in period t is $[m + (1 - m) z]\, b_t$. Then, income y_t is realized. After income is realized, the government decides whether to default or not $d_t \in \{0, 1\}$. If the government does not default, it issues $[b_{t+1} - (1 - m)b_t]$ debt in the primary market to the high valuation investors at a price $q_{ND}^H(y_t, b_{t+1})$. If the government decides to default, consumption this period is $c_t = y_t - \phi(y_t)$. The investors who started the period as low valuation investors will find an intermediary with probability λ and will sell at a price $q_{ND}^{Sale}(y_t, b_{t+1})$. Then, with probability ζ the high valuation investors will receive a liquidity shock, so they will start bearing a holding cost $h_c > 0$. If the government decides to default in period t, it will re-access the debt market in period $t + 1$ with debt $\mathcal{R}(b_t)$ with probability θ.

In period t, if the government has no credit access (is in default), it starts the period with current defaulted debt b_t. Income y_t is realized and consumption c_t is given by $y_t - \phi(y_t)$. Investors who started the period as low valuation investors will find an intermediary with probability λ and will sell at a price $q_D^{Sale}(y_t, b_{t+1})$. With probability ζ the

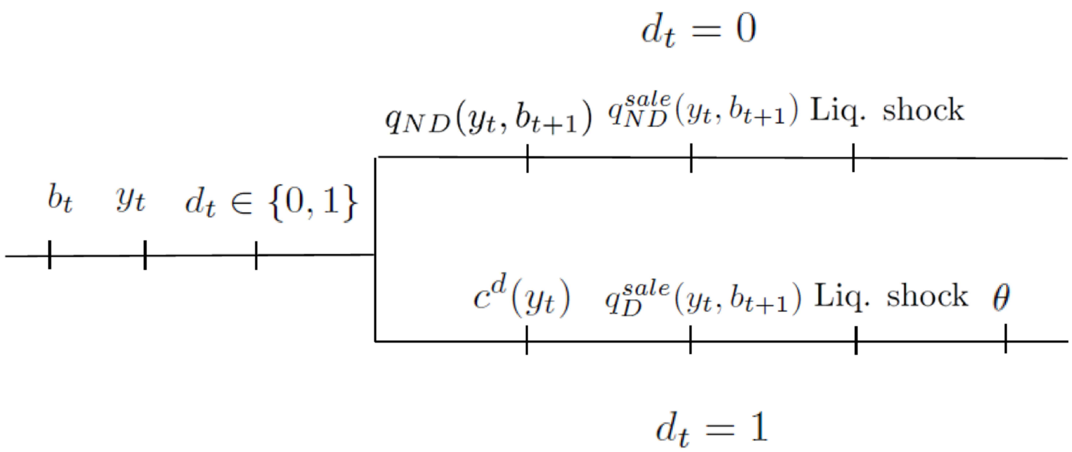

Figure 2.1: This figure summarizes the timing before and after default in period t. The government enters the period with bonds b_t. Then income y_t is realized and the government chooses whether or not to default. The upper branch depicts what happens when the government does not default. In this case, it issues debt in the primary market to the high valuation investors. The new face value of debt after the issue is b_{t+1}. Then, liquidity constrained investors can sell their debt positions if they meet an intermediary. Finally the liquidity shock is realized. The lower branch depicts what happens in the case that the government defaults. In this case, consumption is equal to $c^d(y_t) = y_t - \phi(y_t)$. Thus, liquidity constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Finally, the government will re-access the debt market next period with probability θ. If the government is in default, the timing is depicted by the lower branch of the figure.

high valuation investors receive a liquidity shock, so they will start bearing a holding cost $h_c > 0$. With probability θ the government will reaccess the international debt market in $t+1$ with outstanding debt $\mathcal{R}(b_t)$. Figure 2.4 summarizes the timing.

2.5 Decision Problem of the Government

We represent the infinite horizon decision problem of the government as a recursive dynamic programming problem. The model has one endogenous state variable b and one exogenous state variable y. We focus on a Markov equilibrium with state variables (b, y). The value of a government that has the option to default V^{ND} is the maximum between the values of defaulting on its debt and repayment. At a particular state (b, y) this value is given by

$$V^{ND}(b, y) = \max_{\{D,C\}} \left\{ V^D(b, y), V^C(b, y) \right\}$$

where $V^D(b,y)$ and $V^C(b,y)$ are the values of defaulting and repaying respectively. The value of a government that defaults on its debt is

$$V^D(b,y) = u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta V^{ND}(\mathcal{R}(b), y') + (1-\theta) V^D(b,y) \right].$$

The first term measures the flow utility: because the government defaults, the household consumes $y - \phi(y)$ instead of y. In the next period, with probability θ the government will regain access to the international debt market with an outstanding debt of b. With probability $(1-\theta)$ it will remain in default. The value of a government that chooses to repay its debt is given by

$$V^C(b,y) = \max_{b'} \left\{ (1-\beta) u(c) + \beta \mathbb{E}_{y'} \left[V^{ND}(b', y') \right] \right\}$$

where consumption is given by the budget constraint

$$c = y - [m + (1-m)z]\, b + q^H_{ND}(y, b') \left[b' - (1-m) b \right].$$

In addition, the government faces an upper bound in the ex-ante expected default probability when issuing debt. That is

$$\delta(y, b') = \mathbb{E}_{y'|y}(d(y', b')) \leq \bar{\delta}$$

when $b' - (1-m)b > 0$. As we explain in Section 2.9 for liquidity and credit risk to be jointly determined we need a setting with long term debt and recovery. In this setting, because the government knows that it will only repay a fraction $\mathcal{R}(b)$ of the defaulted debt, it has an incentive to issue debt at low prices right before default; i.e. an incentive to completely dilute bond holders. However, this behavior is not observed in the data. Thus, in order to dis-incentive this behavior, on top of the standard assumption in the literature of a cost of default that depends on output (see for example Arellano (2008) and Mendoza and Yue (2012)), we introduce an upper bound on the ex-ante default probability of issued bonds as in Chatterjee and Eyigungor (2015). As noted in Chatterjee and Eyigungor (2015), there are constraints on the implied default probability at which can be issued in the public market.

The default policy can be characterized by default and repayment sets. Let $D(b)$ be the income levels such that the government prefers to default on its debt

$$D(b) = \left\{ y \in Y : V^C(b,y) < V^D(b,y) \right\}.$$

When the borrower repays its debt, the policy function for debt issue is given by

$$b' = b'(b, y).$$

2.6 Valuations of Debt: Before Default

In this section we define the valuations of the high and low valuation investors before default. Suppose that the government has not decided to default in the state (b, y). The value of debt for the *high valuation investors* if the government wants to issue $b' - (1-m)b$ so that total debt increases to b' is $q^H_{ND}(b', y)$ solves the following functional equation

$$q^H_{ND}(b', y) = \mathbb{E}_{y'} \left\{ (1 - d(b', y')) \frac{m + (1-m)\left[z + \zeta q^L_{ND}(b'', y') + (1-\zeta) q^H_{ND}(b'', y')\right]}{1+r} \right.$$
$$\left. + d(b', y') \frac{\zeta q^L_D(b', y') + (1-\zeta) q^H_D(b', y')}{1+r} \right\}. \tag{2.3}$$

The payoffs for the investor are as follows. If the government does not default on its debt in the next period, $d(b', y') = 0$, the investors will receive the fraction m of the debt that is maturing and the coupon on the remaining fraction $(1-m)$ given by $z(1-m)$. With probability ζ in the next period they will receive a liquidity shock, so their remaining debt $(1-m)$ will have a value $q^L_{ND}(b'', y')$ for them. With probability $(1-\zeta)$ they will receive no liquidity shock and will value debt at $q^H_{ND}(b'', y')$. Note that b'' is the optimal policy for the government in the next period in the event they do not default. Should default occur, the government cannot borrow but keeps the defaulted debt b'. If the government does default on its debt in the next period, $d(b', y') = 1$, the investors will receive neither principal nor coupon payment; the debt will be valued $q^L_D(b', y')$ and $q^H_D(b', y')$ if they receive a liquidity shock and if they do not, respectively. Note that, since these investors are not currently liquidity constrained, they discount at the rate r_U.

The price of debt for a *low valuation investor* solves the following functional equation:

$$q^L_{ND}(b', y) = \mathbb{E}_{y'} \left\{ (1 - d(b', y')) \frac{-h_c + m + (1-m)\left[z + (1-\lambda) q^L_{ND}(b'', y') + \lambda q^{Sale}_{ND}(b'', y')\right]}{1+r} \right.$$
$$\left. + d(b', y') \frac{-h_c + (1-\lambda) q^L_D(b', y') + \lambda q^{Sale}_D(b', y')}{1+r} \right\} \tag{2.4}$$

If the government does not default on its debt in the next period, $d(y', b') = 0$, investors will receive the fraction of debt m that is maturing and the coupon on the remaining fraction of debt $(1-m)$ given by $z(1-m)$. With probability λ in the next period they will find an intermediary to trade their debt and will sell it at a price $q_D^{Sale}(y', b'')$. Otherwise, the investor will keep the debt and his valuation for it will be given by $q_{ND}^L(y', b'')$. Again b'' is the optimal policy for the government in the next period. The sale price is the outcome from the bargaining with the intermediary and is given by

$$q_{ND}^{Sale}(b', y) = \alpha q_{ND}^H(b', y) + (1-\alpha) q_{ND}^L(b', y)$$

If the government does default on its debt next period, $d(y', b') = 1$, the investors will receive neither debt nor coupon payment; the debt will be valued $q_D^L(y', b'')$ and $q_D^H(y', b'')$ if they receive a liquidity shock and if they do not, respectively. The sale price in this case is

$$q_D^{Sale}(b', y) = \alpha_D q_D^H(b', y) + (1-\alpha_D) q_D^L(b', y)$$

Note that we assume that the probability of finding a counterparty to trade is lower when the government is in default.

2.7 Valuations of Debt: After Default

Suppose that the government decides to default or enters the period without market access, with current outstanding debt b, and the income realization is y. The value of debt for the high valuation investors when the government is in default solves the following functional equation

$$q_D^H(b, y) = \frac{1-\theta}{1+r} \mathbb{E}_{y'} \left[\zeta q_D^H(b, y') + (1-\zeta) q_D^L(b, y') \right] + \theta \frac{\mathcal{R}(b)}{b} q_{ND}^H(y, \mathcal{R}(b)) \tag{2.5}$$

With probability $(1-\theta)$ the default does not get resolved. Therefore, the value of the debt *next period* will be $q_D^H(y', b)$ and $q_D^L(y', b)$ if they receive or not the liquidity shock, respectively. With probability θ default gets resolved and the investors receive a fraction f for every dollar of debt they have. They value this debt at $q_{ND}^H(y, \mathcal{R}(b) \times b)$ given by (2.3).

The value of debt for the low valuation investors when the government is in default solves the following functional equation

$$q_D^L(y, b) = \frac{1-\theta}{1+r} \mathbb{E}_{y'} \left[-h_c + \lambda q_D^{Sale}(b, y') + (1-\lambda) q_D^L(b, y') \right] + \theta \frac{\mathcal{R}(b)}{b} q_{ND}^L(y, \mathcal{R}(b)) \tag{2.6}$$

With probability $(1-\theta)$ the default does not get resolved. With probability λ the liquidity

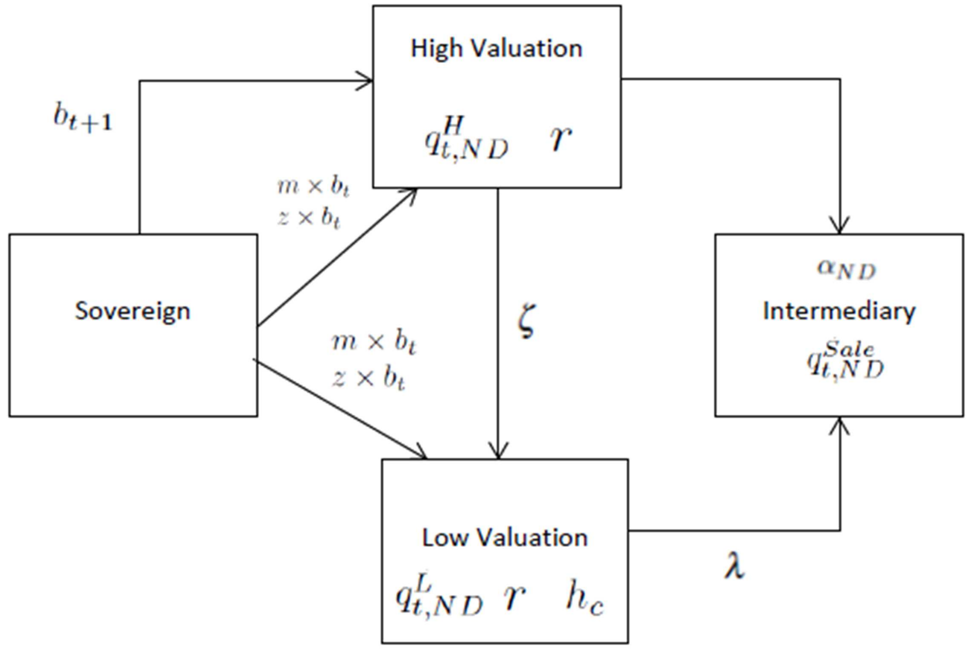

Figure 2.2: This figure details the bond market if the sovereign is not in default and does not default in period t. It starts by issuing debt b_{t+1}. This debt is bought by the high valuation investors in the primary market. After that, with probability λ the low valuation investors will meet an intermediary. They will sell their bonds at the price $q^{Sale}_{t,ND} = q^L_{t,ND}$. After selling their bonds they exit the market. The low valuation investors that do not meet an intermediary will try to sell their bonds next period. Then, with probability ζ, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond next period in the secondary market. Both the high and low valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

constrained investors find an intermediary and they will sell the defaulted bond at a price $q^{Sale}_D(y', b)$ given by

$$q^{Sale}_D(b, y) = \alpha_D q^H_D(b', y) + (1 - \alpha_D) q^L_D(b', y)$$

With probability $(1 - \lambda)$ they do not find an intermediary so they keep the unit of debt which they value it at $q^L_D(b, y')$. With probability θ the default gets resolved, they collect $\mathcal{R}(b)$ for every unit of debt they had. Their valuation for this debt is $q^L_{ND}(\mathcal{R}(b) \times b, y)$ given by equation (2.4).

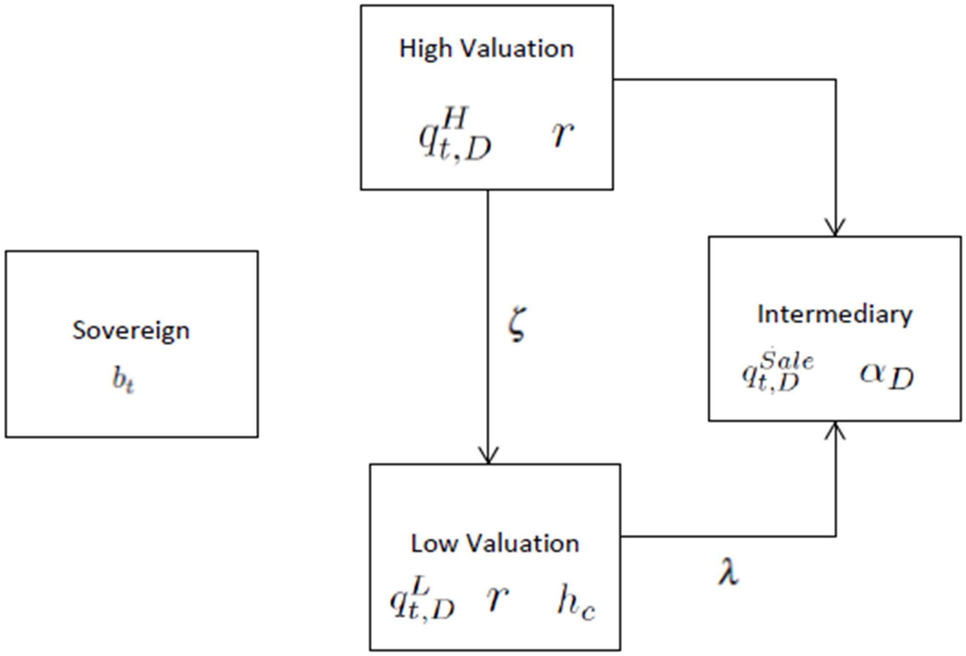

Figure 2.3: The figure details the bond market if the government is in default or defaults in period t. There is no debt issue or debt service. The sovereign has an outstanding balance of debt b_t. The low valuation investors will meet an intermediary with probability λ_D. They will sell their bonds at the price $q^{Sale}_{t,D} = q^L_{t,D}$. After they sell the bond they exit the market. The low valuation investors that do not meet an intermediary will try to sell next period. Then, with probability ζ, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell next period in the secondary market. Finally, with probability θ, the government resolves the default and re-accesses the next period with face value of debt $b_t \times \mathcal{R}(b_t)$.

2.8 Equilibrium

We focus on a Markov equilibrium with state variables (b, y). An *equilibrium* is a set of policy functions for consumption $c(b, y)$, default $d(b, y)$, and debt $b'(b, y)$ such that: taking as given the bond valuation q^H_{ND}, the policy function for consumption $c(b, y)$, debt issue $b'(b, y)$ and the default set $D(b)$, solve the borrowers optimization problem; the bond valuation functions

$$q^H_{ND}(b, y), q^L_{ND}(b, y), q^H_D(b, y), q^L_D(b, y)$$

satisfy (2.3) (2.4), (2.5) and (2.6) when default $d(b', y')$ is consistent with $D(b')$.

2.9 Discussion

We now explain why each one of the modelling assumptions in the model are needed. Our main objective is to provide a framework to study debt and default policy when credit and liquidity risk are jointly determined. For this joint determination in our model we need two equilibrium outcomes: positive bid ask spreads during credit access and also during default. Our modeling choices have the aim of developing the simplest model that generates these two features and can account for debt capacity and spreads of sovereign countries. In Appendix 4 we make these points explicit with a simple jump to default model in which policies are fixed and there is an exogenous default probability.

First, in order to generate bid ask spreads before default we need a friction on the secondary market, $(\lambda, \alpha_D, \alpha_{ND}) \in (0,1)^3$ and long term debt $m > 1$. If all the debt matures in one period then the investors face no liquidity risk. If the investors can sell the asset immediately then there is no loss of receiving a liquidity shock. The consequence of these frictions will be a difference in valuation between liquidity constrained and unconstrained investors; this difference is measured by $q^H_{ND} - q^L_{ND}$. Note that there is substantial evidence of frictions in the secondary market both for developed and developing countries (for recent evidence see Pelizzon et al. (2013)). Long term debt has been studied in several papers in the quantitative literature of sovereign debt (see for example Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009)) and in corporate finance (see for example Leland and Toft (1996)) due to the fact that sovereigns and corporations use a composite of maturities to smooth income and cash flow shocks.

Second, in order to generate bid ask spreads during default we need a positive recovery value, $\mathcal{R}(b) > 0$. If there is no recovery, $\mathcal{R}(b) = 0$, bond prices will be zero during default $q^H_D = q^L_D = 0$. This implies that the bid ask spread during default is zero, and more importantly, as we approach to default the liquidity premium will approach to zero. There is substantial evidence for positive recovery in the data (see for example Cruces and Trebesch (2013) and Yue (2010)).

Finally, an additional feature that we will take into account is that liquidity conditions worsen after default. We introduce this in reduced form by an increased bargaining power of the intermediaries $\alpha_D > \alpha_{ND}$. This will imply that bid ask spreads are higher during default. There is substantial evidence of this for corporate Bonds (see for example, He and Milbradt (2013) and references therein). For sovereign bonds, to the best of our knowledge, there is no systematic study. Still, using data from Bloomberg, we find that for defaulted bonds prices are updated less frequently and are for some bonds and on ocassions constant for weeks; both indicative of lower trading.

3 Numerical Results

We follow a discrete state space method to solve for the equilibrium. As is discussed in Chatterjee and Eyigungor (2012) grid based methods have poor convergence properties when there is long-term debt. To overcome this problem we follow their prescription and compute a "slightly" perturbed version of the model described in this section. The details are given in the Numerical Appendix.

3.1 Calibration

We calibrate the model developed in Section 2 to account for the main features of Argentina's default in 2001. We choose to work with Argentina in the period of 1993:I and 2001:IV for three reasons. First, it makes the comparison with previous studies in the literature that focused on this case and period easy; see for example Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), Arellano (2008). Second, this is a recent episode of default and with secondary market trading of defaulted debt. Third, during the period of 1993:I to 2001:IV Argentina had a fixed exchange rate with the dollar and was borrowing in international debt markets. The fact that Argentina was on a fixed exchange regime will provide a better mapping to the model developed on Section 2.

Functional Forms and Stochastic Processes. The utility function is CRRA $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The endowment process follows

$$\begin{aligned} y_t &= e^{z_t} + \epsilon_t \\ z_t &= \rho_z z_{t-1} + \sigma_z u_t \end{aligned}$$

with $\rho_z \in (0,1)$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2), u_t \sim N(0,1)$. So, the endowment process is given by the sum of an AR(1) process z_t plus a continuous shock ϵ_t.[6] The loss in terms of output during default is given by

$$\phi(y) = \max\left\{0, d_y y + d_{yy} y^2\right\}.$$

This loss function follows the one proposed by Chatterjee and Eyigungor (2012). This default cost function nests several cases in the literature. As is explained in Chatterjee and Eyigungor (2012), when $d_y < 0$, $d_{yy} > 0$, the cost is zero when $0 \leq y \leq -\frac{d_y}{d_{yy}}$ and

[6] In the Appendix we provide the details of the iid random shock to income that is introduced to guarantee convergence of the numerical procedure.

rises more than proportionally with output $y > -\frac{d_y}{d_{yy}}$. Alternatively, when $d_y > 0$ and $d_{yy} = 0$ the cost is a linear function of output. The case studied in Arellano (2008) features consumption in default that is given by mean output if output is over the mean and equal to output if output is less than the mean. This implies a cost function $\phi^A(y) = \max\{y - \mathbb{E}(y), 0\}$, which closely resembles the case of $d_y > 0$ and $d_{yy} = 0$. The convexity of output costs is crucial to obtain spreads with, simultaneously, a high mean and a low volatility. The intuition is that during good times the probability of default is low because the costs of default are high and therefore spreads are high. Borrower's impatience implies that debt is built up during good times. However, during bad times, spreads increase quickly because the default costs decrease and the option of defaulting is more attractive. In our settting the component that depends on the amount of defaulted debt is crucial to disincentive a behavior in which debt is issued in high quantities right before default.

One of the special features of our model with respect to other papers featuring long term bonds is that we introduce recovery.[7] Recovery is crucial for our mechanism; but at the same time, it affects the incentives to borrow right before default. In particular, the government has an incentive to borrow at high interest rates prior to default because it effectively knows that it will repay only a fraction of the face value. This borrowing at high interest rates is counterfactual and generates spikes in the volatility of spreads by introducing an outlier. In order to rule out this behavior our model introduced a maximum default probability and state dependent recovery $\mathcal{R}(b)$. The first one, a maximum default probability follows Chatterjee and Eyigungor (2015). The second one, captures in reduced form endogenous bargaining as in Yue (2010).

With these functional forms, the model has 9 parameters that are standard in the literature of long-term debt: β, γ are preference parameters; ρ_y, σ_y are the parameters for the process of output; σ_ϵ the standard deviation of the randomization variable; m, z rate at which debt matures and coupon rate; d_y, d_{yy} output costs parameters. Our paper introduces an over-the-counter market and endogenous time in autarky after default. So, we introduce 7 additional parameters: r is the discount factor of the unconstrained and the constrained investor; ζ is the probability of receiving a liquidity shock; α_{ND}, α_D are the bargaining powers of the intermediaries before default and after default; λ is the probability of meeting an intermediary; $\bar{b}$ is the maximum face value recovery; θ is the probability of re-access.

[7] See for example Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012).

Externally Calibrated Parameters. Risk aversion γ is set to 2 and this is a standard value in the Real Business Cycles literature and the sovereign debt literature. The parameters for output are estimated from linearly detrended data adjusted for seasonality of the real GDP of Argentina. The data is quarterly and the period is 1980:I and 2001:IV; the source is Neumeyer and Perri (2005). The estimated values at a monthly basis are $\rho_z = 0.983$, $\sigma_z = 0.0151$ and we fix $\sigma_\epsilon = 0.004$.[8] In the computations, we approximate the AR(1) process with Rouwenhorst (1995) in 200 states for output. The discount rate of unconstrained investors is $r = 0.3$ percent per month, to match the risk free real monthly return of the 3 month treasury bill in the period of study. For debt maturity we match the average maturity and coupon information in Broner et al. (2013) as used in Chatterjee and Eyigungor (2012). The maturity $m = \frac{1}{60}$ is chosen to match the median maturity of Argentina's bonds that is equal to 60 months (20 quarters) reported in Chatterjee and Eyigungor (2012). The coupon rate is set to $z = 0.01$ implying a coupon rate of 12 percent per year close to the 11 percent value weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012). We fix the re-entry probability at $1 - \theta = 0.0128$ following Chatterjee and Eyigungor (2012). This implies an average exclusion period of 6.5 years.[9] Maximum default probability: $\overline{\delta} = 0.75$. That is, the one month ahead probability of default cannot exceed 75 percent. Finally, we will fix $\lambda = 0.8647$ following the continous time calibration of He and Milbradt (2013).[10]

Matching Moments. The parameters that remain to be calibrated are

$$\Theta = \left[\beta, d_y, d_{yy}, h_c, \alpha_D, \alpha_{ND}, \zeta\right].$$

On the other hand, the targets are: average debt to GDP ratio, mean and volatility of spreads, mean bid-ask spreads during default and before default, the unconditional turnover,

[8]Total monthly volatility is then $\sqrt{0.004^2 + 0.0151^2} = 0.0156$. At a quarterly frequency, this implies an autocorrelation of $0.983^3 = 0.95$ and an output volatility of $0.0156 \times \sqrt{3} = 0.027$.

[9]Beim and Calomiris (2001), report that for the 1982 default episode, Argentina spent until 1993 in a default state. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in default starting in 2001 until 2005 when it settled with most of its bondholders.

[10]In our model $\lambda = 0.8647$ is the meeting probability in one month. So, this number implies, that on average it will take two weeks. In continous time He and Milbradt (2013) choose $\lambda^{CT} = 40$ for the meeting intensity, that implies a meeting every two weeks. The expected meeting time is 1/40 years, and that amounts to 1.3 weeks.

Parameter	Description	Value
β	Sovereign's discount rate	0.9841
γ	Sovereign's risk aversion	2
ρ_z	Persistence of output	0.983
σ_y	Volatility of output	0.0156
m	Rate at which debt matures	0.0167
z	Coupon rate	0.01
$1-\theta$	Probability of reentry	0.0128
d_y	Output costs for default	-0.264
d_{yy}	Output costs for default	0.337
r	Discount rate of international investors	0.0033
h_c	Holding costs for constrained investors	0.0014
ζ	Probability of getting a liquidity shock	0.139
λ	Probability of meeting a market maker	0.865
α_{ND}, α_D	Bargaining power of market maker	0.875, 1
$\bar{b}$	Maximum recovrey rate for sovereign bonds	0.83
$\bar{\delta}$	Max. Default Probability	0.75

Table 1: Baseline

and mean recovery rate. Thus, we choose Θ in order to match

$$\left[\mathbb{E}\left[\frac{b_t}{y_t}\right], \mathbb{E}\left[sprd_t\right], \sigma\left(sprd_t\right), \mathbb{E}\left[BA_t^{S,ND}\right], \mathbb{E}\left[BA_t^{S,D}\right], \mathbb{E}[Turnover], \mathbb{E}\left[\frac{\min\left\{\bar{b}, b_{def}\right\}}{b_{def}}\right]\right].$$

Targets. First, for the target debt capacity we use the average total external debt of Argentina as a fraction of GNP that is equal to 100 percent over the period 1993:I and 2001:IV. Because our setting features recovery, we will target total debt 100 instead of the portion of 70 percent used in Chatterjee and Eyigungor (2012) that was eventually lost in the default.[11] Second, for the target mean and volatility of spreads we use the series in Neumeyer and Perri (2005). Over the period 1993:I and 2001:IV the mean and standard deviation of spreads was 0.0815 and 0.0443, per year respectively. The internal rate of return of bonds issued in the primary market is computed as $q^H(y,b') = [m+(1-m)z] / \left[m + r^H(y,b')\right]$. The spread is then computed as $(1+r^H(y,b'))^{12} - 1$ minus $(1+r)^{12} - 1$. We will match this with the analogs in the data. Third, we will use 50 basis points as target bid ask spread before default. For European bonds Pelizzon et al. (2013) find that the bid ask spreads have a median of 43 basis points and can rise up to

[11] As is explained in Chatterjee and Eyigungor (2012), the database of World Bank development finance does not take into account coupon payments as debt because they only measure obligations at the face value. Therefore, the model analog of debt as reported in this database is just the face value of current obligations b.

Moment	Data	Model	CE (2012), Table 3
Mean Debt to Gdp	1.0	1.0	0.7
Mean Sovereign Spread	0.0815	0.0815	0.0815
Vol. Sovereign Spread	0.0443	0.0437	0.0443
Mean Bid-Ask Spread, ND	0.0050	0.0049	-
Mean Bid-Ask Spread, D	0.0500	0.0503	-
Mean Turover,	0.12	0.12	-
Expected Recovery,	0.30	0.297	-

Table 2: **Model moments.**

125 basis points (period June 2011 to November 2012). For US corporate bonds Chen et al. (2013) report bid ask spreads of 50 basis points during normal times for junk bonds and 218 during bad times. [12] Fourth, for the bid ask spreads during default, due to data limitations we will rely on data for US Corporate bonds. There are two sources. On the one hand, Edwards et al. (2007) document that the 200 basis points; on the other hand Chen et al. (2013) report 620 basis points during recessions. Therefore, we think that a bid ask spread during default target of 500 basis points is conservative. The model counterparts are computed according to $\left(q^H - q^S\right) / \frac{1}{2}\left(q^H + q^S\right)$. Fifth, again due to data limitations, we will use the average turnover of US corporate bonds. Here we follow 12 percent per month reported in Bao et al. (2011).[13] Finally, we use a target of mean recovery of 30 percent, in line with Yue (2010).

Calibration Results. The final parameter values are reported in Table 1. The results from our baseline calibration are summarized in Table 3.1. The last column lists the baseline results from Chatterjee and Eyigungor (2012) for comparison. Our baseline model generates mean debt to gdp of 100 per-cent exactly as the empirical target of 100 percent. Our model's mean spreads and volatility of spreads are almost perfectly in line with the data. Finally, note that bid-ask spreads before default are around 50 basis points, exactly as the data. The bid ask spreads during default 503 basis points are also close to the target 500 basis points.

What Follows. For the parameter values calibrated in Table 3.1 we analize pricing functions in the primary market and bid ask spreads, we structurally decompose credit spreads and discuss the implications of liquidity in welfare and debt capacity, we pursue

[12] In Appendix C we report bid ask spreads for Argentinean bonds. Using data from Bloomberg, we find that bid ask spreads in some bonds listed can be as much as 10 percent. In our data, if for each day we select the bond with the lowest bid ask spread, and compute the average bid ask spread for the period 2005 to 2014, we obtain a value of 140 basis points; this is clearly a lower bound.

[13] The number corresponds the turnover in Table 1 of Bao et al. (2011).

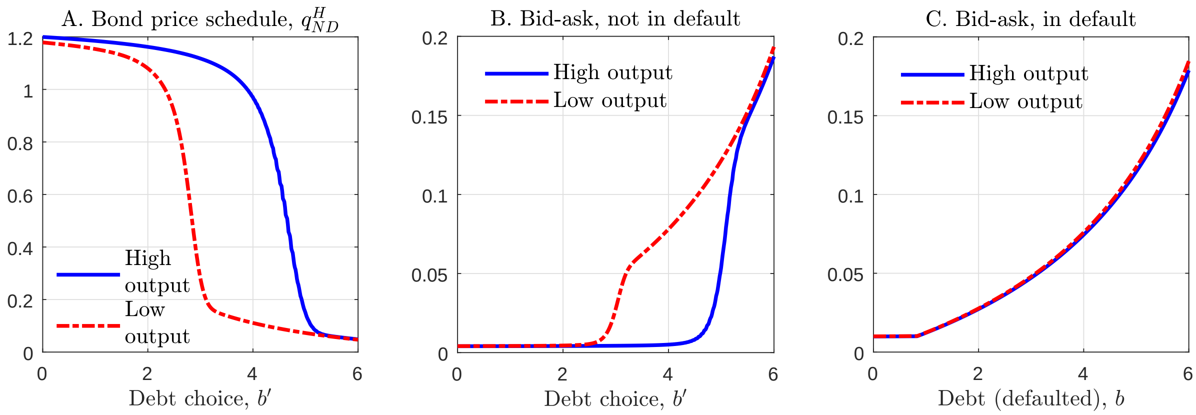

Figure 3.1: **Bond prices and bid-ask spreads.** This figure plots bond prices at issue q^H and bid-ask spreads which are defined as $\frac{q^H - q^S}{\frac{1}{2}(q^H + q^S)}$. All prices are a function (y, b'). Panel A plots prices in the primary market for the valuation of the high valuation investors. Panels B and D plot bid ask spreads during credit access and during autarky.

a case study for Argentina, and report business cycles statistics.

3.2 Bond Prices, Bid-Ask Spreads, Feedback

Figure 3.1 plots model implied bond prices and bid-ask spreads as a function of output y and debt choice b'. First, panel A plots bond prices in the primary market $q^H(b', y)$ during the credit access regime. Note that standard comparative statics apply for bond prices; they are increasing in output and decreasing in debt. Also, they are always positive no matter how big is the debt issuance; this follows because the model features positive recovery upon default.

Second, panels B and C plot bid-ask spreads during the credit access and autarky regimes. Note that bid-ask spreads are around 50 basis points, unresponsive to output movements when output is sufficiently high and default is not a concern, and rise as output falls and default becomes more likely. This is because prices are forward looking and take into account the possibility of worsening liquidity conditions for defaulted bonds. Panel C plots bid-ask spreads for defaulted bonds. Because during default there are no bond issues, the state variable is the amount of debt in default b (and not debt choice b'). As in the credit access regime, bid-ask spreads are also higher when output is lower; this is due to default concerns after reaccessing credit markets. In Figure 3.1 we can observe that the liquidity-credit feedback loop highlighted in He and Milbradt (2013) for corporate bonds. For example, from panel B, is clear that bid-ask spreads increase as the

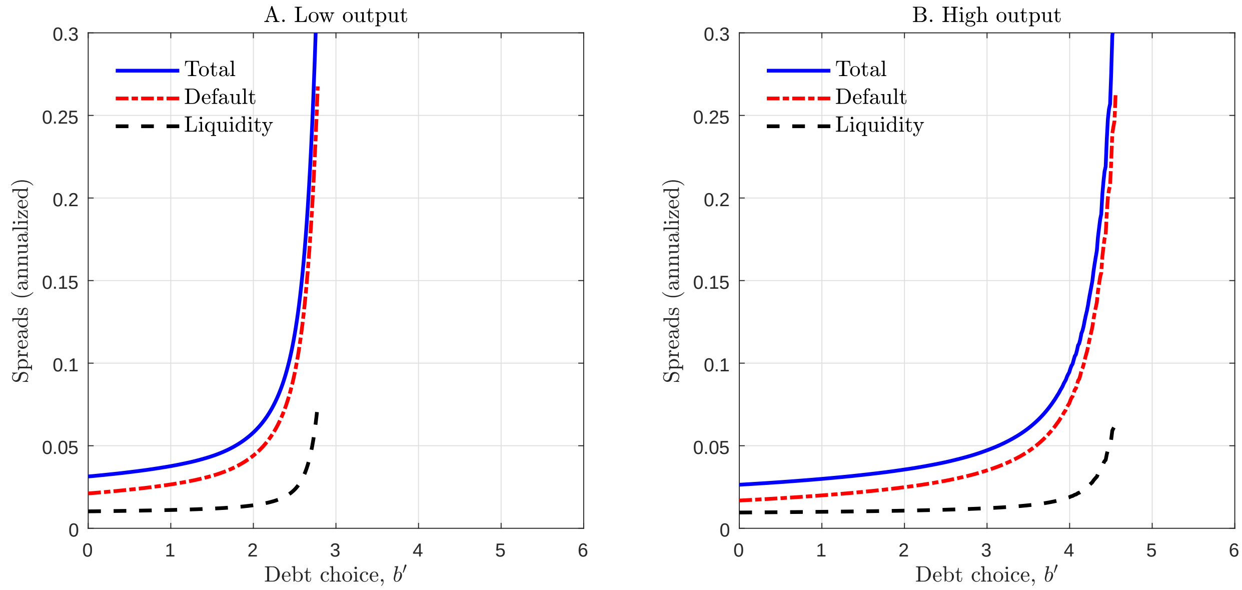

Figure 3.2: **Sovereign spread decomposition.** This figure decomposes total sovereign spreads CS into a default component CS_{DEF} and a liquidity component CS_{LIQ}. Panel A: the bond price schedule $q^H_{ND}(y, b')$ for 2 values of y. The values of $\log(y)$ are chosen to be ± 2 standard deviations of the unconditional distribution for $\log(y)$. The actual values are 0.851 and 1.176. Panel B: the corresponding annualized sovereign spread. Annualized spreads are defined as $cs \equiv \left[\frac{m+(1-m)z+(1-m)q^H_{ND}}{q^H_{ND}}\right]^{12} - (1+r)^{12}$. In addition, the total default and liquidity components are plotted.

country nears default. On the one hand, wider liquidity spreads traduce in higher ex-ante borrowing costs for the country. This in turn leads to increased debt rollover costs and increases default incentives. On the other hand, higher default risk implies that worse liquidity conditions are forecasted in the event of a default, because bid-ask spreads are higher during default. These effects are nonlinear, in particular the feedback mechanism is stronger when output is low and/or when debt levels are high.

3.3 Sovereign Spread Decomposition

One of the advantages of our model in which liquidity and credit risk are jointly determined is that we can decompose spreads into a credit and liquidity component. This permits us to quantify the contribution of theses frictions in the secondary market to the total spreads. We will decompose the total spreads, cs, in two terms

$$cs \equiv cs_{DEF} + cs_{LIQ}. \tag{3.1}$$

The first component, cs_{DEF}, is the default component of the spread, and the second one, cs_{LIQ}, is defined as the liquidity component. The default component of the sovereign spread cs_{DEF} is computed as follows. Denote by $\mathbb{B}, \mathbb{D}$ the debt and default policies of the baseline calibration; detailed in Table 3.2. Denote by $cs(\mathbb{B}, \mathbb{D}, \xi)$ the function that maps policies and a probability of liquidity shocks intro a credit spread.[14] The default component is defined as $cs_{DEF} = cs(\mathbb{B}, \mathbb{D}, \xi = 0)$. The bond price associated with cs_{DEF} is still computed using equilibrium default and debt policies that take into account liquidity spreads, but discounting is done by an investor who faces no liquidity problems. The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account in choosing debt and default policies), individual investors are heterogeneous and in particular there may be some investors without liquidity concerns who discount at the risk free rate. The liquidity component is just the residual $cs_{LIQ} = cs - cs_{DEF}$.

The above decomposition is plotted in Figure 3.2. Panels A and B plot respectively, total spreads, credit risk premium, and liquidity premium for two levels of output; the values of $\log(y)$ are chosen to be ± 2 standard deviations of the unconditional distribution for $\log(y)$. There are two features of the decomposition that are worth noting. First, panels A and B show that liquidity component is increasing as the debt choice increases. This is due to higher default probabilities. Second, note further that as a percentage of total spreads, the liquidity component is sizable: when default risk is low (i.e. when output is high and/or debt levels are low) the liquidity component is predominant while the liquidity component as a fraction of total spreads becomes smaller, but still first order, as overall default risk increases. For example, in panel B we can observe that the fraction of the total sovereign spreads attributable to liquidity is around 45 per cent for a debt choice of 1, for high levels of output. On the other hand, close to default, for example when output is low and debt issue is around 0.7, is around 25 per cent of total spreads. These magnitudes are in line with CDS-basis based calculations in Longstaff et al. (2005) and structural decompositions in He and Milbradt (2013) for corporate Bonds.

Jump to Default: Where is Liquidity Premia Coming From? In our model liquidity premia cs_{LIQ} maps one to one to bid ask spreads. To illustrate this link we use a jump to

[14]More precisely, the function is defined as

$$cs(\mathbb{B}, \mathbb{D}, \xi) \equiv \left[\frac{m + (1-m)\, z + (1-m)\, q^H_{ND}}{q^H_{ND}} \right]^{12} - (1+r)^{12}$$

where q^H_{ND} is the valuation defined in equation (2.3) when evaluated with bond and default policies $\mathbb{B}, \mathbb{D}$ and the probability of a liquidity shock is equal to ξ.

default model. Suppose that the model is the same as the one developed in Section 2 but debt is fixed and the default probability is exogenous and given by p_d.[15] In this case, from equation (2.3), the valuation of the unconstrained investor is given by

$$q^H_{ND} = \frac{(1-p_d)\left(m + (1-m)\left(z + \zeta q^L_{ND} + (1-\zeta)\, q^H_{ND}\right)\right) + p_d\left(\zeta q^L_D + (1-\zeta)\, q^H_D\right)}{1+r}.$$

Define the liquidity premium of bonds in the primary market, ℓ^H_{ND}, as the discount in addition to r that an investor that only faces credit risk needs to impute in order to match the price q^H_{ND}; that is:

$$q^H_{ND} = \frac{1}{1+r+\ell^H_{ND}}\left[(1-p_d)\left(m + (1-m)\left(z + q^H_{ND}\right)\right) + p_d q^H_D\right].$$

Rewriting the previous expression we obtain that the liquidity premium is a combination of bid ask spreads

$$\ell_{ND} = (1-p_d)(1-m)\,\zeta \overbrace{\frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}}}^{\text{BA Sp. NoDef}} + p_d\zeta \overbrace{\frac{q^H_D - q^L_D}{q^H_{ND}}}^{\text{BA Sp. Def}}.$$

Two observations. First, the value of the liquidity premium is pinned down by bid ask spreads. So, as long as the calibration has bid ask spreads that are in line with the data, the liquidity premia that we obtain is disciplined by the friction we observe empirically. Second, there will be a positive comovement between liquidity and credit premia as long as bid ask spreads during the default are larger than bid ask spreads before default. During bad times, liquidity premia will increase.

Disgression: Fixed Debt and Outside Option. Suppose that debt policy is fixed and that there is permanent reversion to autarky after a default. Then we can decompose spreads in four terms

$$cs = cs_{Def,Def} + cs_{Liq \to Def} + cs_{Def \to Liq} + cs_{Liq,Liq}. \tag{3.2}$$

Denote by $\mathbb{D}_1, \mathbb{D}_0$ the optimal policies with and without liquidity shocks. Without loss of generality debt policy is fixed at the optimal policy with liquidity shocks $\mathbb{B}_1$. Define the first one as a pure default component $cs_{Def,Def} = cs(\mathbb{D}_0, \zeta_0)$; default policies

[15] In Appendix B we present this jump to default model in detail.

of a market without liquidity frictions and valued by an investor not subject to liquidity shocks. Define the second one as a liquidity induced default component; it will be given by $cs_{Liq\to Def} = cs(\mathbb{D}_1, \xi_0) - cs(\mathbb{D}_0, \xi_0)$. Note that the total default component is $cs^{Def} = cs(\mathbb{D}_1, \xi_0)$; using the full policies, but valued by an investor that cannot receive liquidity shocks, as in (3.1). The total liquidity component will be defined as $cs^{Liq} = cs - cs^{Def}$. Therefore, the third term will be a pure liquidity component is $cs_{Liq,Liq} = cs(\mathbf{0}, \xi_1)$. This is the spread of a bond valued by an investor that can receive liquidity shocks but cannot be defaulted. Finally, the fourth term, is a liquidity induced default will be given by $cs_{Def\to Liq} = cs^{Liq} - cs_{Liq,Liq}$. We can show that all the terms in the decomposition are greater than or equal to zero.

Claim. Suppose that there is permanent reversion to autarky after a default and debt policy is fixed. Then, $cs_{Def,Def} \geq 0, cs_{Def\to Liq} \geq 0, cs_{Liq\to Def} \geq 0, cs_{Liq,Liq} \geq 0$.

The case with a fixed debt and an outside option that does not depend on debt resembles the setting in structural corporate finance models. In particular, He and Milbradt (2013) shares this feature. In the corporate setting, the default for equity-holders corresponds to bankruptcy, the value of which is usually exogenously fixed at zero (they optimally liquidate the firm when it has no value for them). Furthermore, a fixed debt policy is standard in a corporate setting; the rational is that the bond issue might have a covenant that restricts further issues, and this covenant is enforceable in a court. This assumption of a constant debt structure is usually for simplicity. As we will show next, in the sovereign setting, debt policy and default tresholds are both responding to liquidity frictions.

Full Decomposition: Endogenous Response of Policies. For all the terms related to the default component in the previous decomposition to be positive, we need to keep debt policies fixed. In the sovereign setting this is not a feasible choice since there is no enforceable contract that can be signed so that the sovereign does not issue more debt. In addition, the autarky continuation value depends on future conditions. So, an increase in the liquidity friction might imply a decrease in credit risk due to a more conservative debt policy. In fact, that is the case in our current calibration.

We now argue how the liquidity driven default component with endogenous policies, $cs^{EP}_{Liq\to Def}$, can be negative. Define the full default component as $cs^{EP}_{Def} = cs\left(\mathbb{B}_1, \mathbb{D}_1, \zeta_0\right)$ where $\mathbb{B}_1, \mathbb{D}_1$ are the optimal policies when there are liquidity frictions. The pure default component $cs^{EP}_{Def\to Def} = cs\left(\mathbb{B}_0, \mathbb{D}_0, \zeta_0\right)$ where $\mathbb{B}_0$ and $\mathbb{D}_0$ are the optimal policies from solving the model when we shut down friction on the secondary market and there are no

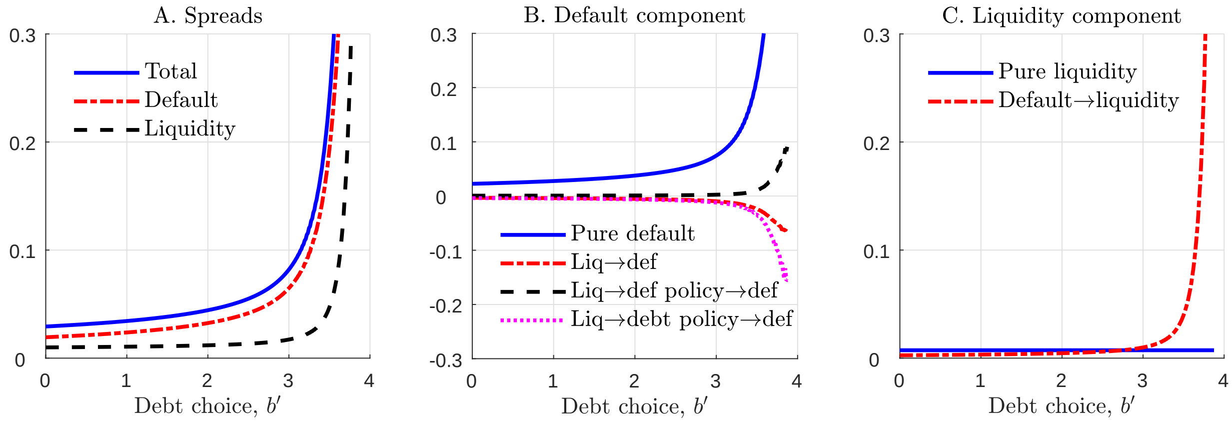

Figure 3.3: **Sovereign spread decomposition.** This figure plots the different components of the default credit spread. The values of $\log(y)$ are chosen to be ± 2 standard deviations of the unconditional distribution for $\log(y)$. The actual values are 0.851 and 1.176.

liquidity shocks. The liquidity driven default component is given, as before, by

$$cs^{EP}_{Liq\to Def} \equiv cs^{EP}_{Def} - cs^{EP}_{Def\to Def}. \tag{3.3}$$

We decompose $cs^{EP}_{Liq\to Def}$ further. In particular,

$$cs^{EP}_{Liq\to Def} = cs\left(\mathbb{B}_1, \mathbb{D}_1, \zeta_0\right) - cs\left(\mathbb{B}_1, \mathbb{D}_0, \zeta_0\right) + cs\left(\mathbb{B}_1, \mathbb{D}_0, \zeta_0\right) - cs\left(\mathbb{B}_0, \mathbb{D},_0 \zeta_0\right)$$

where we now further define

$$cs^{EP}_{Liq\to Def,Pol\to Def} \equiv cs\left(\mathbb{B}_1, \mathbb{D}_1, \zeta_0\right) - cs\left(\mathbb{B}_1, \mathbb{D}_0, \zeta_0\right) \tag{3.4}$$
$$cs^{EP}_{Liq\to Debt,Pol\to Def} \equiv cs\left(\mathbb{B}_1, \mathbb{D}_0, \zeta_0\right) - cs\left(\mathbb{B}_1, \mathbb{D}_1, \zeta_0\right) \tag{3.5}$$

When debt is fixed, there is only a first component $cs^{EP}_{Liq\to Def,Pol\to Def}$ which contributes positively to spreads. The intuition is that under the same debt policy, debt rollover is more costly when liquidity conditions are bad, and as a result, default is more likely to occur. Alternatively, in our setting, we have an additional component $cs^{EP}_{Liq\to Debt,Pol\to Def}$ which captures the effect of liquidity on default through *endogenous debt policy changes*. This component can be negative; in fact, in our calibration is negative. Since debt rollover is more expensive when *secondary* debt market are illiquid, debt policy becomes more conservative and this reduces default probabilities. The sign of the sum of these two effects are ambiguous.

Moment	Data	$h_c = 0$	$h_c = 0.0005$	$h_c = 0.001$	$h_c = 0.00145$
Mean Debt to GDP	1.00	1.017	1.012	1.007	1.002
Mean Spread	0.0815	0.0767	0.0785	0.0802	0.0815
Vol. of Spread	0.0443	0.0474	0.0466	0.0450	0.0436
Mean Bid-Ask Spread	0.0050	0	0.0017	0.0034	0.0049
Welfare	-	1.0164	1.0158	1.0152	1.0147

Table 3: Comparative Statistics

These components are presented in Figure 3.3. First, note that as we mentioned, in our calibration the component $cs^{EP}_{Liq \to Debt, Pol \to Def}$ is negative. This is because worse liquidity conditions, given default policies, will imply a more precautionary debt policy. Also, note that the component $cs^{EP}_{Liq \to Def, Pol \to Def}$ is always positive; this is as in the case with fixed debt policy. Second, it is interesting to point out how these two components vary across the business cycle. As a *fraction* of the total default component, $cs^{EP}_{Liq \to Def, Pol \to Def}$ is more important during good times (when output is high); during bad times, default becomes very likely regardless of liquidity conditions so the change of default policy because of different liquidity conditions becomes a second order determinant of spreads. Regardless of the business cycle, $cs^{EP}_{Liq \to Debt, Pol \to Def}$ is the larger component. This shows the importance of debt policy in alleviating liquidity problems. Quantitatively, this component is more important during bad times since spreads are more sensitive to debt levels during bad times.

3.4 Liquidity, Debt Capacity, Welfare

We now study how increases in the severity of a liquidity shock change debt capacity, spreads and bid ask spreads and welfare. In particular, we introduce changes in the cost of liquidity discount factor of liquidity constrained investors. We start from a value of h_c equal to 0; that is, when a liquidity shock is not costly, and we slowly increase it. We compute mean spreads, debt capacity, and bid ask spreads. In addition we compute the welfare costs of secondary market frictions.

Define $V^o(0, y; h_c)$ as the value of a government which starts with zero debt and income y when the cost of carry of liquidity constrained investors is h_c. Let $\Pi(y)$ be the invariant distribution of output implied by $\mathbb{P}(y_{t+1} = y' \mid y_t = y)$. The welfare of alternative frictions in the secondary markets in terms of consumption equivalent will be defined as c such that

$$\frac{c^{1-\sigma}}{(1-\beta)(1-\sigma)} = \sum_{y' \in \mathbb{Y}} V^o(0, y; h_c)\Pi(y).$$

In particular, it is the value of quarterly consumption of a constant stream of consumption value by the consumer.

The results of the exercise and summarized in Table 3.4. We also report a measure of welfare in consumption equivalent. The value of $h_c = 0.00145$ is our baseline calibration. Note that an increase of holding costs has a small impact over spreads and debt capacity. However, increasing the holding cost, as expected, has a large impact on the bid ask spreads that go from zero with no frictions to 50 basis points in the baseline.

There are substantial gains for decreasing secondary market frictions. In particular, the difference in welfare from the baseline calibration calibration to the case with $h_c = 0$ is equal to 0,17 percentage points. To put this number in perspective, note that Lucas (2003) finds that the welfare gains of eliminating business cycles in the US in the case of log utility are 0.05 percentage points of consumption. For the case of Argentina, in the CRRA framework, the welfare gain in terms of consumption equivalent of eliminating the business cycle of a representative agent would be $\gamma\sigma_c^2\frac{1}{2}$ and, assigning a volatility of consumption 50 percent higher than output, this can be bounded by $\gamma\sigma_y^2 1.5\frac{1}{2} = 2\frac{1}{2}1.5\,(0.0156)^2 = 0.24$ percentage points of consumption. Thus, the welfare gain of making the secondary market frictionless are around two thirds of this number.

There are two things worth noting regarding the welfare gains. First, note that the small welfare gains in absolute value are a consequence of CRRA framework that we are using. Second, note that even though in our calibrated example there are welfare gains, it need not to be the case. The reason is that the models exhibits two frictions: limited commitment and liquidity. Therefore, decreasing one of the frictions need not to increase welfare.[16]

3.5 Case Study: Argentina's Default in 2001

In this subsection we conduct and event study of Argentina's default in 2001:IV. For the policy functions of the baseline calibration we feed in the shocks that Argentina received in the 1990's given an its initial level of debt. Panel A plots the level of GDP in logs. Panel B plots the total spreads generated by our model in comparison with the data. Note that our model can replicate the most salient feature of the series of spreads. The first spike is the Tequila crisis in Mexico in 1996, where there is a sharp increase in the spreads coming from a recession. In addition, we can see that in 2001 the model correctly account for the spike in spreads and default. Note that even though the recession started

[16]The intuition is that secondary market frictions can act as a commitment device or the sovereign that actually has a problem taking debt. This idea parallels the results in Amador et al. (2006).

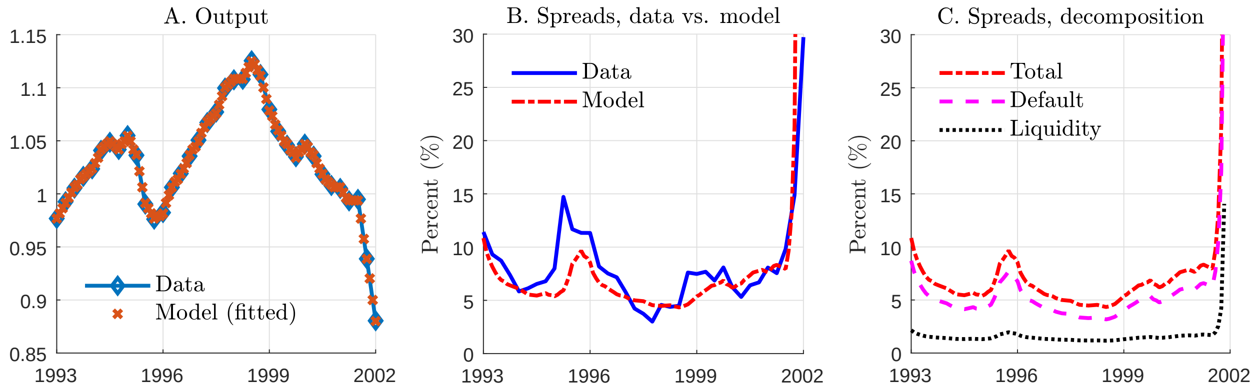

Figure 3.4: **Event Study: Argentina's Default, 2001:IV.** The first figure plots output, Argentinean spreads in the data, and Argentinean spreads generated by the model when the output realizations for Argentina are fed in. The second figure plots total spreads and the credit and liquidity components as defined in 3.1.

in 1998:III, spreads continued to be below 800 basis points until the beginning of 2001. Panel C depicts how, given bid ask spreads in the data, total spreads of Argentinan bonds can be decomposed into a liquidity and credit component using the decomposition we introduced in (3.1). When the spreads started to spike in the beginning of 2001 before default, the liquidity component was not a negligible component of the credit spreads. In fact, in our calibration it could explain 30 percent given of the total spreads, during good times. Confirming the results for Corporate Bonds, this magnitude is smaller close to default.

3.6 Business Cycle Properties

The model's business cycle properties are summarized in Table 4. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012), for comparison. Qualitatively, the model performs well. As in the data, consumption is as volatile as output and nearly perfectly correlated with output. The volatility of the current account relative to output volatility is 0.09 in the model which is close to its empirical counterpart of 0.17. Qualitatively the model does a good job of capturing counter-cyclical sovereign credit risk although but quantitatively the -0.65 correlation still falls short of the empirical moment of -0.79. The model generates debt service (as a fraction of output) of 7.9 per-cent and a default frequency of 6.6 per-cent. Finally, note that the baseline model generates a negative correlation of -0.497 between the current account and output, in line with Chatterjee and Eyigungor (2012) but still short from its empirical counterpart.

Variable	Data	Model	CE (2012), Table 4
$\sigma(c)/\sigma(y)$	1.09	1.15	1.11
$\sigma\left(\frac{NX}{y}\right)/\sigma(y)$	0.17	0.09	0.20
$corr(c,y)$	0.98	0.98	0.99
$corr\left(\frac{NX}{y},y\right)$	-0.88	-0.497	-0.44
$corr(r-r^f,y)$	-0.79	-0.65	-0.65
Debt service	0.053	0.079	0.055
Default frequency	0.125	0.066	0.068

Table 4: **Business cycle properties.**

4 Conclusion

We studied debt policy of emerging economies taking into account credit and liquidity risk. To account for credit risk, we followed the quantitative literature of sovereign debt in studying an incomplete markets model with limited commitment and exogenous costs of default. To account for liquidity risk, we introduced search frictions in the market for sovereign bonds. By introducing liquidity risk in an otherwise standard model of sovereign debt, default and liquidity risk are now jointly determined. To quantify the role of liquidity on sovereign spreads, debt capacity, and welfare, we performed quantitative exercises when our model is calibrated to match key features of the Argentinean default. We find that liquidity premia can be a substantial component of spreads, that given reasonable friction in the secondary market this risk premia increases during bad times, and that reductions in secondary market frictions would imply increases in welfare. The model can also account for key features of observed sovereign defaults and matches business cycle fluctuations in the data.

References

Afonso, Gara and Ricardo Lagos, “Trade dynamics in the market for federal funds,” Technical Report, Staff Report, Federal Reserve Bank of New York 2012.

Aguiar, Mark and Gita Gopinath, “Defaultable debt, interest rates and the current account,” *Journal of International Economics*, 2006, *69* (1), 64 – 83.

— **and Manuel Amador**, “Sovereign Debt: A Review,” Working Paper 19388, National Bureau of Economic Research August 2013.

Amador, Manuel, Iván Werning, and George-Marios Angeletos, “Commitment vs. flexibility,” *Econometrica*, 2006, *74* (2), 365–396.

Arellano, Cristina, “Default Risk and Income Fluctuations in Emerging Economies,” *The American Economic Review*, 2008, *98* (3), 690–712.

— **and Ananth Ramanarayanan**, “Default and the Maturity Structure in Sovereign Bonds,” *Journal of Political Economy*, 2012, *120* (2), pp. 187–232.

Ashcraft, Adam B. and Darrell Duffie, “Systemic Illiquidity in the Federal Funds Market,” *American Economic Review*, 2007, *97* (2), 221–225.

Asquith, Paul, Andrea S Au, Thomas Covert, and Parag A Pathak, “The market for borrowing corporate bonds,” *Journal of Financial Economics*, 2013, *107* (1), 155–182.

Atkenson, Andrew, Andrea Eisfeldt, and Rierre Olivier Weill, “The Market for OTC Derivatives,” *Working Paper*, 2013.

Bai, Jennie, Christian Julliard, and Kathy Yuan, “Eurozone Sovereign Bond Crisis: Liquidity or Fundamental Contagion,” *Working Paper*, 2012.

Bao, Jack, Jun Pan, and Jiang Wang, “The Illiquidity of Corporate Bonds,” *The Journal of Finance*, 2011, *66* (3), 911–946.

Beber, Alessandro, Michael W. Brandt, and Kenneth A. Kavajecz, “Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market,” *Review of Financial Studies*, 2009, *22* (3), 925–957.

Beim, DO. and CO Calomiris, “Emerging Financial Markets,” *McGraw-Hill*, 2001.

Benjamin, D and M Wright, “Recovery Before Redemption: A Theory of Sovereign Debt Renegotiation,” *Working Paper*, 2009.

Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler, "WHY DO EMERGING ECONOMIES BORROW SHORT TERM?," *Journal of the European Economic Association*, 2013, *11*, 67–100.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, pp. 2674–2699.

_ and _ , "A seniority arrangement for sovereign debt," *The American Economic Review*, 2015, *105* (12), 3740–3765.

Chen, Hui, Rui Cui, Zhiguo He, and Konstantin Milbradt, "Quantifying liquidity and default risks of corporate bonds over the business cycle," *Preliminary version as of April*, 2013.

Chen, Long, David Lesmond, and Jason Wei, "Corporate Yield Spreads and Bond Liquidity," *The Journal of Finance*, 2007, *62* (1), 119–149.

Cruces, Juan J. and Christoph Trebesch, "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 85–117.

Cui, Wei and Sören Radde, "Search-based endogenous asset liquidity and the macroeconomy," 2016.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, "Over-the-Counter Markets," *Econometrica*, 2005, *73* (6), 1815–1847.

_ , _ , and _ , "Valuation in Over-the-Counter Markets," *Review of Financial Studies*, 2007, *20* (6), 1865–1900.

Eaton, Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 1981, *48* (2), pp. 289–309.

Edwards, Amy, Lawrence Harris, and Michael Piwowar, "Corporate Bond Market Transaction Costs and Transparency," *The Journal of Finance*, 2007, *62* (3), 1421–1451.

Fleming, Michael J., "Are Larger Treasury Issues More Liquid? Evidence from Bill Reopenings," *Journal of Money, Credit and Banking*, 2002, *34* (3), pp. 707–735.

Friewald, Nils, Rainer Jankowitsch, and Marti G. Subrahmanyam, "Illiquidity or credit deterioration: A study of liquidity in the {US} corporate bond market during financial crises," *Journal of Financial Economics*, 2012, *105* (1), 18 – 36.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79* (1), 117 – 125.

_ **and** _ , "Debt dilution and sovereign default risk," *Available at SSRN*, 2015.

He, Zhiguo and Konstantin Milbradt, "Endogenous Liquidity and Defaultable Bonds," *Econometrica*, 2013, *forthcoming.*

Lagos, Ricardo and Guillaume Rocheteau, "Search in Asset Markets: Market Structure, Liquidity, and Welfare," *The American Economic Review*, 2007, *97* (2), pp. 198–202.

_ **and** _ , "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 2009, *77* (2), 403–426.

_ **,** _ **, and Pierre-Olivier Weill**, "Crises and liquidity in over-the-counter markets," *Journal of Economic Theory*, 2011, *146* (6), 2169 – 2205.

Leland, Hayne and Klaus Toft, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *The Journal of Finance*, 1996, *51* (3), 987–1019.

Longstaff, Francis., Sanjay Mithal, and Eric Neis, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *The Journal of Finance*, 2005, *60* (5), 2213–2253.

Lucas, Robert E, "Macroeconomic priorities," *The American Economic Review*, 2003, *93* (1), 1.

Mendoza, Enrique and Vivian Yue, "A general equilibrium model of sovereign default and business cycles," *Quarterly Journal of Economics*, jan 2012.

Neumeyer, Pablo A. and Fabrizio Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 2005, *52* (2), 345 – 380.

Pelizzon, Loriana, Marti Subrahmanyam, Davide Tomio, and Jun Uno, "The Microstructure of the European Sovereign Bond Market: A Study of Euro-zone Crisis," *Working Paper*, 2013.

Rouwenhorst, Geert, "Asset Pricing Implications of Equilibrium Business Cycle Models," in Thomas Cooley, ed., *Frontiers of Business Cycle Research*, Princeton University Press, 1995, pp. 294–330.

Yue, Vivian Z., "Sovereign default and debt renegotiation," *Journal of International Economics*, 2010, *80* (2), 176 – 187.

Appendix A: Numerical Method

It is well known that numerical convergence is often a problem in discrete time sovereign debt models with long-term debt. To get around this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). Should the government choose to repay its debt, total output is given by $y_t = e^{z_t} + \epsilon_t$ where $z_t = \rho_z z_{t-1} + \sigma_z u_t$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. As shown in Chatterjee and Eyigungor (2012), this noise component ϵ_t guarantees the existence of a solution of the pricing function equation. In our quantitative implementation we set set $\sigma_\epsilon = 0.004$, $\rho_z = 0.983$ and $\sigma_z = 0.0151$, to match the output moments from Argentina. Qualitatively, it does not otherwise alter the model. A government that chooses to repay its debt will obtain

$$V^C(b, y, \varepsilon) = \max_{b'} \left\{ (1-\beta) u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}(b', y') \right] \right\} \tag{.1}$$

where the budget constraint is now given by

$$c = y + \varepsilon - b[m + (1-m) z] + q^H_{ND}(y, b') \left[b' - (1-m) b \right] \tag{.2}$$

which contains the randomization component ε. Debt choice is denoted as $b'(b, y, \varepsilon)$. We impose that $\varepsilon_t \equiv 0$ during the autarky regime. The value to defaulting remains the same and is given by

$$V^D(b, y) = (1-\beta) u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta V^{ND}(\mathcal{R}(b), y') + (1-\theta) V^D(b, y) \right] \tag{.3}$$

Note that the lack of choice variables in autarky means that randomization is not necessary for overall numerical convergence. The default decision is given by

$$d(b, y, \varepsilon) = 1_{\left\{ V^C(b,y,\varepsilon) \geq V^D(b,y) \right\}} \tag{.4}$$

and contains the randomization component. The continuation values are now adjusted as follows

$$V^{ND}(b, y) = \mathbb{E}_\varepsilon \left[\max \left\{ V^D(b, y), V^C(b, y, \varepsilon) \right\} \right] \tag{.5}$$

in order to take into account the randomization component. Finally, bond prices are adjusted accordingly so as to take into account the additional randomization variable:

$$q^H_{ND}\left(b',y\right) = \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(b',y',\varepsilon')}{1+r}\left[m+(1-m)\left[\begin{array}{c} z+\zeta q^L_{ND}\left(b'\left(b',y',\varepsilon'\right),y'\right) \\ +\left(1-\zeta\right)q^H_{ND}\left(b'\left(b',y',\varepsilon'\right),y'\right)\end{array}\right]\right] \\ +\frac{d(b',y',\varepsilon')}{1+r}\left[\zeta q^L_D\left(b',y'\right)+\left(1-\zeta\right)q^H_D\left(b',y'\right)\right]\end{array}\right\} \tag{.6}$$

$$q^L_{ND}\left(b',y\right) = \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(b',y',\varepsilon')}{1+r}\left[-h_c+m+(1-m)\left[\begin{array}{c} z+\left(1-\lambda\right)q^L_{ND}\left(b'\left(b',y',\varepsilon'\right),y'\right) \\ +\lambda q^{Sale}_{ND}\left(b'\left(b',y',\varepsilon'\right),y'\right)\end{array}\right]\right] \\ +\frac{d(b',y',\varepsilon')}{1+r}\left[-h_c+\left(1-\lambda\right)q^L_D\left(b',y'\right)+\lambda q^{Sale}_D\left(b',y'\right)\right]\end{array}\right\} \tag{.7}$$

$$q^H_D(b,y) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q^H_D(b,y')+(1-\zeta)q^L_D(b,y')\right]+\theta\frac{\mathcal{R}(b)}{b}q^H_{ND}(\mathcal{R}(b),y) \tag{.8}$$

$$q^L_D(b,y) = \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c+\lambda q^{Sale}_D(b,y')+(1-\lambda)q^L_D(b,y')\right]+\theta\frac{\mathcal{R}(b)}{b}q^L_{ND}(\mathcal{R}(b),y) \tag{.9}$$

$$q^{Sale}_{ND}(b,y) = (1-\alpha_{ND})q^L_{ND}(b,y)+\alpha_{ND}q^H_{ND}(b,y) \tag{.10}$$

$$q^{Sale}_D(b,y) = (1-\alpha_D)q^L_D(b,y)+\alpha_D q^H_D(b,y) \tag{.11}$$

The rest of the numerical scheme is standard and follows the routine outlined in Chatterjee and Eyigungor (2012). We summarize the scheme in 4 steps:

a. Start by discretizing the state space. This involves choosing grids $\{y_i\}_{i=1}^{N_y}$ and $\{b_j\}_{j=1}^{N_b}$ for output and debt. The grid points and transition probabilities for output is chosen in accordance with the Rouwenhorst (1995) method. In the baseline model the number of states for output is chosen to be $N_y = 200$. The grid points for debt values are uniformly distributed over the range $[0, b_{max}]$ with the upper limit b_{max} chosen large enough so as never to be binding in simulations. The baseline calibration has $b_{max} = 6.0$ and $N_b = 450$.

b. Next perform value function iteration. Given bond prices, update value functions V^C and V^D. The debt and default policies $b'(\cdot)$ and $d(\cdot)$ are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, linear interpolation is used to obtain terms involving $\mathcal{R}(b)$.

c. Given debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.

Appendix B: Jump to Default

In this Appendix we spell out a particular case of our model in which default probabilities are exogenous. The idea is to introduce a clear definition of liquidity premium, show how bid ask spreads map into liquidty premium as a function of default risk, and to clarify the role of the features of the model in the results. Assume an unconditional constant default probability p^{LR} each period. Then the pricing equations yield a system of 4 equations and 4 unkowns $\bar{q}^H_{ND}, \bar{q}^L_{ND}, \bar{q}^H_{D}, \bar{q}^L_{D}$. The system is given by

$$\begin{aligned}
\bar{q}^H_{ND} &= \frac{1}{1+r}\Big[\left(1-p^{LR}\right)\left(m+(1-m)\left(z+\zeta\bar{q}^L_{ND}+(1-\zeta)\,\bar{q}^H_{ND}\right)\right) \\
&\quad +p^{LR}\left(\zeta\bar{q}^L_{D}+(1-\zeta)\,\bar{q}^H_{D}\right)\Big] \\
\bar{q}^L_{ND} &= \frac{1}{1+r}\Big[\left(1-p^{LR}\right)\left(-h_c+m+(1-m)\left(z+\lambda\bar{q}^H_{ND}+(1-\lambda)\,\bar{q}^L_{ND}\right)\right) \\
&\quad +p^{LR}\left(-h_c+\lambda\bar{q}^H_{D}+(1-\lambda)\,\bar{q}^L_{D}\right)\Big] \\
\bar{q}^H_{D} &= \frac{1-\theta}{1+r}\left(\zeta\bar{q}^L_{D}+(1-\zeta)\,\bar{q}^H_{D}\right)+\theta f\bar{q}^H_{ND} \\
\bar{q}^L_{D} &= \frac{1-\theta}{1+r}\left(-h_c+\lambda\bar{q}^H_{D}+(1-\lambda)\,\bar{q}^L_{D}\right)+\theta f\bar{q}^L_{ND}.
\end{aligned}$$

The solution to this system yields four value functions that depend on the unconditional default probability, p^{LR}, and the parameters of the model. Fix the unconditional default probability p^{LR} and suppose there is a short run departure. In particular, the current default probability is p_d. After a default, the default probability will be again fixed in the long run probability p^{LR}.[17] Then, current prices are given by

$$\begin{aligned}
q_{ND}^{H} &= \frac{1}{1+r}\left[(1-p_d)\left(m+(1-m)\left(z+\zeta q_{ND}^{L}+(1-\zeta)\,q_{ND}^{H}\right)\right)\right.\\
&\quad \left.+p_d\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)\right] \\
q_{ND}^{L} &= \frac{1}{1+r}\left[(1-p_d)\left(-h_c+m+(1-m)\left(z+\lambda q_{ND}^{H}+(1-\lambda)\,q_{ND}^{L}\right)\right)\right.\\
&\quad \left.+p_d\left(-h_c+\lambda q_{D}^{H}+(1-\lambda)\,q_{D}^{L}\right)\right]\\
q_{D}^{H} &= \frac{1-\theta}{1+r}\left(\zeta q_{D}^{L}+(1-\zeta)\,q_{D}^{H}\right)+\theta f\bar{q}_{ND}^{H}(p^{LR})\\
q_{D}^{L} &= \frac{1-\theta}{1+r}\left(-h_c+\lambda q_{D}^{H}+(1-\lambda)\,q_{D}^{L}\right)+\theta f\bar{q}_{ND}^{L}(p^{LR}).
\end{aligned} \tag{.12}$$

Note that difference between first system of four equations and the next one is that in the first one $\bar{q}_{ND}^{H}(p^{LR})$,$\bar{q}_{ND}^{L}(p^{LR})$ are taken as given. This will permit us to take limits on p_d on $[0,1]$ and still have a well defined system of equations.

Bid Ask Spreads and Liquidity Premia. How the observable frictions in the secondary market, bid ask spreads, map into liquidity premia? The liquidity premium, ℓ_{ND}, is defined as

$$q_{ND}^{H}=\frac{(1-p_d)\left(m+(1-m)\left(z+q_{ND}^{H}\right)\right)+p_d q_{D}^{H}}{1+r+\ell_{ND}}$$

where q_{ND}^{H} is given by (.12). We can then rewrite bond prices using the endogenous liquidity component as follows

$$\ell_{ND}=(1-p_d)\,(1-m)\,\zeta\frac{q_{ND}^{H}-q_{ND}^{L}}{q_{ND}^{H}}+p_d\zeta\frac{q_{D}^{H}-q_{D}^{L}}{q_{D}^{H}}.$$

Thus, ℓ_{ND} is defined as the additional spread that is needed to explain a price q_{ND}^{H} if the investor has no concerns for liquidity. Two observations about this liquidity component. First, it is the expected valuation loss after taking into account a loss upon a having to sell and the probability of incurring a liquidity event and having to sell. Second, it depends on distance to default (or p_d); this is the case because the loss upon having to sell will be higher if the bond is defaulted.

[17] Note that this process captures the idea of mean reversion in the hazard rates, that is common on the literature of credit risk modeling (see for example Longstaff et al. (2005)). Formally, we can think about a irreducible Markov chain with 2 states and transition matix $\mathbf{P}$. The p^{LR} will be defined by the invariant distribution Π.

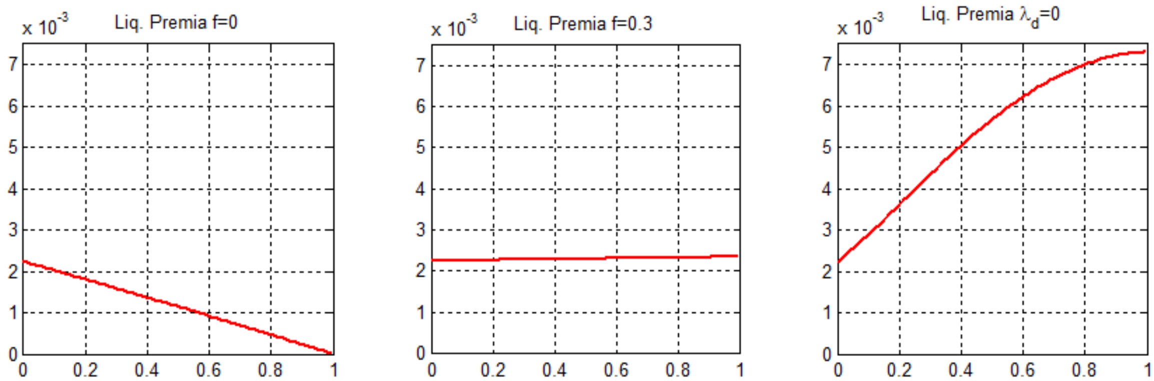

Figure .1: **Liquidity Premium**.

Understanding the model. The jump to default model is also useful to highlight why each one of the pieces in the model is needed. As said before, the requirements for the channel are: (1) long term debt and frictions on the secondary market (2) positive recovery ($\mathcal{R}(b) > 0$) (3) worse liquidity conditions in default ($\alpha_D < \alpha_{ND}$). The first thing is that there is a bid ask spread. For this, there has to be a friction on the secondary market, $\lambda \in (0,1), \alpha \in (0,1)$ and long term debt is needed $m < 1$. The next figure illustrates how the liquidity premium is changing with the default probabilities. We fix the parameters to: $z = 0.03, 1 - \theta = 0.0385, r = 0.01, \zeta = 0.25, \alpha_{ND} = 0.8, \alpha_D = 0, \mathcal{R}(b) = 0.3$. This coincides with our calibration for Argentina. In addition we fix $\bar{p}^{LR} = 0.03$. In the first panel, we show the case in which there is long term debt and frictions in the secondary market, but there is no recovery. This will imply that as we approach to default, the liquidity premium will actually decrease (in absolute value). The panel in the middle includes the case when there is a recovery value for the bonds. In this case there will be a positive bod ask spread, but it could well be the case that the bid ask spread during default is lower as is illustrated in the figure. This will imply that as we approach default, again, the liquidity premium will decrease. Finally, the panel on the right hand side illustrates the case in which $\alpha_D < \alpha_{ND}$. This implies that as we approach to default the liquidity premium is higher.