Illiquidity in Sovereign Debt Markets*

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Abstract

We study sovereign debt and default policies when credit and liquidity risk are jointly determined. To account for both types of risks we focus on an economy with incomplete markets, limited commitment, and search frictions in the secondary market for sovereign bonds. We quantify the effect of liquidity on sovereign spreads and welfare by performing quantitative exercises when our model is calibrated to match key features of the Argentinean default in 2001. From a positive point of view, we find (a) that a substantial portion of sovereign spreads is due to a liquidity premium, and (b) the liquidity premium helps to resolve the "credit spread puzzle," by generating high mean spreads while maintaining a low default frequency. From a normative point of view, we find that reductions in secondary market frictions improve welfare.

Keywords: Credit Risk, Liquidity Risk, Sovereign Debt, Open Economies.

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1 Introduction

Sovereign countries borrow to smooth shocks. One friction that prevents the smoothing of expenses over time and states of nature is governments' inability to commit to future debt and default policies. This lack of commitment implies that the government will repay its debts only if it is convenient to do so and will dilute debt holders whenever it sees fit. To compensate investors for bearing these risks the sovereign pays a *credit risk premium* that reduces the available resources for domestic consumption and can substantially increase borrowing costs during bad times. The sovereign debt literature has helped us to understand how a lack of commitment shapes the outcomes of sovereign countries from a positive point of view and what policies are desirable from a normative point of view.[1]

The recent European debt crisis, however, has underscored that decentralized markets impose additional frictions that prevent smoothing by sovereign countries.[2] Their bonds are traded in over-the-counter markets, where trading is infrequent. Thus, if an investor holds a large position in a sovereign bond, it might take time to find a counterparty willing to trade at a fair price. For this reason, investors need to be compensated not only for the risk of default or dilution but also for illiquidity, which introduces, in addition, a *liquidity risk premium*. This liquidity premium further reduces available resources and constrains sovereign policies. So far, the literature has been silent about this feature of sovereign borrowing. Our objective in this paper is to fill this gap by answering the following questions. How do credit and liquidity premium interact? What portions of total spreads can be explained by credit and by liquidity? What would be the welfare gains of reducing frictions in the secondary market?

Our paper contributes to the literature on sovereign borrowing in two ways. First, we propose a tractable model of sovereign borrowing in which credit and liquidity premia jointly determine borrowing and default decisions. Second, in a quantitative exploration focusing on one of the most studied cases of sovereign default, Argentina's default in 2001, we show that the liquidity premium is a substantial component of total spreads,

[1]See for example Aguiar and Gopinath (2006), Arellano (2008), and see Aguiar and Amador (2014) for a recent review . Recent contributions to the quantitative literature of sovereign debt, among many others, include Arellano and Bai (2014), Chatterjee and Eyigungor (2015), Hatchondo et al. (2016), Pouzo and Presno (2016), Aguiar et al. (2017), Arellano and Bai (2017), Bianchi et al. (2018), Dovis (2018), Roch and Uhlig (2018), Ottonello and Perez (2018), Perez et al. (2019), Sanchez et al. (2018), Arellano et al. (2019) and Bocola and Dovis (Forthcoming).

[2]For example, Pelizzon et al. (2016) documents substantial liquidity frictions for Italian bonds during the recent crises. In addition to the recent evidence from sovereign debt markets, a large literature that studies corporate bonds has documented sizable trading frictions and liquidity premia in the US corporate debt market. See for example Edwards et al. (2007) and Longstaff et al. (2005).

and helps to rationalize the "credit spread puzzle" by simultaneously generating high credit spreads and a low default frequency. Furthermore, we show that, through the lens of our model, the welfare gains from eliminating liquidity frictions are, quantitatively, of the same order of magnitude as the gains from eliminating income fluctuations.

We begin our paper by constructing a model of sovereign debt where debt and default policies take into account credit and liquidity risk. We focus on a small open economy that borrows from international investors to smooth income shocks following the quantitative literature on sovereign debt that builds on Eaton and Gersovitz (1981). A benevolent government issues non-contingent long-term debt and chooses debt and default policies to maximize household utility. The government cannot commit to future debt and default policies and might default in some states of nature. The distinctive feature of our model, in comparison to the previous literature, is the introduction of search frictions in the secondary market for sovereign bonds, following the literature on over-the-counter markets, such as Duffie et al. (2005). In our model, investors buy bonds in the primary market and can receive idiosyncratic liquidity shocks. If a shock occurs, they will bear a cost for holding the bond and therefore become natural sellers. Due to search frictions in the secondary market, it will take time for them to find a counterparty with whom to transact. As a result, bid-ask spreads arise endogenously through the bargaining between investors and dealers in the over-the-counter bond market.

One of the main features of the model we propose is that default and liquidity risk will be jointly determined. On the one hand, the presence of search frictions in the secondary market introduces a liquidity risk premium that affects prices in the primary market, thereby affecting debt and default policies, which in turn affect the credit risk premium. On the other hand, as the credit risk premium increases, the probability of default also increases, and because investors foresee worse liquidity conditions in the future should a default occur, liquidity conditions will also deteriorate, which in turn increase the liquidity risk premium. Therefore, in our model, the default and liquidity risk premia are jointly determined. This joint determination is important because it will enable us to decompose total spreads into liquidity and credit components and to study the effects on welfare of reducing liquidity frictions in the secondary market.

After building a model of sovereign borrowing in which both credit and liquidity premia constrain the choices of the government, we perform quantitative exercises to assess (1) how much of total spreads are due liquidity frictions, and (2) what the welfare gains from eliminating these frictions would be. To do so, we calibrate the model to match key features of Argentina's default in 2001. In particular, we match debt levels, the mean and volatility of spreads, bid-ask spreads, and bond trading turnover, to counterparts in the data.

Our first quantitative finding is that the liquidity premium is a substantial component of total spreads. For the 1993:I to 2001:IV period, our calibrated model attributes to the liquidity premium (on average) roughly one quarter of the total Argentine sovereign spread. Furthermore, the liquidity component of total sovereign spreads generated by our model is time-varying and during good times (when debt levels are low and output is high). This prediction is consistent with the empirical findings in Bai et al. (2012) and Pelizzon et al. (2016).

A corollary of our first quantitative finding is that accounting for the liquid component of total spreads helps to resolve the "credit spread puzzle." Standard sovereign default models fully attribute the sovereign spread to default risk. As a result, such models require a counter-factually high default rate to match observe levels of sovereign spreads.[3] In contrast, our model attributes a significant portion of the total sovereign spread to liquidity risk and is therefore able to match the mean level of Argentine spreads while simultaneously matching a low annual default frequency of 2.8 percent.

Our second quantitative finding is that the welfare gains from eliminating liquidity frictions are substantial. In particular, using our calibrated model, we find that the welfare gains induced from eliminating secondary market frictions are 0.17 percent in consumption equivalent terms. To put this number in perspective, given the volatility of consumption for Argentina in the period of study, a representative agent in an economy as in Lucas (2003) would pay 0.40 percent in consumption equivalent terms to eliminate income fluctuations.

We believe that the distinction between credit and liquidity risk is important for the design of debt policies for two reasons. First, in the long run, the policies to mitigate lack of commitment differ from those to mitigate frictions in the secondary market. For example, Hatchondo et al. (2016) and Chatterjee and Eyigungor (2015) show that fiscal rules and covenants on debt improve welfare in models when the government lacks commitment. However, policies that would decrease the liquidity premium in the long run include the development of a centralized exchange for sovereign bond trading or increasing transparency in the secondary market, as reported in Edwards et al. (2007). Second, the policies implemented during a short-term crisis might also be different. For example, a government could use resources to repay debt or to bail-out financial institutions that hold government debt and are in distress. An alternative policy, focusing on the secondary market, would be to provide liquidity to intermediaries.

[3] A recent exception is Pouzo and Presno (2016), which resolves the credit spread puzzle through an "uncertainty premium."

Literature Review. The key novelty of our paper is to incorporate secondary bond market liquidity frictions into quantitative models of sovereign default (see Aguiar and Gopinath, 2006 and Arellano, 2008 for early examples) that build on Eaton and Gersovitz (1981). Our model incorporates long-term debt (see, e.g., Hatchondo and Martinez, 2009, Arellano and Ramanarayanan, 2012 and Chatterjee and Eyigungor, 2012) and debt recovery after default (see, e.g., Yue, 2010).

Our key contribution is to show that accounting for the liquidity premium allows our model to simultaneously match a high level of credit spread and a low default frequency, thereby providing a potential resolution of the "credit spread puzzle." Furthermore, we show that reducing secondary market liquidity frictions can significantly improve long-run welfare. Pouzo and Presno (2016) provide an alternative resolution of the credit spread puzzle by introducing model uncertainty and ambiguity aversion on the side of the investors. These ambiguity averse investors perceive a probability of default that is higher than the actual default probability. As a consequence, they demand an uncertainty risk premium that increases sovereign spreads for a given default frequency, which rationalizes the "credit spread puzzle." Our paper provides an alternative explanation.

To model secondary market frictions, we build on the random over-the-counter search framework in Duffie et al. (2005). In this setting, bond holders naturally charge a liquidity premium for holding illiquid sovereign bonds. This is due to the possibility of costly delays when bond holders attempt to offload their positions. In subsequent work to our paper, Chaumont (2018) also incorporates liquidity frictions into a model of sovereign borrowing. Chaumont (2018) adopts a framework with competitive search (see, for example, Moen, 1997) in which investors direct their search to different sub-markets. In contrast, our paper adopts a framework with random search, which has a long tradition in studying over the counter markets (see, for example, Duffie et al., 2005 and He and Milbradt, 2014). Our paper shows that a setup with random search can address the credit risk puzzle, and that the long-run welfare gains from reducing liquidity frictions are large.

A recent corporate credit risk literature studies the implications of secondary market search frictions for corporate default risk. He and Milbradt (2014) and Chen et al. (2017) decompose corporate credit spreads into liquidity and credit components. We adapt their decomposition to our sovereign default setting. Our decomposition further accounts for the role of dynamic debt issuance, a feature that is common to sovereign debt models. We show that optimal dynamic debt management by governments can partially mitigate the adverse liquidity-default risk feedback loop. This highlights an additional margin over which active sovereign debt management policies can be of value.

Finally, there is a growing empirical literature which documents a sizeable liquidity component in sovereign spreads.[4] Pelizzon et al. (2016) document a strong relationship between sovereign risk and secondary bond market liquidity for Italian bonds. Bai et al. (2012) documents the presence of a liquidity component in sovereign spreads for a wider set of Eurozone countries. Liquidity risk is documented even for sovereigns with no meaningful default risk. For example, Fleming (2002) finds evidence of liquidity effects in U.S. treasury markets. Our paper complements these empirical studies by quantifying the size of the liquidity premium over the business cycle and to assess the welfare losses due to liquidity frictions.

2 Model

In this section, we present a model of sovereign default with trading frictions in the secondary market. We first describe the setting: section 2.1 describes the macroeconomic environment, section 2.2 describes the secondary bond market, and 2.3 describes the the timing of the model. We then proceed to characterize the equilibrium: section 2.4 characterizes the decisions of the government given prices, sections 2.5 and 2.6 define bond prices and valuations, and section 2.7 defines the equilibrium. Finally, section 2.8 discusses why each of the ingredients in the model are needed to quantify the credit and liquidity component of spreads and to study the welfare implications of frictions in the secondary market.

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. The small open economy receives a stochastic stream of income denoted by y_t. Income follows a first-order Markov process $\mathbb{P}\left(y_{t+1} = y' \mid y_t = y\right)$. The government is benevolent and wants to maximize the utility of the household. To do this, it trades bonds in the international bond market to smooth the household's consumption. The household evaluates consumption streams, c_t, according to:

$$(1-\beta)\,\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right],$$

with time-preference $\beta \in (0,1)$ and utility function $u\left(\cdot\right)$, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The sovereign issues long-term debt when it is not in default. As in Hatchondo

[4] The importance of liquidity risk has also been documented for corporate bond spreads. See, for example, Longstaff et al. (2005), Chen et al. (2007), Edwards et al. (2007), Bao et al. (2011), and Friewald et al. (2012).

and Martinez (2009), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012), each unit of debt matures with probability m each period. Non-maturing bonds pay coupon z. This memoryless formulation of the debt maturity structure means that the face value of outstanding debt is the only relevant state variable for the obligations of the government. The government can issue bonds at a price q_t in the primary bond market. In equilibrium, the price of debt depends on current income, y_t, and next period's bond position, b_{t+1} (our convention is that $b_{t+1} > 0$ denotes debt). The budget constraint for the economy is given by:

$$c_t = y_t - [m + (1-m)\, z]\, b_t + q_t\, [b_{t+1} - (1-m)b_t]\,, \tag{2.1}$$

where mb_t is the repayment of principal for maturing debt, $(1-m)zb_t$ is the total coupon payment for non-maturing debt, and $q_t\,[b_{t+1} - (1-m)b_t]$ represents the proceeds from newly issued debt.

There is limited enforcement of debt; thus, the government can default at its convenience. There are two consequences of default. First, the government loses access to the international credit market and goes into autarky. Second, output is lower during default and is given by $y_t - \phi(y_t)$. That is, there is also a direct output cost of default, $\phi\,(y_t)$, which is a standard assumption in the literature.

The government can regain access to the international credit market with probability θ each period. A fraction of defaulted debt is written off when the government regains access to credit markets. In particular, the new face value of outstanding debt is $\mathcal{R}\,(b_t) \;=\; \min\left\{\bar{b}, b_t\right\}$, where b_t is the face value of defaulted debt and $\bar{b}$ is a maximum recovery value.[5] That is, the fraction of recovered debt in face value terms is $\mathcal{R}\,(b_t)\,/b_t = \min\left\{\bar{b}/b_t, 1\right\}$, and this fraction converges to zero as the amount of defaulted debt goes to infinity. Bond holders of (previously) defaulted bonds receive replacement bonds, with each unit, in face value terms, of defaulted bonds being replaced by new bonds of face value $\mathcal{R}\,(b_t)\,/b_t$. The coupon rate and maturity probability of the replacement bonds remain unchanged at z and m, respectively.

[5]Models of sovereign debt renegotiation such as Yue (2010) and Bai and Zhang (2012) endogenize recovery as the equilibrium outcome of a Nash bargaining game between the government and its creditors. These models produce recovery values of the form $\mathcal{R}(\bar{b}\,(y_t)\,, b_t) \;=\; \min\left\{\bar{b}\,(y_t)\,, b_t\right\}$ with state-dependent maximum recovery values, $\bar{b}\,(y_t)$. In our specification, the maximum recovery value does not vary across states.

2.2 Primary and Secondary Bond Markets

There are two bond markets: the primary market and the secondary market. The government issues debt in the primary bond market. International investors can initially purchase newly issued debt in the primary market. Subsequent trading of bonds occurs in the secondary market. Trading in the secondary bond market is subject to search frictions. We adopt the random search framework as in Duffie et al. (2005) and He and Milbradt (2014).

Each international investor is assumed to be small and can hold at most a single unit of government debt. These investors are risk-neutral and can be either constrained or unconstrained. Unconstrained investors price bonds by discounting future payoffs at the risk-free rate, r. Unconstrained investors can become constrained if they receive a liquidity shock. Liquidity shocks are idiosyncratic in nature and have a per period probability ζ of occurrence. Constrained investors also discount payoffs at the risk free rate, but are additionally subject to per period holding costs $h_c > 0$.[6] As a result, unconstrained investors have a high bond valuation, q^H, while constrained investors have a low bond valuation, q^L (the exact expressions for q^H and q^L are given given in sections 2.5 and 2.6). Therefore, unconstrained investors are the natural buyers of bonds in both the primary and the secondary markets, while constrained investors are the natural sellers of bonds in secondary markets.

Constrained investors try to offload their bond positions in the secondary market. Secondary market trading is intermediated where the per period contact probability between constrained investors and intermediaries is λ.[7] Constrained investors and intermediaries bargain upon making contact. The total surplus is:

$$S = A - q^L,$$

[6]We interpret liquidity events as investor-specific events that prompt an immediate need to sell (e.g. to meet an expenditure). Holding costs represent the utility loss associated with delayed transactions. Valuations of debt are non-negative due to free disposal of the asset. When solving the model numerically, we additionally assume that bonds can be freely disposed. This is necessary to ensure that the presence of holding costs do not imply negative bond prices for some off-the-equilibrium path values of the state space. Note that bond prices are always positive along the equilibrium path.

[7]For simplicity, we assume that the matching function between constrained investors and intermediaries implies a constant matching probability, which is standard in the OTC search literature (see, e.g., Duffie et al. (2005), Lagos and Rocheteau (2007), He and Milbradt (2014), Afonso and Lagos (2015)). Appendix E shows that the fraction of bonds held by constrained investors is approximately constant over time in our calibrated model. This implies that the number of potential sellers (approximately) scale one-for-one with the level of outstanding debt. A constant matching probability would result in standard matching frameworks under the assumption that the number of potential buyers also scale one-for-one with the level of outstanding debt.

where q^L is the low valuation of the constrained investor, and A denotes the ask price at which the intermediary can then offload the bond. Following He and Milbradt (2014), we assume that there is a large mass of competitive unconstrained investors waiting on the sidelines who intermediaries could contact with immediate effect. This simplifying assumption means that we do not have to keep track of the dealer's inventory as intermediaries could instantaneously offload bonds to high-valuation investors. Since the large mass of high-valuation investors on the sidelines act competitively, intermediaries can offload their positions at high valuations, and thus:

$$A = q^H.$$

The total surplus, $S = q^H - q^L$, is then divided according to a Nash bargaining rule with the bargaining power of the constrained low-valuation investors being $\alpha \in [0, 1]$. This implies that the price at which constrained investors sell to intermediaries upon contact is:

$$q^S = q^L + \alpha(q^H - q^L).$$

The dollar bid-ask spread, per unit of principal, is the difference between intermediaries' selling price, q^H, and buying price, q^S,

$$ba^{Dollar} = (1 - \alpha)(q^H - q^L).$$

The proportional bid-ask spread:

$$ba = \frac{(1 - \alpha)(q^H - q^L)}{\frac{1}{2}\left(q^H + q^L\right) + \frac{\alpha}{2}\left(q^H - q^L\right)},$$

is simply the dollar bid-ask spread normalized by the mid price, $\frac{1}{2}\left(q^H + q^S\right)$.

We assume that the bargaining power of constrained low-valuation investors is lower when it comes to trading defaulted bonds:

$$\alpha_D < \alpha_{ND}.$$

The objective of this assumption is to generate (quantitatively) larger bid ask spreads during default. This equilibrium outcome of our model is in line with substantial evidence that bid-ask spreads are higher during default for US corporate Bonds (see, e.g., Edwards et al., 2007 and He and Milbradt, 2014).[8] This assumption is not qualitatively necessary,

[8]To the best of our knowledge, we are unaware of cross-country studies of bid-ask spreads for sovereign

though. In Appendix D, Table 5, we quantitatively show that even in the case in which $\alpha_D = \alpha_{ND}$ the bid ask spreads during default are higher. In Section B of the Appendix, subsections B.2 and B.3, we further discuss the role of the assumption $\alpha_D < \alpha_{ND}$.

2.3 Timing

The timing for the government is as follows and is summarized in Figure 2.1. First, consider the case in which the government has credit access (i.e., is not in default) and begins period t with an amount b_t of outstanding debt. Income, y_t, is then realized. The government then decides whether or not to default $d_t \in \{0, 1\}$. If the government chooses not to default ($d_t = 0$), principal payments for maturing debt, mb_t, and coupon payments for non-maturing debt, $(1-m)\, zb_t$, are made. The government can then issue new debt in the primary market. An issuance with face value $b_{t+1} - (1-m)b_t$ leads to outstanding debt with face value b_{t+1} at the beginning of the next period. As previously mentioned, unconstrained investors are the natural buyers of new bond issues, and thus, bonds are always issued at the high valuation, q_t^H. Finally, consumption takes place and is given by $c_t = y_t - [m + (1-m)\, z]\, b_t + q_t^H\, [b_{t+1} - (1-m)b_t]$.

Next, consider the case in which the government is already in default or chooses to default in the current period ($d_t = 1$). In this case, b_t is the amount of debt that is in default. The government is in autarky and cannot borrow. Consumption is simply equal to income adjusted for the costs of default: $c_t = y_t - \phi\,(y_t)$. Nature determines whether the government regains credit access between the end of period t and the beginning of the next period, $t+1$. The probability of regaining credit access is θ, and in that event, the government re-accesses the debt market with an outstanding debt of $\mathcal{R}(b_t) = \min\left\{\bar{b}, b_{t+1}\right\}$ at the beginning of the next period. Otherwise, the government remains in autarky.

The timing for investors is as follows. Investors pay holding costs h_c for the period if they begin period t constrained. Secondary market trading for outstanding bonds occurs once per period, immediately before the government issues new bonds in the primary market. As mentioned in section 2.2, only constrained bond holders attempt to sell in the secondary market. With probability λ, a constrained investor meets an intermediary and offloads his bond position. The transaction price is $q_{ND}^S\,(y_t, b_{t+1})$ when the government is not in default, and $q_D^S\,(y_t, b_t)$ when the government is in default. Constrained investors who fail to contact intermediaries remain constrained going into the next period $t+1$.

An investor who is unconstrained at the beginning of period t is not subject to holding

bonds that are in default. The challenge stems from the fact that bid ask quotes in many cross country trading platforms, for example Bloomberg, are without commitment so that intermediaries are not obliged to fulfill transactions at quoted prices.

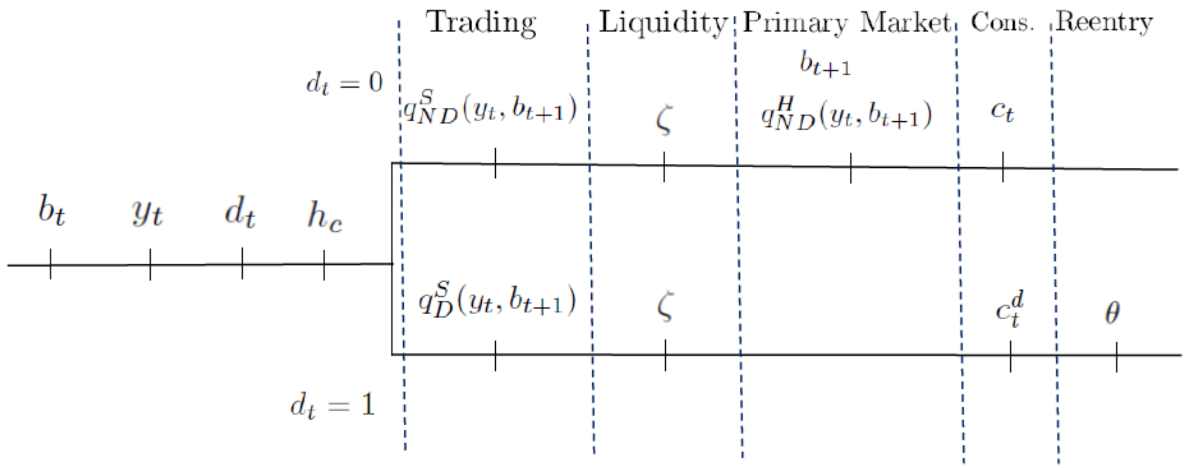

Figure 2.1: This figure summarizes the timing before and after default in period t. The government enters the period with bonds b_t. Then, income, y_t, is realized, and the government chooses whether or not to default, d_t. Constrained investors are subject to holding costs, h_c. The upper branch depicts the sequence of events in the absence of default: secondary market trading of outstanding bonds occurs, and unconstrained investors receive liquidity shocks. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. Then, the liquidity shock is realized. Then, the government issues a face value of debt $b_{t+1} - (1-m)b_t$, facing a price $q^H_{ND}(y_t, b_{t+1})$. Finally, consumption is realized. The lower branch depicts what happens in the case in which the government defaults. First, liquidity-constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Note that the primary market is closed while the government is in autarky. Then, the government will re-access the debt market in the next period with probability θ. Finally, consumption is equal to $c^d(y_t) = y_t - \phi(y_t)$.

costs for the period. However, such an investor can become constrained for the beginning of the next period $t+1$ if he receives a liquidity shock during period t. Liquidity shocks occur with probability ζ and take place after the conclusion of trading in the secondary market. This means that a newly constrained investor in period t is unable to immediately offload his position in the same period. In addition, liquidity shocks occur prior to new bond issuances in the primary market. This implies that the unconstrained investors who purchased newly issued bonds during period t will still be unconstrained at the beginning of period $t+1$.

2.4 The Government's Decision Problem

The government takes the bond price schedule as given and chooses debt and default policies to maximize household welfare. This infinite-horizon decision problem can be cast as a recursive dynamic programming problem. We focus on a Markov equilibrium with income, y, as the exogenous state variable and debt, b, as the endogenous state variable. The value for a government with an option to default, V^{ND}, is the larger of the value of defaulting, V^D, and the value of repayment, V^C,

$$V^{ND}(y,b) = \max_{d\in\{0,1\}} dV^D(y,b) + (1-d)\,V^C(y,b)\,.$$

The solution to this problem yields the government's default policy:

$$d = \boldsymbol{D}(y,b) = 1_{\left\{V^D(y,b)>V^C(y,b)\right\}}. \tag{2.2}$$

That is, the government defaults whenever the value of defaulting is higher than the value of repayment.

The value of defaulting is:

$$V^D(y,b) = (1-\beta)\,u(y-\phi(y)) + \beta\mathbb{E}_{y'|y}\left[\theta V^{ND}\left(y',\mathcal{R}(b)\right) + (1-\theta)V^D\left(y',b\right)\right],$$

where the flow utility is determined by household consumption in default, $y-\phi(y)$, while the continuation value takes into account the possibility of regaining credit market access with debt level $\mathcal{R}(b)$.

The value of repaying is:

$$V^C(y,b) = \max_{b'}\left\{(1-\beta)u(c) + \beta\mathbb{E}_{y'|y}\left[V^{ND}\left(y',b'\right)\right]\right\} \tag{2.3}$$

and is subject to two constraints. The first constraint is the budget constraint,

$$c = y - \left[m+(1-m)z\right]b + q^H_{ND}\left(y,b'\right)\left[b'-(1-m)b\right], \tag{2.4}$$

in which the government issues bonds by running an auction with commitment, as is standard in Eaton and Gersovitz (1981) settings. The government obtains an auction price that is equal to the valuation of (unconstrained) high valuation investors, $q^H_{ND}(y,b')$.[9]

In addition, the government faces an upper bound in the ex ante one-period-ahead

[9]The budget constraint (2.4) treats bond prices in the event of a debt buyback (i.e. $b' < (1-m)\,b$) symmetrically. In principle, the government could potentially benefit from interacting with constrained investors in order to repurchase outstanding debt at the low valuation price q^L_{ND}. In our quantitative analysis, however, we find that debt buybacks do not occur on the equilibrium path (see Appendix A). This is because debt buybacks imply costly transfers to outstanding creditors. See, for example, Bulow and Rogoff (1991) and Aguiar et al. (2019) for proofs of no buyback resultsin sovereign debt settings. See, also, Admati et al. (2018) for similar results in a corporate finance setting.

expected default probability:

$$\delta\left(y,b'\right) \equiv \mathbb{E}_{y'|y}\left[d\left(y',b'\right)\right] \leq \bar{\delta} \tag{2.5}$$

whenever there is positive debt issuance, $b' - (1-m)b > 0$. As explained in Chatterjee and Eyigungor (2015), in long-term debt models with positive recovery, the government has incentives to dilute existing bond holders by issuing large amounts of debt just prior to default. Since the liability of the government upon regaining credit access is at most $\overline{b}$, the government will then issue an infinite amount of debt just prior to default to fully dilute existing bond holders. Constraint (2.5) is a simple modeling device to rule out such counterfactual behavior.[10]

The solution to the repayment problem (2.3) yields the debt policy of the government:

$$b' = \boldsymbol{B}\left(y,b\right). \tag{2.6}$$

2.5 Debt Valuations Before Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is not in default. The debt market before default is summarized in Figure 2.2. Let y be current income, and suppose that b' is the post-issuance face value of outstanding debt. The value of one unit of debt for an unconstrained investor with a high valuation is:

$$\begin{aligned} q^H_{ND}\left(y,b'\right) = \mathbb{E}_{y'|y}\Bigg\{ &\left[(1-d\left(y',b'\right)\right] \frac{m + (1-m)\left[z + \zeta q^L_{ND}\left(y',b''\right) + (1-\zeta) q^H_{ND}\left(y',b''\right)\right]}{1+r} \\ &+ d\left(y',b'\right) \frac{\zeta q^L_D\left(y',b'\right) + (1-\zeta) q^H_D\left(y',b'\right)}{1+r} \Bigg\}, \end{aligned} \tag{2.7}$$

which reflects the state-contingent payoffs of the bond. An investor receives principal m and coupon $(1-m)\,z$ in the absence of default during the next period, $d\left(y',b'\right) = 0$. In

[10] In the quantitive section we set this number to 0.75, which amounts to an upper bound on the one month default probability of 75 percent. In Appendix D we show that even in the case in which this constraint caps the default probability at 99 percent per month, the quantitive properties of the model are almost unchanged. In addition, as noted in Chatterjee and Eyigungor (2015), sovereign bonds issued in financial centers (e.g. New York) have to be underwritten by some investment bank, and. reputational concerns may prevent them from issuing bonds with very high probabilities of immediate default. Flandreau et al. (2009), Figure 3b, reports that sovereign spreads at issuance where at most 1000 basis points for the period 1993 to 2007. For a 10 year zero coupon bond this spread amount to a 1 month (risk neutral) default probability of less than 1 percent, which is a fraction of the upper bound that we use.

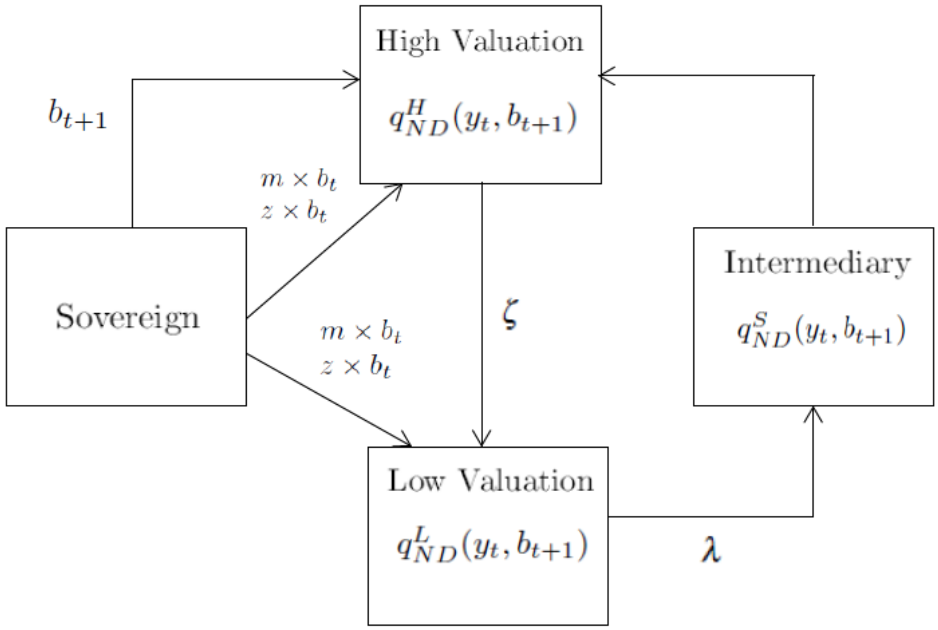

Figure 2.2: This figure details the bond market if the sovereign is not in default and does not default in period t. The sovereign begins by issuing debt b_{t+1}. The high-valuation investors buy this debt in the primary market. After that, with probability λ, the low-valuation investors will meet an intermediary. They will sell their bonds at the price $q^S_{ND}(y_t, b_{t+1})$. After selling their bonds, they exit the market. The low-valuation investors who do not meet an intermediary will attempt to sell their bonds in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond in the next period in the secondary market. Both the high- and low-valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

this case, the continuation value of the $1 - m$ non-maturing fraction of the bond depends on next period's optimal debt policy, $b'' = \boldsymbol{B}(y', b')$, and the realization of the idiosyncratic liquidity shock. A liquidity shock arrives with probability ζ, in which case the investor obtains a low continuation value, $q^L_{ND}(y', b'')$. Otherwise, the investor remains unconstrained and assigns a high continuation value, $q^H_{ND}(y', b'')$, to the bond. An investor does not receive any cashflow in the event of a default, $d(y', b') = 0$. In this case, the government defaults on b' units of debt, and the per unit price of defaulted debt is $q^H_D(y', b')$ and $q^L_D(y', b')$ for investors who do not receive and receive liquidity shocks, respectively. The value of defaulted bonds will be described in the next section 2.6.

The price of debt for a constrained investor with a low valuation is:

$$q^L_{ND}(y, b') = \mathbb{E}_{y'|y} \left\{ \left[(1 - d(y', b')\right] \frac{-h_c + m + (1-m)\left[z + (1-\lambda)q^L_{ND}(y', b'') + \lambda q^S_{ND}(y', b'')\right]}{1+r} \right.$$

$$+d\left(y',b'\right)\frac{-h_c+(1-\lambda)q_D^L\left(y',b'\right)+\lambda q_D^S\left(y',b'\right)}{1+r}\Bigg\}. \tag{2.8}$$

The valuation of a constrained investor is similar to that of an unconstrained investor, but reflects the following differences. First, a constrained investor is assessed holding costs h_c, that lower the effective value of a bond. Second, the continuation value for a constrained investor depends on trading outcomes in the secondary market. As described in section 2.2, a constrained investor sells with probability λ. The selling price (in the next period) is:

$$q_{ND}^S\left(y',b''\right)=\alpha_{ND}q_{ND}^H\left(y',b''\right)+(1-\alpha_{ND})q_{ND}^L\left(y',b''\right)$$

in the absence of default and $q_D^S(y',b')$ in the event of default.

2.6 Debt Valuations After Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is in default. The debt market after default is summarized in Figure 2.3. Let the current income be y, and let b be the amount of debt in default. In this case, the value of one unit of debt for unconstrained high-valuation investors is:

$$q_D^H\left(y,b\right)=\frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q_D^H\left(y',b\right)+(1-\zeta)q_D^L\left(y',b\right)\right]+\theta\frac{\mathcal{R}\left(b\right)}{b}q_{ND}^H\left(y,\mathcal{R}\left(b\right)\right). \tag{2.9}$$

The government regains credit access in the next period with probability θ, in which case $\mathcal{R}\left(b\right)/b$ is the fraction recovered for each unit of defaulted debt. The value of recovered bonds, $q_{ND}^H\left(y,\mathcal{R}\left(b\right)\right)$, is given by (2.7) and reflects the new value of total outstanding debt, $\mathcal{R}\left(b\right)$. Otherwise, investors receive no payments if default is not resolved and the continuation value reflects the probability ζ of receiving a liquidity shock.

Similarly, the per unit value of debt for constrained low-valuation investors is:

$$q_D^L(y,b)=\frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c+\lambda q_D^S\left(y',b\right)+(1-\lambda)q_D^L\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q_{ND}^L\left(y,\mathcal{R}\left(b\right)\right). \tag{2.10}$$

This valuation is analogous to that of unconstrained investors (2.9), but accounts for holding costs and trading frictions in the secondary market. Constrained investors sell during the next period with probability λ. The selling price for a defaulted bond is:

$$q_D^S(y,b)=\alpha_D q_D^H(y,b)+(1-\alpha_D)q_D^L(y,b),$$

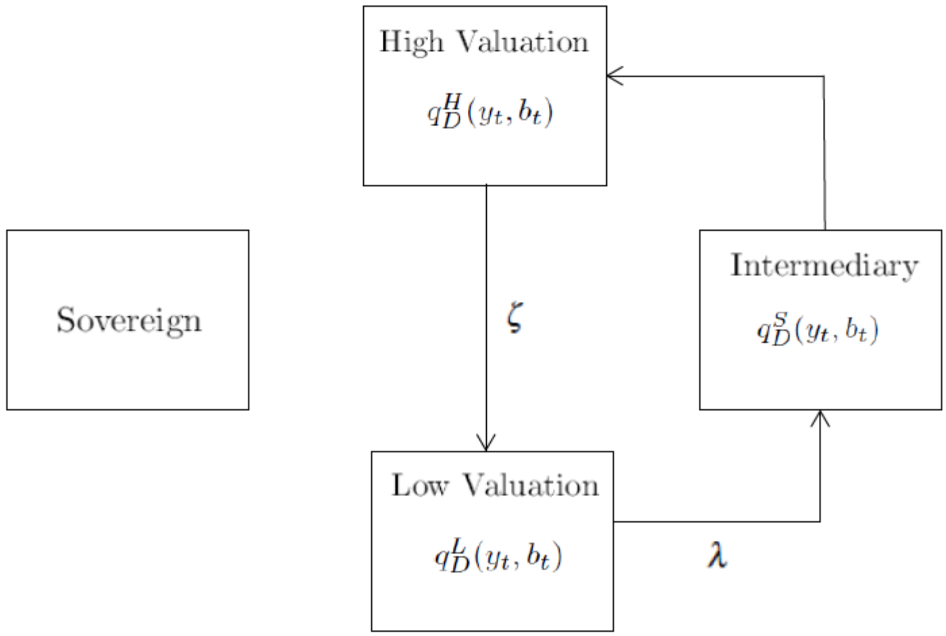

Figure 2.3: This figure details the bond market if the government is in default or defaults in period t. There is no debt issue or debt service. The sovereign has an outstanding balance of debt b_t. The low-valuation investors will meet an intermediary with probability λ. They will sell their bonds at the price $q_D^S(y_t, b_t)$. After they sell the bonds, they exit the market. The low-valuation-investors who do not meet an intermediary will attempt to sell in the next period. Then, with probability ζ, the high-valuation investors will receive a liquidity shock. They will have the opportunity to sell in the next period in the secondary market. Finally, with probability θ, the government resolves the default and re-accesses the credit market in the next period with a total outstanding debt of $b_t \times \mathcal{R}(b_t)$.

and reflects the lower investor bargaining power during default: $\alpha_D < \alpha_{ND}$.

2.7 Equilibrium

We focus on a Markov equilibrium with state variables y and b. An equilibrium consists of a set of policy functions for consumption $\boldsymbol{C}(y, b)$, default $\boldsymbol{D}(y, b)$, and debt $\boldsymbol{B}(y, b)$, as well as bond valuations $q_{ND}^H(y, b')$, $q_{ND}^L(y, b')$, $q_D^H(y, b)$ and $q_D^L(y, b)$ such that: (1) the policies solve the government's problem (2.3) taking bond valuations as given, and (2) the bond valuations satisfy equations (2.7) (2.8), (2.9) and (2.10).

2.8 Discussion

In this subsection, we discuss our modeling assumptions. Our goal is to provide a parsimonious framework to study debt and default policy in a setting where default and liquidity risk are jointly determined.

There is substantial evidence of trading frictions in the secondary market for sovereign bonds (see, e.g., Pelizzon et al. 2013 for recent evidence for sovereign bonds and Edwards et al. 2007 for evidence on US corporate bonds). To model these frictions, we adopt the framework of random search following a large literature on OTC markets that builds on the seminal contribution of Duffie et al. (2005). These secondary market frictions endogenously generate bid-ask spreads and a liquidity premium.

We introduce long-term debt in order to generate realistic levels ofliquidity risk premium. In the extreme case that outstanding debt matures every period, bid-ask spreads are equal to zero. This is because there is no need to trade when investors can simply wait and receive principal payouts from the government in the next period. In addition, long-term debt is now a standard feature of models of sovereign debt following the contributions of Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012) and is also needed to generate realistic debt levels and sovereign spreads.

We also introduce a positive debt recovery, $\mathcal{R}(b) > 0$, after a default. Positive recoveries are an important feature of the data (see, e.g., Yue (2010), Bai and Zhang (2012), and Cruces and Trebesch (2013)). In our model debt recovery is necessary in order to generate realistic behavior of bid-ask spreads. This is because the absence of recovery, $\mathcal{R}(b) = 0$, implies no future cash-flows and therefore zero valuations for both constrained and unconstrained investors during default, $q_D^H = q_D^L = 0$. In turn, this implies that the (dollar) bid-ask spread is zero during default. More importantly, it would also imply that bid-ask spreads approach zero as default probabilities approach one, which is at odds with a widening of bid-ask spreads during sovereign crises documented in Pelizzon et al. (2013) and the evidence for corporate bonds Edwards et al. (2007).

We assume a decrease in the bargaining power of investors after a sovereign default, $\alpha_D < \alpha_{ND}$, in order to quantitatively match the observed increase in bid-ask spreads in the lead up to a default (see, e.g., Pelizzon et al. (2016); there is also similar evidence in the context of corporate bonds, see, e.g., Edwards et al. (2007) and He and Milbradt (2014)). Qualitatively, this assumption is not necessary to for generating a positive comovement between bid-ask spreads and sovereign default probabilities. Indeed, we show that bid-ask spreads are higher for defaulted bonds even when $\alpha_D = \alpha_{ND}$ (see row (2) of Table 5 in

Appendix D).[11] The assumption $\alpha_D < \alpha_{ND}$ quantitatively magnifies the difference in bid-ask spreads between defaulted and non-defaulted bonds, and allows our model to match the large difference in bid-ask spreads between defaulted and non-defaulted bonds. An alternative modeling choice for generating this result is to have investor-dependant bond recovery rates, as in He and Milbradt (2014), with recovery rates being lower for constrained investors.

In Appendix B we further discuss our modeling choices using a simple jump-to-default model in which debt policies are fixed and there is an exogenous default probability.

3 Results

We numerically solve a discretized version of the model. As discussed in Chatterjee and Eyigungor (2012) grid-based methods have poor convergence properties when there is long-term debt. To overcome this problem, we follow their prescription and use randomization methods to ensure convergence. To ensure numerical accuracy, we choose a dense grid with 200 points for the persistent component of output and 450 points for debt. We implement the model in CUDA and numerically compute the model on a Tesla K80 GPU. Appendix C discusses the details.

3.1 Calibration

We calibrate the model developed in section 2 to account for the main features of Argentina's default in 2001. We focus on Argentina over the period from 1993:I to 2001:IV for three reasons. First, using this period facilitates comparison to prior studies in the sovereign debt literature (see, for example, Hatchondo and Martinez (2009), Arellano (2008) and Chatterjee and Eyigungor (2012)), and in this way, we can be transparent about the contribution of our paper. Second, this sample satisfies our model's main assumptions: (1) our model is real and Argentina had a fixed exchange rate vis-a-vis the dollar during this period, and (2) Argentina's bonds were traded in an illiquid secondary market during this period. We calibrate and simulate the model at a monthly frequency because liquidity is inherently a short-run phenomenon. However, we report results at a quarterly

[11] This is because a sovereign default results in an effective extension of debt maturity (as payments only resume after the government regains credit access) which, in turn, magnifies the difference in valuation between constrained and unconstrained investors.

frequency to facilitate comparison to previous studies.[12]

Functional Forms and Stochastic Processes. As is standard in the literature, we specify the household utility to be CRRA $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We set the endowment process to be:

$$y_t = e^{z_t} + \epsilon_t,$$
$$z_t = \rho_z z_{t-1} + \sigma_z u_t,$$

where z_t is a discretized AR(1) process with persistence ρ_z, volatility σ_z, and normally distributed innovations $u_t \sim N(0,1)$. We also add a small amount of noise $\epsilon_t \sim trunc\, N(0, \sigma_\epsilon^2)$ that is continuously distributed. As shown in Chatterjee and Eyigungor (2012), this is necessary to achieve numerical convergence. We set the output loss during default to:

$$\phi(y) = \max\left\{0, d_y y + d_{yy} y^2\right\}.$$

This loss function is proposed by Chatterjee and Eyigungor (2012) and nests several cases in the literature. When $d_y < 0$ and $d_{yy} > 0$, the cost is zero for the range $0 \leq y \leq -\frac{d_y}{d_{yy}}$ and rises more than proportionally with output for $y > -\frac{d_y}{d_{yy}}$. Alternatively, when $d_y > 0$ and $d_{yy} = 0$ the cost is a linear function of output.[13] As explained in Chatterjee and Eyigungor (2012), the convexity of output costs is necessary to match the volatility of sovereign spreads.

A priori set parameters. We set risk aversion to $\gamma = 2$, which is standard in the sovereign debt literature. The parameters for output are estimated from (linearly) detrended and seasonally adjusted data for Argentina for the quarterly sample from 1980:I to 2001:IV available from Neumeyer and Perri (2005). After estimating an AR (1) model for output at a quarterly frequency, we obtain monthly values $\rho_z = 0.983$, and $\sigma_z = 0.0151$, and we fix $\sigma_\epsilon = 0.004$.[14] We set the risk free rate to $r = 0.0033$ per month so that the quarterly risk free rate is 1 percent, which is standard. We set $m = 1/60$ to match an average debt matu-

[12]To be precise about this conversion, the quarterly debt to output ratio in our paper is the stock of debt at the end of the quarter divided by the sum of monthly output within the quarter. This implies that the average debt to quarterly output will be one-third of average debt to monthly output.

[13]The case studied in Arellano (2008) features consumption in default that is given by the mean output if the output is over the mean and equal to output if the output is less than the mean. This implies a cost function $\phi^A(y) = \max\{y - \mathbb{E}(y), 0\}$, which closely resembles the case of $d_y > 0$ and $d_{yy} = 0$.

[14]The conversion is as follows. Total monthly volatility is given by $\sqrt{0.004^2 + 0.0151^2} = 0.0156$. Then, total monthly volatility and the monthly autocorrelation are converted to a quarterly frequency. This implies an autocorrelation of $0.983^3 = 0.95$ and an output volatility of $0.0156 \times \sqrt{3} = 0.027$. The latter values are those recovered from the data.

Parameter	Description	Value
	Apriori set parameters	
γ	Sovereign's risk aversion	2
ρ_z	Persistence of output	0.983
σ_y	Volatility of output	0.0156
r	Discount rate of international investors	0.0033
m	Rate at which debt matures	0.0167
z	Coupon rate	0.01
θ	Probability of reentry	0.0128
$\overline{\delta}$	Max. Default Probability	0.75
λ	Probability of meeting a market maker	0.865
α_D	Bargaining power of the low-valuation investor in default	0
	Calibrated parameters	
β	Sovereign's discount rate	0.9841
$\overline{b}$	Maximum recovery for sovereign bonds	0.83
d_y	Output costs for default	-0.264
d_{yy}	Output costs for default	0.337
h_c	Holding costs for constrained investors	0.0014
α_{ND}	Bargaining power of the low-valuation investor, not in default	0.125
ζ	Probability of getting a liquidity shock	0.139

Table 1: Baseline parameters.

rity of 5 years based on values reported in Chatterjee and Eyigungor (2012) and Broner et al. (2013). We set the coupon rate to $z = 0.01$, so that the annualized coupon rate is 12 percent, which is close to the 11 percent value-weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012). We fix the reentry probability at $1 - \theta = 0.0128$, following Chatterjee and Eyigungor (2012). This implies an average exclusion period of 6.5 years.[15] We fix $\lambda = 0.8647$ such that the average time required for a constrained investor to offload his position to an intermediary is two weeks, as in Chen et al. (2017). Following Chatterjee and Eyigungor (2015), we set the maximum one-month-ahead default probability to $\overline{\delta} = 0.75$. Appendix D shows that the choice of $\overline{\delta}$ is (quantitatively) innocuous for our channel. For example, the targeted moments remain almost unchanged when we increase $\bar{\delta}$ to 0.99. This constraint is only necessary to preclude the government to issue an infinite amount of debt right before a default, which implies unbounded first and second moments for spreads.

Calibrated parameters. The remaining seven parameters are exactly identify based on seven moments. We choose time preference $\beta = 0.9841$ to target an average debt to

[15] Beim and Calomiris (2001) report that for the 1982 default episode, Argentina remained in a default state until 1993. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in the default state from 2001 until 2005, when it settled with most of its bondholders.

quarterly output ratio of 100 percent based on the mean debt to quarterly output ratio for Argentina for the period between 1993:I and 2001:IV.[16] We set the maximal debt recovery to be $\bar{b} = 0.83$ to match a mean recovery of $\mathbb{E}\left[\frac{\min\{\bar{b}, b_{def}\}}{b_{def}}\right] = 0.3$ in the model. This is based on a realized recovery rate of 30 percent for the 2001 Argentine default. In addition, we choose the default cost parameters $d_y = -0.264$ and $d_{yy} = 0.337$ to match the first two moments for the quarterly behavior of (annualized) sovereign spreads. Following Chatterjee and Eyigungor (2012), this involves targeting mean spreads of 0.0815 and a quarterly volatility of 0.0443.[17]

We calibrate parameters relating to secondary market frictions as follows. We first normalize the bargaining power of investors during periods of default to $\alpha_D = 0$. We then choose a holding cost of $h_c = 0.0014$ to target a mean proportional bid-ask spread of 500 basis points during default. Due to data limitations, we base this target on the bid-ask spreads of defaulted US bonds which are documented to range between 200 basis points during normal times (see, e.g. Edwards et al. 2007) and 620 basis points during recessions (see, e.g Chen et al., 2017). We then set the bargaining power of investors outside of periods of default to be $\alpha_{ND} = 0.125$ to target a mean proportional bid-ask spread of 50 basis points outside of default. For European bonds, Pelizzon et al. (2013) find a median bid-ask spread of 43 basis points in their sample, which increased to 125 basis points during the period June 2011 to November 2012. For non-investment grade US corporate bonds, Chen et al. (2017) report bid-ask spreads of 50 basis points during normal times and 218 during bad times. In addition, following the literature of market micro-structure, we compute a measure of effective spreads introduced by Roll (1984). In line with the evidence for corporate bonds, we find that for Argentinean bonds the mean and median effective spreads are 84.1 and 66.0 basis points respectively (see Appendix G for details). Appendix E shows that turnover, or the fraction of outstanding bonds being traded each period, is approximately $\lambda\zeta / [m + (1-m)(\lambda + \zeta)]$ on average in the model. We set the probability of receiving a liquidity shock each period to $\zeta = 0.139$ to match an average turnover of 12 percent per month based on the average turnover rate for US corporate bonds (see Bao et al. (2011)).[18] This value implies that, on average, an unconstrained investor becomes constrained every 7 months. Appendix D conducts a sensitivity analysis for the key parameters of the model and illustrates the relationship

[16] Chatterjee and Eyigungor (2012) target a debt to quarterly output ratio of 70 percent because their model does not feature recovery. We target the full 100 percent because our model features recovery.

[17] The annualized sovereign spread is given by $cs(y, b') = (1 + r^H(y, b'))^{12} - (1 + r)^{12}$ where the yield to maturity is given by $r^H(y, b') = [m + (1 - m)z] / q^H(y, b') - m$.

[18] To the best of our knowledge, turnover data for Argentinean bonds are unavailable for our period of interest.

Moment	(1) Target	(2) Baseline	(3) CE (2012)
Mean Debt to GDP	1.0	1.0	0.7
Expected Recovery	0.30	0.297	0
Mean Sovereign Spread	0.0815	0.0815	0.0815
Vol. Sovereign Spread	0.0443	0.0437	0.0443
Mean Bid-Ask Spread, ND	0.0050	0.0049	-
Mean Bid-Ask Spread, D	0.0500	0.0503	-
Mean Turnover	0.12	0.12	-
Default frequency (annual)	-	0.028	0.068

Table 2: **Model moments.**

between parameter values and target moments.

Calibration Results. The baseline parameters are shown in Table 1. Column (2) of Table 2 displays the corresponding model-implied moments. Our calibrated model closely matches all targeted moments. Our baseline model generates a mean debt to (quarterly) GDP ratio of 100 percent and an average recovery rate of 29.7 percent. The average sovereign spread is 0.0815 and the volatility is 0.0437. The average bid-ask spread before and after default is 50 and 503 basis points, respectively. The mean turnover rate is 12 percent. Finally, the annualized default rate is 0.028.

Coming Next. In subsection 3.2 we discuss the implications of liquidity in solving the (sovereign) credit spread puzzle. In subsection 3.3, we report the pricing functions in the primary market and the bid-ask spreads. In subsections 3.4 and 3.5, we provide a structural decomposition of total spreads into liquidity and default components. In subsection 3.6 we quantify the welfare implications of liquidity frictions. In subsection 3.7 we use our calibrated model to impute the liquidity component of Argentinean spreads in the lead up to Argentina's 2001 default. Finally, subsection 3.8 reports business cycle statistics of the model.

3.2 Credit Spread Puzzle

Table 2 shows that our baseline model matches the mean and volatility sovereign spreads while having an annual default frequency of 2.8 percent. Our model-implied default frequency is in agreement with the consensus default frequency used by the literature. For example, Arellano (2008), Hatchondo et al. (2010), Lizarazo (2013) and Pouzo and Presno (2016) all target an annual default frequency of 3 percent; Yue (2010) and Mendoza and Yue (2012) target an annual default frequency of 2.7 and 2.78 percent, respectively. Our

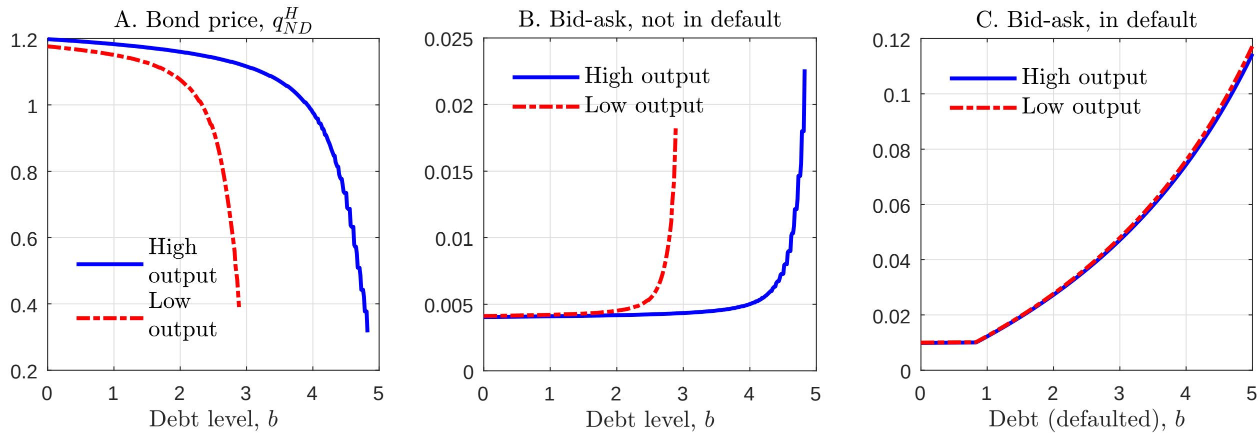

Figure 3.1: **Bond prices and bid-ask spreads.** This figure plots bond prices at issue, q^H, and bid-ask spreads which are defined as $\left(q^H - q^S\right) \Big/ \left[\frac{1}{2}\left(q^H + q^S\right)\right]$. All prices are a function of output y and current debt level b, with next period's debt b' being given by the optimal debt policy $b' = \boldsymbol{B}(b, y)$, which is given by equation (2.6). Panel A plots prices in the primary market for high-valuation investors. Panels B and C plot bid-ask spreads during credit access and during autarky. All plots are for high ($y = 0.920$) and low ($y = 1.087$) output; these output values correspond to ± 1 standard deviations of the unconditional distribution for (log) output. Panels A and B are for the range of debt level for which default does not occur. Given our monthly calibration, a 3 on the horizontal axis of debt choice corresponds to an average debt to quarterly output ratio of 1. For panels B and C, a 0.05 in the vertical axis corresponds to 500 basis points.

model is therefore able to address the "credit spread" puzzle by simultaneously matching both credit spreads and default rates. The underlying reason for our model's ability to address the credit spread puzzle is simple: a portion of the credit spread is not due to default risk, but rather liquidity risk. In contrast, models that match credit spreads without consideration for the role of liquidity frictions would result in a counterfactually high model-implied default rate (e.g. column (4) of Table 2 reports results from Chatterjee and Eyigungor (2012), which does not consider illiquidity for sovereign bonds).[19] In the coming sections, we further investigate the quantitative importance of liquidity for the credit spread puzzle. We show that, on average, liquidity is responsible for roughly a quarter of credit spreads, with this fraction changing depending on business cycle conditions.

3.3 Bond Prices, Bid-Ask Spreads, and Feedback

Figure 3.1 plots the model-implied bond prices and bid-ask spreads as a function of

[19] A recent exception is Pouzo and Presno (2016). They address the credit spread puzzle by introducing model uncertainty.

the level of debt b at the beginning of the period. The plots are for high and low values of output y, which correspond to values at plus and minus one standard deviation of the unconditional distribution for (log) output, respectively.

Panel A plots bond prices in the primary market during the credit access regime, $q^H_{ND}(y, b')$, where b' denotes the optimal choice of debt at state (b, y), and is given by $\boldsymbol{B}(b, y)$. The presence of bond recovery following a default implies that bond prices are always strictly positive. Standard comparative statics apply: bond prices are increasing in output and decreasing in debt levels, as is usually the case in quantitative models of sovereign debt. Because the model is calibrated at a monthly frequency the debt to quarterly output ratio is approximately $b/3$.

Panel B plots proportional bid-ask spreads, $\left(q^H - q^S\right) / \left[\frac{1}{2}\left(q^H + q^S\right)\right]$, during periods in which the government is not in default. The bid-ask spread is stable and lies between 40 and 50 basis points for low levels of debt (e.g., $b \leq 2.5$) regardless of income levels. Bid-ask spreads increase as the likelihood of default increases. This occurs when output is low or the debt level is high. For example, the bid-ask spread reaches 183 basis points when the debt level is $b = 2.89$ and output is low. The logic for this increase in bid ask spreads is as follows. First, a default results in an extension in the effective maturity of promised payments which, in turn, magnifies valuation differences across constrained and unconstrained investors. Second, the decrease in investor bargaining power for trading default bonds further increases bid-ask spreads in default.[20] Therefore, since prices are forward looking, bid-ask spreads (for non-defaulted bonds) increase as the probability of default increases, reflecting the increased likelihood of encountering worse liquidity conditions associated with trading defaulted bonds.

Panel C shows that the proportional bid-ask spread for defaulted bonds is also increasing in the amount of debt in default. The reasoning is as follows. To a first-order approximation the dollar bid-ask spread during default, $q^H_D - q^S_D$, is proportional to holding costs. Since holding costs are constant, the proportional bid-ask spread increases as the value of bonds decrease. The latter is true if the amount of debt in default is high and the fractional recovery per unit of defaulted debt, $\mathcal{R}(b)/b$, is low. Panel C also shows that there is little difference in the proportional bid-ask spreads of defaulted bonds across different output states. From equations (2.9) and (2.10), we see that the value of defaulted bonds depends on the current output state only through its influence on the value of recovered debt upon the government reentering international credit markets. In our calibration, periods of autarky last 6.5 years on average. As a result of mean reversion in output over such a horizon, the influence of current output on the eventual recovered debt value is

[20] The larger bid-ask spread during periods of default is shown in Panel C.

weak. As a result, the proportional bid-ask spread for defaulted bonds therefore mainly depends on the amount of debt in default.

In combination, Panels B and C of Figure 3.1 demonstrate a liquidity-default feedback loop by which incentives to default are higher during bad times due to liquidity frictions. This margin was first highlighted by He and Milbradt (2014) in the context of corporate bonds. To see this feedback loop, consider the net proceeds from debt rollover, the difference in the value of newly issued bonds and the repayment of the principal of maturing debt, $q^H_{ND,t}[b_{t+1} - (1-m)b_t] - mb_t$. A larger debt issuance is necessary to break even when rolling over debt if the probability of default is high and the price of newly issued bonds, $q^H_{ND,t}$, is low. This leads to higher levels of indebtedness and further increases the government's incentives to default in future periods. This rollover risk channel is already understood (see, .e.g, Chatterjee and Eyigungor, 2012). The presence of liquidity frictions further amplifies this rollover risk channel. Higher debt levels further increase bid-ask spreads in the secondary market (Panel B), which results in an even lower issuance price in the primary bond market. In turn, this further exacerbates the government's default incentives and results in a liquidity-default feedback loop.[21]

3.4 Sovereign Spread Decomposition

In this subsection, we present a decomposition of total spreads into a liquidity and a credit component. Our main result is that liquidity premia can be a substantial component of total spreads.

As a first step towards the decomposition, we start by defining *total spreads*. Consider a government with debt and default policies given by $\widetilde{\boldsymbol{B}}(y,b)$ and $\widetilde{\boldsymbol{D}}(y,b)$, respectively. Let $q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ be the corresponding value of this government's debt to an unconstrained investor who receives liquidity shocks with probability ζ. That is, $q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} = q^H_{ND}\left(y,\widetilde{\boldsymbol{B}}(y,b)\right)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ where $q^H_{ND}(y,b')\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}$ is the solution to equation (2.7) under debt policy $\widetilde{\boldsymbol{B}}$ and default policy $\widetilde{\boldsymbol{D}}$. Since our calibration is monthly, the corresponding annualized sovereign spread is defined as:

$$cs(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} \equiv \left(1 + r^H(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)}\right)^{12} - (1+r)^{12}, \tag{3.1}$$

where r is the risk free rate, and the bond's yield to maturity is given by $r^H(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} = [m + (1-m)z] \Big/ q^H_{ND}(y,b)\Big|_{(\widetilde{\boldsymbol{B}},\widetilde{\boldsymbol{D}},\zeta)} - m$.

[21] In Appendix B we discuss this feedback mechanism in a simple model with fixed debt and exogenous default probabilities.

Next, we decompose the total spread (3.1) into a credit and a liquidity component.[22] Let $\boldsymbol{B}$ and $\boldsymbol{D}$ respectively be the debt and default policy from the baseline calibration (i.e. the parameters listed in Table 1).

The *default component* of the sovereign spread is defined as:

$$cs_{DEF}\left(y,b\right) \equiv cs\left(y,b\right)\Big|_{(\boldsymbol{B},\boldsymbol{D},0)}. \tag{3.2}$$

The pricing of the default component uses the baseline debt $\boldsymbol{B}$ and default $\boldsymbol{D}$ policies, but the investor that prices the bond has a zero probability of receiving a liquidity shock.[23] More precisely, the policy pair $(\boldsymbol{B},\boldsymbol{D})$ is the optimal policy of a government that is subject to price schedule $q^H_{ND}(y,b') \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)}$ in the primary market. However, the bond price associated with $cs_{DEF}(y,b)$ is given by $q^H_{ND}(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)}$.

The *liquidity component* is then defined as the residual:

$$cs_{LIQ}(y,b) \equiv cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y,b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} \tag{3.3}$$

and accounts for the portion of the total sovereign spread that is not explained by the default component. These two definitions amount to decomposing sovereign spreads as:

$$cs\left(y,b\right) = cs_{DEF}\left(y,b\right) + cs_{LIQ}\left(y,b\right). \tag{3.4}$$

What portion of total spreads is explained by each component? Figure 3.2 plots the decomposition above. Panels A and B plot the total spreads, credit risk premium, and liquidity premium for two levels of output (chosen to be ±1 standard deviations of the unconditional distribution of (log) output). We highlight two features of this decomposition. First, Panels A and B show that the liquidity component is increasing as the debt choice increases. This increase reflects a higher default probability combined with higher bid-ask spreads during default. Second, the liquidity component represents a sizable fraction of total spreads. In particular, when default risk is low (i.e. when output is high and/or debt levels are low) the liquidity component is predominant; however, as the overall default risk increases, the liquidity component as a fraction of total spreads

[22] The decomposition is analogous to the decomposition provided in He and Milbradt (2014) in the context of corporate bond spreads. As it will be made clear in the next subsection, one important difference is that our decomposition takes into account the endogenous response of debt policy, while the decomposition in He and Milbradt (2014) is for a fixed debt policy.

[23] The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account when choosing debt and default policies), individual investors are heterogeneous so that there may be some investors without liquidity concerns who discount at the risk-free rate.

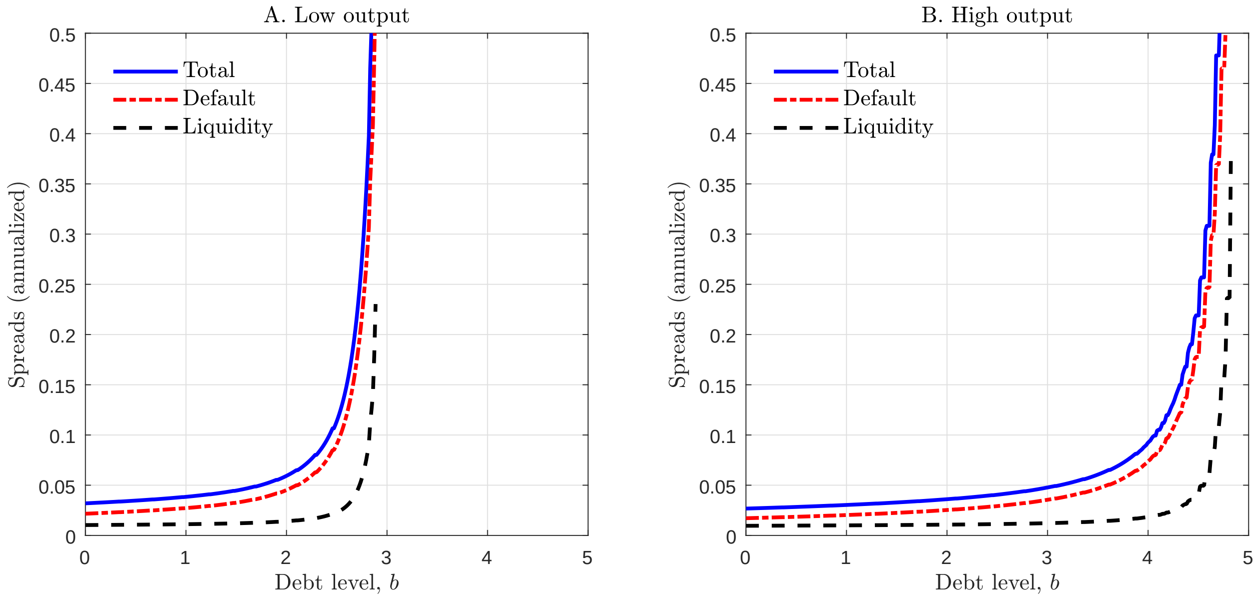

Figure 3.2: **Sovereign spread decomposition.** This figure decomposes total sovereign spreads, CS, into a default component, CS_{DEF}, and a liquidity component, CS_{LIQ}. The default component is defined in equation (3.2), and the liquidity component is defined in (3.3). Panels A and B plot the decomposition for low ($y = 1.087$) and high ($y = 0.920$) output values, respectively. These output values correspond to ± 1 standard deviations of the unconditional distribution for (log) output.

becomes smaller, but remains a first-order factor. For example, in Panel B, the fraction of the total sovereign spreads attributable to liquidity is about ones third for debt levels less than 1 and a high level of output. However, close to default, for example when output is low and debt choice is around 2, liquidity is responsible for 24 percent of total spreads. These magnitudes are in line with the CDS-basis based calculations in Longstaff et al. (2005) and structural decompositions in He and Milbradt (2014) for non-investment grade corporate bonds.

How does default risk affect the liquidity premium? To see this, consider a "jump-to-default" model similar to the one developed in section 2 except that debt is fixed, and the default probability is exogenous and given by p_d (we present the jump-to-default model in detail in Appendix B). In this case, the valuation of the unconstrained investor (2.7) can be written as

$$q^H_{ND} = \frac{1}{1 + r + \ell^H_{ND}} \left[(1 - p_d) \left(m + (1 - m) \left(z + q^H_{ND} \right) \right) + p_d q^H_D \right]$$

where

$$\ell_{ND} = (1 - p_d)(1 - m) \zeta \frac{q^H_{ND} - q^L_{ND}}{q^H_{ND}} + p_d \zeta \frac{q^H_D - q^L_D}{q^H_{ND}}$$

is the liquidity premium that is needed to equate the market price q_{ND}^{H} to the valuation of an investor who is not subject to liquidity concerns.

Three observations are in order here. First, conditional on default probabilities, the value of the liquidity premium before default, ℓ_{ND}, is pinned down by the bid-ask spreads. Thus, as long as the calibration generates bid-ask spreads that are in line with the data, the liquidity premia that we obtain are disciplined by the friction we observe empirically. Second, the liquidity premium will endogenously vary over the business cycle, depending on default probabilities and the bid ask spreads. Third, there will be a positive co-movement between liquidity and credit premia as long as bid-ask spreads are larger in default. When this holds, liquidity premia increases during bad times when default risk (p_d) is high.

How does liquidity risk drive default incentives? Worse liquidity conditions exacerbate default incentives by increasing debt rollover costs. To see this, consider a stationary debt structure $b_{t+1} = b_t$. In this case, the budget constraint (2.1) becomes $c_t = y_t - [m + (1-m) z] b_t + m b_t q_{ND,t}^{H}$. In the jump-to-default setting, the debt issuance price can also be written as

$$q_{ND}^{H} = \frac{(1-p_d)\left[m + (1-m)\left(z + q_{ND}^{H}\right)\right] + p_d q_D^{H}}{1+r} - EL,$$

where

$$EL \equiv \zeta \frac{(1-p_d)(1-m)\left(q_{ND}^{H} - q_{ND}^{L}\right) + p_d\left(q_D^{H} - q_D^{L}\right)}{1+r}$$

is the price discount that compensates investors for expected losses due to secondary market illiquidity. Worse liquidity conditions lead to more substantial price discounts (EL) and, as a consequence, lower consumption through increased debt rollover costs. In turn, this generates more significant sovereign default incentives.

3.5 Liquidity, Policies, and Spreads

We now examine the default and liquidity components of spreads more closely. We would like to quantitatively answer the following two questions. First, what are the determinants of the credit and liquidity components of sovereign spreads? Second, how much of the default premium is explained by liquidity frictions, and what portion of the liquidity premium is due to default risk?

A Closer Examination of the Default Risk Premium. We begin by further decomposing the default component (3.2). Let $\boldsymbol{B}_0$ and $\boldsymbol{D}_0$ respectively denote the debt and default

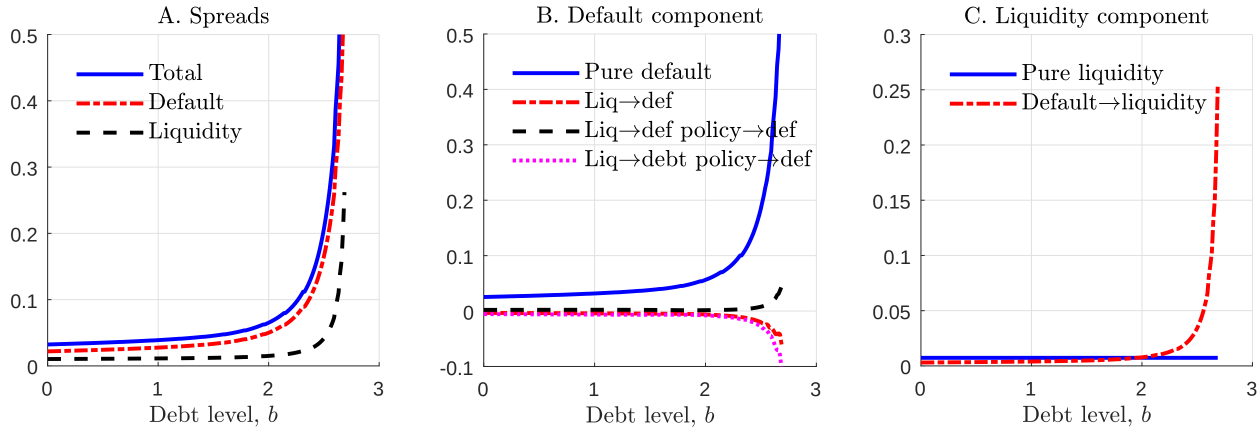

Figure 3.3: **Sovereign spread decomposition.** This figure plots the different components of spreads when output is at its median value ($y = 1$). Panel A plots total spreads, the default component defined in equation (3.2), and the liquidity component defined in (3.3). Panels B and C further decomposes the default component and liquidity component, respectively.

policy of a government operating in an economy that is not subject to liquidity frictions. This corresponds to a special case of our model where investors' probability of receiving a liquidity shock is set to zero (i.e. $\zeta = 0$). We can then decompose:

$$cs_{DEF}(y,b) = cs_{DEF,DEF}(y,b) + cs_{LIQ\rightarrow DEF}(y,b), \tag{3.5}$$

where the two terms in the decomposition are defined as

$$cs_{DEF,DEF}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)}, \tag{3.6}$$

$$cs_{LIQ\rightarrow DEF}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} - cs(y,b)\mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)}. \tag{3.7}$$

The first term, equation (3.6), is the "pure default" component. It is the spread that the sovereign would pay if there were no liquidity frictions (as in, e.g., Chatterjee and Eyigungor, 2012). The second term, equation (3.7), is the liquidity-driven component of default spreads. This is the portion of the default component caused by liquidity-induced changes in debt and default policies (relative to the policies that would prevail in an economy absent liquidity frictions).

The sign of the liquidity-induced default component, $cs_{LIQ\rightarrow DEF}(y,b)$, depends on the change in debt and default policies, from $(\boldsymbol{B}_0, \boldsymbol{D}_0)$ to $(\boldsymbol{B}, \boldsymbol{D})$, after liquidity shocks are switched on. On the one hand, holding default policies fixed, total spreads will decrease as the government takes on less debt in response to the increased borrowing costs associated with higher liquidity frictions. On the other hand, fixing debt policies, total spreads will increase as the government defaults in more states of nature due to the higher costs of borrowing induced by liquidity frictions.[24] To quantify these two effects, we further decompose $cs_{LIQ \to DEF}(y, b)$ into two terms:

$$cs_{LIQ \to DEF}(y, b) \equiv cs_{LIQ \to DEF,Debt}(y, b) + cs_{LIQ \to DEF,Def}.(y, b),$$

where we define

$$cs_{LIQ \to DEF,Debt}(y, b) \equiv cs(y, b) \mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)} - cs(y, b) \mid_{(\boldsymbol{B}_0,\boldsymbol{D}_0,\zeta=0)}, \tag{3.8}$$

$$cs_{LIQ \to DEF,Def}(y, b) \equiv cs(y, b) \mid_{(\boldsymbol{B},\boldsymbol{D},\zeta=0)} - cs(y, b) \mid_{(\boldsymbol{B},\boldsymbol{D}_0,\zeta=0)} . \tag{3.9}$$

The first term, equation (3.8), measures the component of spreads due to liquidity-driven changes to debt debt policy alone, holding default policy fixed. Similarly, the second term, equation (3.9), measures the component of spreads due to liquidity-driven changes to default policy, holding debt policy fixed.

Does an increase in liquidity frictions unambiguously increase spreads? The overall response along with each of the components $cs_{DEF,DEF}$, $cs_{LIQ \to DEF}$, $cs_{LIQ \to DEF,Debt}$, and $cs_{LIQ \to DEF,Def}$ are depicted in Panel B of Figure 3.3. Note that the component $cs_{LIQ \to DEF,Def}$ is always positive. Furthermore, in our calibration, the component $cs_{LIQ \to DEF,Debt}$ is negative. This is because given default policies, worse liquidity conditions will imply a more precautionary debt policy. Finally, we find that the overall liquidity-driven-default component is negative. This is mainly due to the effects of a more conservative debt policy in response to liquidity risk. This illustrates the effectiveness of optimal debt management in reducing the impact of liquidity risk and why it can be the case that liquidity frictions could actually decrease spreads through its effect on optimal debt policy.[25]

[24] The intuition is as follows. Suppose that the default policy remains fixed when the friction ζ changes. On the one hand, an increase in the liquidity frictions might increase interest rates in the primary market, driving debt issuance down, and this might, in turn, imply a decrease in total spreads. On the other hand, this increase in the liquidity friction might induce the country to borrow even more, to sustain consumption, which in turn will exacerbate the increase in spreads caused by the liquidity friction. The same can occur with default policies: fixing debt policies, a change in the liquidity friction might induce the sovereign to default in a higher (lower) number of states of nature, which in turn implies higher (lower) spreads.

[25] As we mentioned before, in our a model there is a combination of two frictions: lack of commitment and liquidity frictions. When we increase one of the frictions it can theoretically be welfare improving. The reason is that higher liquidity frictions might directly increase spreads, but these higher spreads could lead to a more conservative debt and default policy. Even though in our calibrated exercise we will find that higher liquidity frictions will decrease welfare, the channel by which liquidity frictions *might* increase welfare is precisely the one we are documenting. This is in line with theoretical models such as Amador et al. (2006).

A Closer Examination of the Liquidity Risk Premium. We can further decompose the liquidity premium into two terms:

$$cs_{LIQ}(y,b) = cs_{DEF\to LIQ}(y,b) + cs_{LIQ,LIQ}(y,b), \tag{3.10}$$

where we define:

$$cs_{LIQ,LIQ}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B}_0,\boldsymbol{D}=0,\zeta)}, \tag{3.11}$$

$$cs_{DEF\to LIQ}(y,b) \equiv cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D},\zeta)} - cs(y,b)\mid_{(\boldsymbol{B},\boldsymbol{D},0)} - cs(y,b)\mid_{(\boldsymbol{B}_0,\boldsymbol{D}=0,\zeta)}. \tag{3.12}$$

The first component, equation (3.11), is a "pure liquidity" term corresponding to the spread for a government which never defaults, but whose bonds are held by investors subject to liquidity shocks (as in Duffie et al. (2005)). The second residual term, equation (3.12), is a "default-induced liquidity" component that measures the portion of the liquidity component that is due to default risk. Panel C of Figure 3.3 illustrates these two components. The pure liquidity component does not vary with debt levels because this component is not sensitive to the default probabilities, which depends on debt levels. The default-induced liquidity component is increasing as debt level increases and default becomes more likely. It is a result of the default-liquidity feedback mechanism of our model.

Relationship to the Literature. Combining (3.5) and (3.10), we can write total spreads as:

$$cs \equiv cs_{DEF,DEF} + cs_{LIQ\to DEF} + cs_{DEF\to LIQ} + cs_{LIQ,LIQ}, \tag{3.13}$$

which is the decomposition found in He and Milbradt (2014) and Chen et al. (2017), which assumes fixed debt levels. A key difference in our setting, which features dynamic debt issuance, we show that the liquidity driven default component, $cs_{DEF\to LIQ}$, additionally depends on liquidity-driven changes to debt policy (in addition to liquidity-driven changes to default policy). Panel B of Figure 3.3 shows that liquidity-driven changes to debt policy decreases spreads as debt issuance becomes more conservative in response to the higher costs of financing for illiquid bonds. This partially offsets the increase in credit spreads due to liquidity-driven increases in default.

Moment	Data	$h_c = 0$	$h_c = 0.0005$	$h_c = 0.001$	$h_c = 0.00145$
Mean Debt to GDP	1.00	1.017	1.012	1.007	1.002
Mean Spread	0.0815	0.0767	0.0785	0.0802	0.0815
Vol. of Spread	0.0443	0.0474	0.0466	0.0450	0.0436
Mean Bid-Ask Spread	0.0050	0	0.0017	0.0034	0.0049
Welfare	-	1.0164	1.0158	1.0152	1.0147

Table 3: Comparative Statistics.

3.6 Secondary Market Frictions and Welfare

In this section, we examine the implications of secondary market illiquidity for household welfare. We define welfare as the certainty equivalent consumption:

$$c\left(h_c\right) \equiv u^{-1}\left(\mathbb{E}\left[V^C\left(y, b=0; h_c\right)\right]\right)$$

obtained by a sovereign with no initial debt ($b = 0$) operating in an economy in which constrained international investors are subject to holding costs h_c. We vary the illiquidity of secondary bond markets, as measured by bid-ask spreads, by varying the severity of liquidity shocks for international investors and consider the resulting welfare implications for the sovereign. In particular, we vary international investors' holding costs conditional on receiving a liquidity shock, h_c, starting with an economy in which international investors never become constrained (i.e. $h_c = 0$), and we then slowly worsen the severity of liquidity shocks by increasing holding costs, h_c.

The results of this exercise are summarized in Table 3. We see that the average bid-ask spread is increasing in holding costs. Importantly, household welfare decreases as the secondary bond market becomes more illiquid. In particular, decreasing bid-ask spreads from 50 basis points to zero results in a 0.17 percent increase in household welfare. To put this number in perspective, the welfare gain from eliminating the business cycle in a representative agent setting with CRRA preferences is 0.40 percent of the consumption equivalent. Thus, the welfare gains from removing secondary market frictions are substantial. In particular, through the lens of our model, in that period, secondary market frictions can account for 42.5 percent of the cost of the business cycle for Argentina.[26]

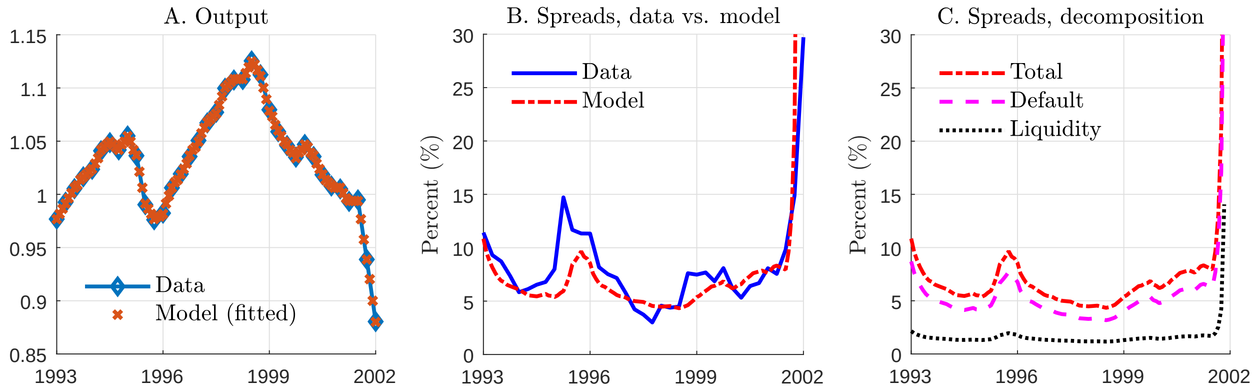

Figure 3.4: **Event Study: Argentina's Default, 2001:IV.** The first figure plots output, Argentinean spreads in the data, and Argentinean spreads generated by the model when the output realizations for Argentina are fed in. The second figure plots total spreads and the credit and liquidity components as defined in 3.4.

3.7 Case Study: Argentina's Default in 2001

In this subsection, we conduct an event study of Argentina's default in December of 2001. For the policy functions of the baseline calibration, depicted in Table 1, we input the output shocks that Argentina received in the 1990s and the initial level of debt. Panel A plots the level of GDP in logs. Panel B plots the total spreads generated by our model in comparison with the data. Finally, Panel C plots total spreads and the liquidity and credit components computed using the decomposition in (3.5).

First, note that our model can replicate the most salient features of the series of spreads. The first spike is the Tequila crisis in Mexico in 1996, where there is a sharp increase in the spreads coming from a recession. In addition, we can see that for 2001 the model can correctly account for the spike in spreads and default. Note that although the recession started in 1998:III, spreads continued to be below 800 basis points until the beginning of 2001.

Second, in Panel C, we observe that the liquidity premium is a larger portion of total spreads during good times, when total spreads are low. However, when the spreads began to spike in 1999, the liquidity component started to be a smaller *portion* of total spreads. In fact, in our calibration on average the liquidity component explains 23 percent of the total spreads during the period of 1993:I to 2001:IV.

[26] Appendix F details the calculations of the welfare cost of the business cycles for Argentina following Lucas (2003).

Variable	Data	Model	CE (2012), Table 4
$\sigma(c)/\sigma(y)$	1.09	1.15	1.11
$\sigma\left(\frac{NX}{y}\right)/\sigma(y)$	0.17	0.09	0.20
$corr(c,y)$	0.98	0.98	0.99
$corr\left(\frac{NX}{y},y\right)$	-0.88	-0.50	-0.44
$corr(r-r^f,y)$	-0.79	-0.65	-0.65
Debt service	0.053	0.08	0.055

Table 4: **Business cycle properties.**

3.8 Business Cycle Properties

The model's business cycle properties are summarized in Table 4. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model performs well. As in the data, consumption is as volatile as output, and nearly perfectly correlated with it. The volatility of the current account relative to output volatility is 0.09 in the model, which is close to its empirical counterpart of 0.17. The model performs well at capturing countercyclical sovereign credit risk, with a correlation of -0.65 between the sovereign spread and output. In addition, there is a negative correlation, -0.50, between the current account and output. Finally, debt service (as a fraction of output) is 8 percent. Overall, the business cycle properties generated by the calibrated model are similar to those generated by the model in Chatterjee and Eyigungor (2012).

4 Conclusion

The quantitative literature on sovereign debt has helped us to understand debt capacity, spreads, and welfare when the central friction is lack of commitment. The recent Sovereign Debt Crisis in Europe, however, has highlighted that there is substantial liquidity risk associated with sovereign lending. Motivated by these facts, we study debt and default policy when the government lacks commitment, and there are search frictions in the secondary market for sovereign bonds.

After proposing a tractable framework in which both credit and liquidity risk constrain the debt and default choices of the government, we proceeded to study the quantitative importance of liquidity in sovereign spreads and welfare. To do so, we calibrated our model to match the main features of the Argentinean experience during the 1990's, one of the most widely studied cases of sovereign default. Our first result is that liquidity risk is a substantial component of sovereign spreads. In particular, in a model-based decomposition, we find that it accounts for 23 percent of total sovereign spreads. Our second result is that liquidity matters for welfare. In particular, through the lens of our model, a representative agent would be willing to pay 42 percent of the cost of income fluctuations to shut down liquidity frictions.

Why should we distinguish between credit and liquidity risk? Why should we incorporate liquidity risk into models of sovereign borrowing? An important reason to distinguish between credit and liquidity risk are the different normative implications of each friction. There are at least three reasons why the introduction of liquidity would matter for debt management policy. First, long-run policies to fight spreads caused by lack of commitment might be different from those to combat spreads due to the lack of liquidity. Second, policies in the short run might also be different; for example, one strategy during bad times could be to capitalize financial intermediaries as opposed to decreasing government spending or repaying debt. Third, maturity and currency management policies might also differ. The benefit of spreading debt across currencies and maturities is to complete the market. However, a possible unintended consequence would be to lower liquidity in the market for these bonds. The increase in the cost of borrowing due to illiquidity could potentially undo the insurance benefits of a more complete debt market. All of these are topics for further research.

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Appendix

A Debt issuance

This section shows that, in simulations of our baseline calibration (Table 1 lists the parameters), in equilibrium the government does not buy back debt. Monthly proportional debt issuance, measured as a fraction of outstanding debt at the start of each period, is given by

$$issue_t = \frac{b_{t+1} - (1-m)\, b_t}{b_t}. \tag{A.1}$$

We simulate a long time series for our baseline calibration and report results. In the simulation, proportional debt issuance has a mean of $\mathbb{E}\left[issue_t\right] = 0.017$ and a standard deviation of $\sigma\left(issue_t\right) = 0.0045$. The mean proportional debt issuance is slightly higher than the debt maturity probability of $m = 0.0167$, which indicates that in most months, the government is simply rolling over maturing debt. The minimum and maximum values for proportional debt issuances in simulations are $min\left\{issue_t\right\} = 0.0051$ and $\max\left\{issue_t\right\} = 0.1095$, respectively. The strictly positive minimum value indicates that there are no debt buybacks on the equilibrium path.

B Jump to Default

In this appendix, we describe a particular case of our model in which default probabilities are exogenous, and debt is fixed. The idea is to introduce a clear definition of the liquidity premium, show how bid-ask spreads map onto the liquidity premium as a function of default risk, and to clarify the role of the features of the model in the results. Assume an unconditional constant default probability p^{LR} in each period. Then the pricing equations yield a system of 6 equations and 6 unknowns $\bar{q}^H_{ND}, \bar{q}^L_{ND}, \bar{q}^H_{D}, \bar{q}^L_{D}, \bar{q}^S_{ND}, \bar{q}^S_{D}$. The system is given by:[27]

$$
\begin{aligned}
\bar{q}^H_{ND} &= \frac{1}{1+r}\left[\left(1-p^{LR}\right)\left(m+(1-m)\left(z+\zeta\bar{q}^L_{ND}+(1-\zeta)\,\bar{q}^H_{ND}\right)\right)\right.\\
&\qquad \left.+p^{LR}\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)\right], \\
\bar{q}^L_{ND} &= \frac{1}{1+r}\left[\left(1-p^{LR}\right)\left(-h_c+m+(1-m)\left(z+\lambda\bar{q}^S_{ND}+(1-\lambda)\,\bar{q}^L_{ND}\right)\right)\right.\\
&\qquad \left.+p^{LR}\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)\right], \\
\bar{q}^H_D &= \frac{1-\theta}{1+r}\left(\zeta\bar{q}^L_D+(1-\zeta)\,\bar{q}^H_D\right)+\theta\mathcal{R}\bar{q}^H_{ND}, \\
\bar{q}^L_D &= \frac{1-\theta}{1+r}\left(-h_c+\lambda\bar{q}^S_D+(1-\lambda)\,\bar{q}^L_D\right)+\theta\mathcal{R}\bar{q}^L_{ND}, \\
\bar{q}^S_{ND} &= \bar{q}^L_{ND}+\alpha_{ND}\left(\bar{q}^H_{ND}-\bar{q}^L_{ND}\right), \\
\bar{q}^S_D &= \bar{q}^L_D+\alpha_D\left(\bar{q}^H_{ND}-\bar{q}^L_{ND}\right),
\end{aligned}
\tag{B.1}
$$

where $\mathcal{R} \in [0,1]$ denotes the fraction recovered after a default. The solution to this system yields four value functions that depend on the unconditional default probability, p^{LR}, and the parameters of the model. Fix the unconditional default probability, p^{LR}, and suppose there is a short-run departure. In particular, the current default probability is p_d. After a default, the default probability will remain fixed at the long run probability of default,

[27]For ease of exposition, the equations do not include the free asset disposal conditions that guarantee non-negative prices. Bond prices in the presence of free asset disposal can be characterized as the solution to a linear complementarity problem.

p^{LR}.[28] Then, current prices are given by:

$$\begin{aligned}
q^H_{ND} &= \frac{1}{1+r}\left[(1-p_d)\left(m+(1-m)\left(z+\zeta q^L_{ND}+(1-\zeta)\,q^H_{ND}\right)\right)\right. \\
&\quad \left. +p_d\left(\zeta q^L_D+(1-\zeta)\,q^H_D\right)\right], \\
q^L_{ND} &= \frac{1}{1+r}\left[(1-p_d)\left(-h_c+m+(1-m)\left(z+\lambda q^S_{ND}+(1-\lambda)\,q^L_{ND}\right)\right)\right. \\
&\quad \left. +p_d\left(-h_c+\lambda q^S_D+(1-\lambda)\,q^L_D\right)\right], \\
q^H_D &= \frac{1-\theta}{1+r}\left(\zeta q^L_D+(1-\zeta)\,q^H_D\right)+\theta\mathcal{R}\bar{q}^H_{ND}(p^{LR}), \\
q^L_D &= \frac{1-\theta}{1+r}\left(-h_c+\lambda q^S_D+(1-\lambda)\,q^L_D\right)+\theta\mathcal{R}\bar{q}^L_{ND}(p^{LR}), \\
q^S_{ND} &= q^L_{ND}+\alpha_{ND}\left(q^H_{ND}-q^L_{ND}\right), \\
q^S_D &= q^L_D+\alpha_D\left(q^H_D-q^L_D\right).
\end{aligned} \tag{B.2}$$

Note that difference between the first system of four equations and the second system is that in the second one $\bar{q}^H_{ND}(p^{LR})$,$\bar{q}^L_{ND}(p^{LR})$ are taken as given. This will permit us to take limits on p_d over $[0,1]$ and still have a well-defined system of equations. For both systems of equations, which define valuations (B.1) and (B.2), free disposal of the asset implies that all the valuations are non-negative.

B.1 Bid-Ask Spreads and Liquidity Premia

How do the observable frictions in the secondary market, the bid-ask spreads, map onto liquidity premia? To see the connection, rewrite the bond price (B.2) as:

$$q^H_{ND} = \frac{(1-p_d)\left(m+(1-m)\left(z+q^H_{ND}\right)\right)+p_d q^H_D}{1+r+\ell_{ND}},$$

where the endogenous liquidity component is:

$$\ell_{ND} = (1-p_d)\,(1-m)\,\zeta\frac{q^H_{ND}-q^L_{ND}}{q^H_{ND}}+p_d\zeta\frac{q^H_D-q^L_D}{q^H_D}. \tag{B.3}$$

[28] Note that this process captures the idea of mean reversion in the hazard rates, which is common in the literature on credit risk modeling (see, for example Longstaff et al. (2005)). Formally, we can consider an irreducible Markov chain with 2 states and transition matrix $\mathbf{P}$. The p^{LR} will be defined by the invariant distribution Π.

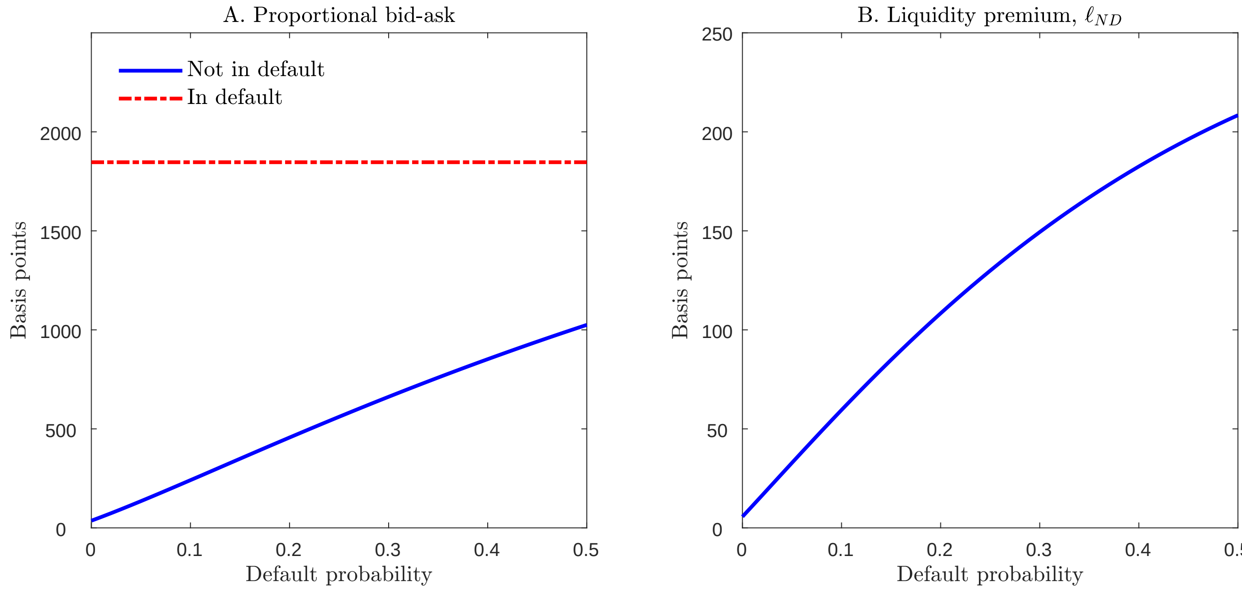

Figure B.1: **Liquidity Premium**. Panel A plots proportional bid-ask spreads $\left(q^H - q^S\right)/q^H$ in and out of default. Panel B plots the corresponding liquidity premium (B.3).

That is, ℓ_{ND} is the additional liquidity spread that is needed to equate the market price q^H_{ND} to the valuation of an investor who is not subject to liquidity concerns. Two observations regarding this liquidity component are in order. First, it is the expected valuation loss after taking into account the loss conditional on having to sell and the probability of incurring a liquidity event and having to sell. Second, it depends on distance to default (or p_d); this is the case because the loss conditional on selling is higher during a default episode. Figure B.1 illustrates the relationship between proportional bid-ask spreads in and out of default, default probabilities, and the liquidity premium (B.3).[29]

B.2 Difference in proportional bid ask Spreads

From expression (B.3), we see that the liquidity spread ℓ_{ND} depends on the difference in proportional bid-ask spreads in default, $\left(q^H_D - q^L_D\right)/q^H_D$, and out of default, $\left(q^H_{ND} - q^L_{ND}\right)/q^H_{ND}$. In our model, this difference can arise from two channels.

The first channel is an "endogenous maturity extension" channel, whereby the effective maturity of bonds increase after a sovereign default. Since payments only resume after the government regains credit access, the extension in effective maturity for defaulted bonds is equal to the expected amount of time spent in autarky, $1/\theta$. Panel A of Figure B.2 plots the difference in proportional bid ask spread (i.e. $\left(q^H_D - q^L_D\right)/q^H_D -$

[29]The numerical examples in this section use the following parameters: $z = 0.03$, $\theta = 0.0128$, $r = 0.0033$, $\zeta = 0.139$, $\alpha_{ND} = 0.125$, $\alpha_D = 0$, $\mathcal{R} = 0.3$, and $\bar{p}^{LR} = 0.03$.

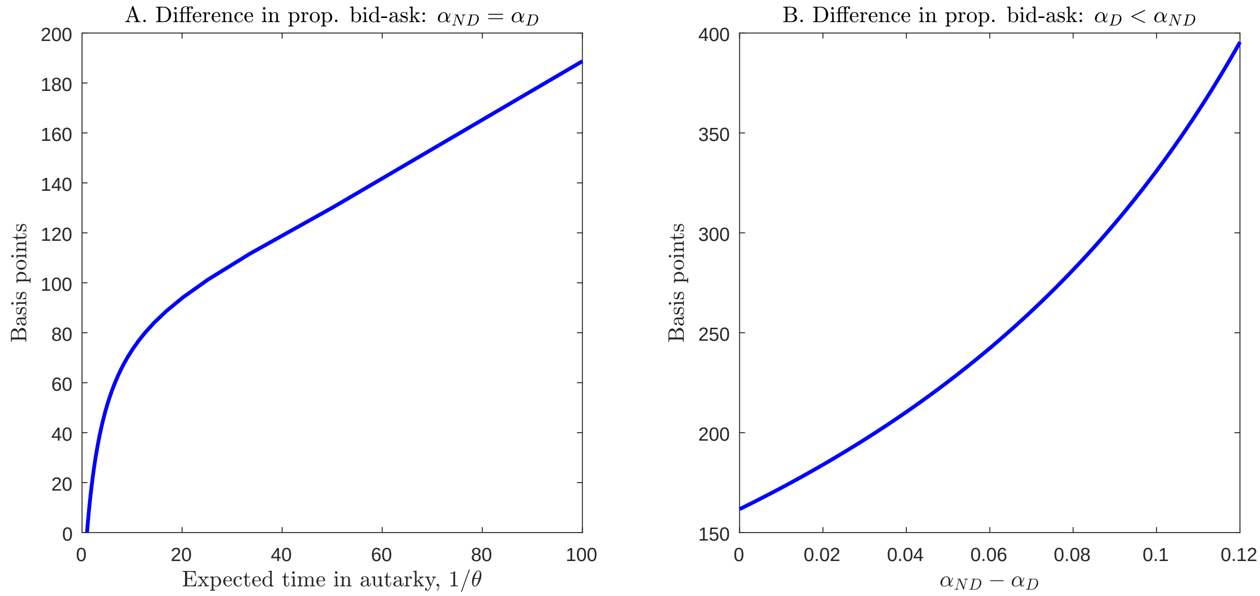

Figure B.2: **Difference in proportional bid-ask spreads**. The figures plot the difference in bid-ask spreads in and out of default: $\left(q_D^H - q_D^S\right)/q_D^H - \left(q_{ND}^H - q_{ND}^S\right)/q_{ND}^H$. Panel A plots this difference by as a function of the amount of time spent in autarky. Panel B plots the difference as a function of the decrease in investors' bargaining power when trading defaulted bonds.

$\left(q_{ND}^H - q_{ND}^L\right)/q_{ND}^H)$ as a function of the expected time spent in autarky. In this exercise, we set the short and long run default probabilities to be equal, and additionally set $\alpha_{ND} = \alpha_D$ so that there are no differences in bargaining power for trading defaulted and non-defaulted bonds. We see that the difference in proportional bid-ask spread is increasing in the expected time spent in autarky. In other words, a longer extension of the effective maturity of defaulted bonds results in a larger increase in proportional bid-ask spreads after a sovereign default. Intuitively, the valuation difference between constrained and unconstrained investors increases for long maturity bonds whose distant payoffs are less valuable to constrained investors. The effective maturity extension following a default will therefore adversely affect constrained investors to a greater extent, and result in an increase in proportional bid ask spreads as a result.

The second channel through which differences in proportional bid-ask spreads arise is an "investor bargaining channel" in which investors' bargaining power for trading defaulted bonds are lower: $\alpha_D < \alpha_{ND}$. Panel B of Figure B.2 plots the difference in proportional bid ask spread as a function of the decrease in investors' bargaining power (i.e. $\alpha_{ND} - \alpha_D$). We see that larger decreases in investors' bargaining power result in higher differences in proportional bid ask spreads.

In our baseline calibration, which features both channels, the average difference in proportional bid-ask spreads in and out of default is around 450 basis points (see section 3.1). When we switch off the "investor bargaining" channel by setting $\alpha_{ND} = \alpha_D$ while keeping all other parameters unchanged from our baseline calibration, the average difference in proportional bid-ask spreads in and out of default drops to 170 basis points (see row (2) of Table 5). Therefore, the "endogenous maturity extension" channel is roughly responsible for 40 percent of the difference in proportional bid-ask spreads in and out of default, while the "investor bargaining channel" is responsible for the remaining 60 percent.

B.3 Liquidity and Default Incentives

In our baseline model with endogenous default, worsening liquidity conditions exacerbate default incentives by increasing debt rollover costs. To see this, consider the stationary setting in which debt is fixed $b_t = b$. In this case, the budget constraint (2.1) becomes

$$c = y - [m + (1-m)\,z]\,b + mbq^H_{ND}.$$

From this, we see that worsening liquidity conditions will increase default incentives if it leads to lower debt issuance prices q^H_{ND}. The link between liquidity conditions and issuance prices can be illustrated using our simple jump-to-default model by rewriting the bond price (B.2) as

$$q^H_{ND} = \underbrace{\frac{(1-p_d)\left[m + (1-m)\left(z + q^H_{ND}\right)\right] + p_d q^H_D}{1+r}}_{\text{Valuation for } \zeta=0} - EL,$$

where

$$EL \equiv \zeta \frac{(1-p_d)\,(1-m)\left(q^H_{ND} - q^L_{ND}\right) + p_d\left(q^H_D - q^L_D\right)}{1+r} \tag{B.4}$$

is the compensation for expected losses due to secondary market illiquidity.

A liquidity-default spiral will be present whenever increases in the likelihood of default (p_d) increase expected liquidity losses (EL). For this to happen, each of the model's ingredients are needed: long-term debt and frictions on the secondary market; positive recovery ($\mathcal{R}(b) > 0$); and, worse liquidity conditions during a default ($\alpha_D < \alpha_{ND}$). First, long-term debt and frictions in the secondary market are needed to generate a bid-ask spread. That is, the following conditions need to hold: $\lambda \in (0,1)$ and $m < 1$. Second, recovery is needed for the bonds to have a positive price during default. If this were not to be the case, bid-ask spreads would not defined during the default state. Third, worse

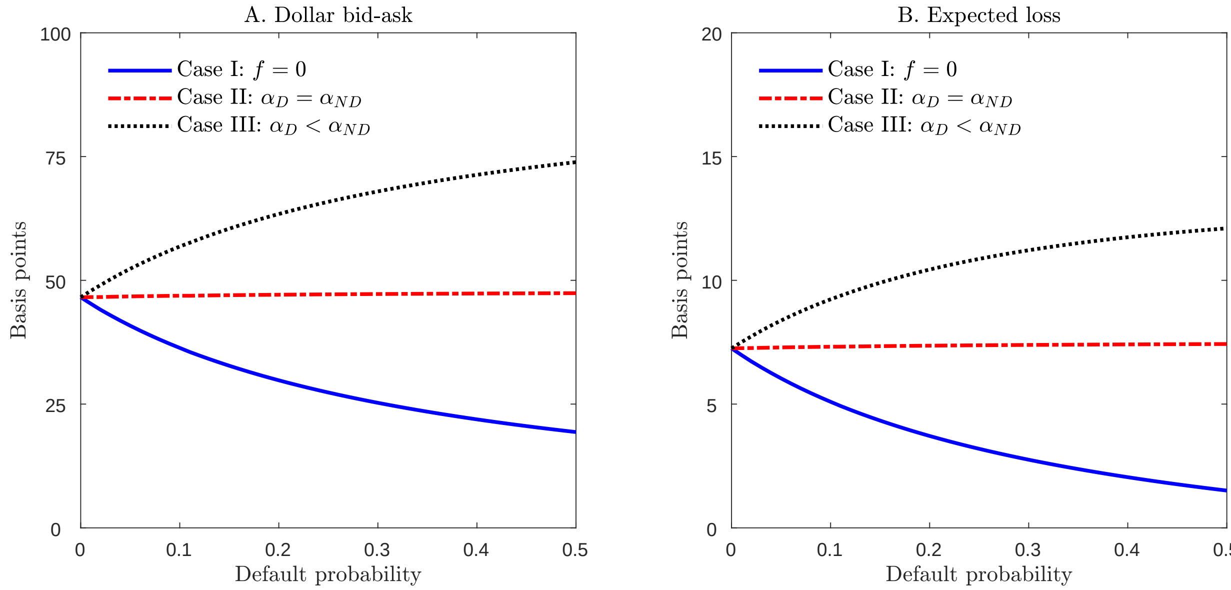

Figure B.3: **Liquidity Premium**. Panel A plots dollar bid-ask spreads $q^H_{ND} - q^S_{ND}$. Panel B plots expected loss due to secondary market illiquidity (B.4).

liquidity conditions during default are needed to generate higher dollar bid-ask spreads during default.

Figure B.3 illustrates the role of the model's ingredients by plotting the relationship between liquidity premia and default probabilities for three different scenarios. The solid lines plot the case in which there is long-term debt and frictions in the secondary market, but no recovery ($f = 0$). In this case, we see in Panels A and B that both dollar bid-ask spreads and expected losses decrease as default probabilities increase. Next, the dash-dot lines illustrate the case for which there is a positive recovery value for bonds ($f > 0$), but bondholders' bargaining does not decrease in default ($\alpha_D = \alpha_{ND}$). In this case, both dollar bid-ask spreads and expected losses remain constant as default probabilities increase. Finally, the dotted lines show the case for which both positive recovery and a post-default decrease in bondholders' bargaining power (i.e. $\alpha_D < \alpha_{ND}$) are present. Here, we see that liquidity premia increase as default becomes more likely. This implies the presence of the default-liquidity feedback channel: in the baseline model with endogenous default rates, increased liquidity premia increase the cost of rolling over debt which, in turn, further increases default incentives.

C Numerical Method

It is well-known that numerical convergence is often a problem in discrete-time sovereign debt models with long-term debt. To circumvent this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). This involves altering total output to be: $y_t + \varepsilon_t$, where $\varepsilon_t \sim trunc\, N(0, \sigma_\varepsilon^2)$ is continuously distributed. As shown in Chatterjee and Eyigungor (2012), the noise component ε_t guarantees the existence of a solution of the pricing function equation. The government's repayment problem (2.3) is altered as follows:

$$V^C(y,b,\varepsilon) \quad = \quad \max_{b'} \left\{ (1-\beta)\, u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}(y',b') \right] \right\},$$

where the budget constraint is now given by,

$$c \quad = \quad y + \varepsilon - b\,[m + (1-m)\, z] + q^H_{ND}(y,b')\,[b' - (1-m)\, b]$$

and the maximum default probability is given by

$$\delta(y,b') = \mathbb{E}_{\varepsilon',y'|y}\left[d(y',b',\varepsilon') \right] \leq \bar{\delta}.$$

Debt choice is denoted as $b'(b,y,\varepsilon)$. We impose that $\varepsilon_t \equiv 0$ during the autarky regimes, meaning that the expression for the value to default remains the same; i.e.,

$$V^D(y,b) \quad = \quad (1-\beta)\, u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta V^{ND}(y', \mathcal{R}(b)) + (1-\theta)\, V^D(y,b) \right].$$

The default decision is given by:

$$d(y,b,\varepsilon) \quad = \quad 1_{\left\{ V^C(y,b,\varepsilon) \geq V^D(y,b) \right\}},$$

and depends on the randomization component. The continuation value is adjusted as follows:

$$V^{ND}(y,b) \quad = \quad \mathbb{E}_\varepsilon \left[\max \left\{ V^D(y,b), V^C(y,b,\varepsilon) \right\} \right],$$

to take into account the randomization component. Finally, bond prices are also adjusted to take into account the additional randomization variable:

$$
\begin{aligned}
q_{ND}^{H}\left(y,b'\right) &= \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(y',b',\varepsilon')}{1+r}\left[m+(1-m)\left[\begin{array}{c} z+\zeta q_{ND}^{L}\left(y',b'\left(b',y',\varepsilon'\right)\right) \\ +\left(1-\zeta\right)q_{ND}^{H}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right] \\ +\frac{d(b',y',\varepsilon')}{1+r}\left[\zeta q_{D}^{L}\left(b',y'\right)+\left(1-\zeta\right)q_{D}^{H}\left(y',b'\right)\right]\end{array}\right\} \\
q_{ND}^{L}\left(b',y\right) &= \mathbb{E}_{y',\varepsilon'|y}\left\{\begin{array}{c} \frac{1-d(y',b',\varepsilon')}{1+r}\left[-h_c+m+(1-m)\left[\begin{array}{c} z+\left(1-\lambda\right)q_{ND}^{L}\left(y',b'\left(b',y',\varepsilon'\right)\right) \\ +\lambda q_{ND}^{S}\left(y',b'\left(b',y',\varepsilon'\right)\right)\end{array}\right]\right] \\ +\frac{d(y',b',\varepsilon')}{1+r}\left[-h_c+\left(1-\lambda\right)q_{D}^{L}\left(y',b'\right)+\lambda q_{D}^{S}\left(y',b'\right)\right]\end{array}\right\} \\
q_{D}^{H}\left(y,b\right) &= \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[\zeta q_{D}^{H}\left(y',b\right)+(1-\zeta)q_{D}^{L}\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q_{ND}^{H}\left(y,\mathcal{R}(b)\right) \\
q_{D}^{L}\left(y,b\right) &= \frac{1-\theta}{1+r}\mathbb{E}_{y'|y}\left[-h_c+\lambda q_{D}^{S}\left(y',b\right)+(1-\lambda)q_{D}^{L}\left(y',b\right)\right]+\theta\frac{\mathcal{R}(b)}{b}q_{ND}^{L}\left(y,\mathcal{R}(b)\right) \\
q_{ND}^{S}\left(y,b\right) &= (1-\alpha_{ND})q_{ND}^{L}\left(y,b\right)+\alpha_{ND}q_{ND}^{H}\left(y,b\right) \\
q_{D}^{S}\left(y,b\right) &= (1-\alpha_{D})q_{D}^{L}\left(y,b\right)+\alpha_{D}q_{D}^{H}\left(y,b\right)
\end{aligned}
$$

The rest of the numerical scheme is standard and follows the routine outlined in Chatterjee and Eyigungor (2012). We summarize the scheme in 4 steps:

a. Start by discretizing the state space. This involves choosing grids $\{y_i\}_{i=1}^{N_y}$ and $\{b_j\}_{j=1}^{N_b}$ for output and debt. The grid points and transition probabilities for output are chosen in accordance with the Tauchen (1986) method and encompass ± 3 standard deviations of the unconditional distribution of output. In the baseline model the number of states for output is chosen to be $N_y = 200$. The grid points for debt values are uniformly distributed over the range $[0, b_{max}]$, with the upper limit, b_{max}, being chosen large enough so that this (implicit) constraint never binds in simulations. The baseline calibration has $b_{max} = 6.0$ and $N_b = 450$.

b. Next perform value function iteration. Given bond prices, update value functions V^C and V^D. The debt and default policies, $b'(\cdot)$ and $d(\cdot)$, are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, linear interpolation is used to obtain terms involving $\mathcal{R}(b)$.

c. Given the debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.

D Sensitivity Analysis

In this section, we conduct a sensitivity analysis to illustrate the role of various parameters of our model. The dependence of our targeted moments on the model parameters

	Debt/ GDP	Rec- overy	Spread, mean	Spread, vol.	Bid-ask, ND	Bid-ask, D	Turn- over	Def freq
(1) Baseline	1	0.297	0.0815	0.0436	0.0049	0.0503	0.12	0.028
(2) $\alpha_D = \alpha_{ND}$	1	0.293	0.0819	0.0452	0.0048	0.0221	0.12	0.0308
(3) $\overline{\delta} = 0.99$	1	0.274	0.0874	0.0508	0.005	0.0569	0.12	0.03
(4) $\zeta = 0.1$	1	0.296	0.0806	0.0435	0.0058	0.0669	0.09	0.029
(5) $\zeta = 0.2$	1	0.298	0.0827	0.0438	0.004	0.0364	0.163	0.028
(6) $h_c = 0.001$	1.01	0.293	0.0802	0.0449	0.0034	0.0305	0.12	0.031
(7) $h_c = 0.002$	0.99	0.301	0.0835	0.0432	0.0068	0.0839	0.12	0.026
(8) $\alpha_{ND} = 0.05$	1	0.298	0.0834	0.0438	0.0070	0.0508	0.12	0.028
(9) $\alpha_{ND} = 0.2$	1	0.296	0.0804	0.0434	0.0036	0.05	0.12	0.029
(10) $\overline{b} = 0.6$	0.96	0.227	0.0862	0.0461	0.005	0.0781	0.12	0.028
(11) $\overline{b} = 1.0$	1.03	0.344	0.0781	0.0423	0.0049	0.0408	0.12	0.029
(12) $\lambda = 0.75$	1	0.297	0.0821	0.0438	0.0052	0.0504	0.117	0.029
(13) $\lambda = 0.95$	1	0.297	0.0813	0.0436	0.0047	0.0502	0.122	0.028

Table 5: **Sensitivity analysis.**

is shown in Table 5. For reference, row (1) shows the model-implied moments for the baseline calibration whose parameters are listed in Table 1.

In row (2) of Table 2 shows the model-implied moments when we set $\alpha_D = \alpha_{ND} (= 0.125)$ so that investors' bargaining power are the same for trading defaulted and non-defaulted bonds. We still obtain higher bid-ask spreads for bonds in default. This is due to the aforementioned lengthening of the effective maturity of outstanding payments after a default which, in turn, leads to larger valuation differences between constrained and unconstrained types. The mean bid-ask spread in default is now 221 basis points, roughly half that of the baseline calibration. Therefore, having a decrease in investors' bargaining power after defaulting ($\alpha_D < \alpha_{ND}$) is necessary only for the purposes of quantitatively generating higher bid-ask spreads in default. All other model-implied moments are very similar compared to that of the baseline model.

In row (3), we purposefully set the maximum one-period-ahead default probability to be very close to one: $\overline{\delta} = 0.99$. In our model with positive recovery, an increase in $\overline{\delta}$ allows the government to issue larger amounts of debt to dilute bondholders prior to defaulting. This results in an increase in both the mean and volatility of spreads. However, we see that the mean and volatility of spreads remain finite as long as the maximum default probability is less than 1. Importantly, bid-ask spreads still increase during periods of default indicating that the default-liquidity feedback channel continues to hold.

Rows (4) and (5) vary the probability of receiving a liquidity shock ζ. Higher values of ζ results in a higher turnover for the secondary market trading of bonds.

Rows (6) and (7) show that higher holding costs for constrained investors h_c results in higher bid-ask spreads, both in and out of default.

Rows (8) and (9) show that increasing the bargaining power of investors α_{ND} for trading non-defaulted bonds decreases bid-ask spreads for those bonds.

Rows (10) and (11) show that the value the maximum debt recovery $\overline{b}$ is identified from the mean recovery rate, with larger values of $\overline{b}$ resulting in a higher mean recovery rate.

Finally, rows (12) and (13) vary the probability for a constrained investor to meet a market maker λ. Increases in λ increase turnover but decrease bid-ask spreads. As a result, given the presence of the other liquidity parameters ζ, h_c, and α_{ND}, λ cannot be additionally identified based on bid-ask spreads and turnover. For this reason, we apriori fix the value of λ based on the literature.

E Distribution of bond holder types and turnover

The fraction of outstanding debt held by constrained investors, f_t^L, has law of motion

$$f_{t+1}^L = \begin{cases} (1-\lambda)\, f_t^L + \zeta\left(1 - f_t^L\right) & \text{if gov. is in default} \\ \frac{\left[(1-\lambda) f_t^L + \zeta\left(1-f_t^L\right)\right](1-m) b_t + \zeta\left[b_{t+1} - (1-m) b_t\right]}{b_{t+1}} & \text{if gov. is not in default.} \end{cases} \tag{E.1}$$

When the government is in default, the law of motion (E.1) reflects changes in the composition due to high types receiving liquidity shocks (occurring with probability ζ) as well as low types offloading their position to high types through the market maker (this occurs with probability λ). When the government is not in default, the law of motion (E.1) reflects changes in the composition of bond holder types for previously issued debt, and the fraction of newly issued bonds held by low type investors one period after issuance. In addition, expression (E.1) reflects the fact that the government does not buy back bonds in equilibrium (see Appendix A), so that the $b_{t+1} - (1-m)\, b_t$ term is always positive. Turnover, or the fraction of outstanding debt transacted in secondary markets each period, is then given by

$$Turnover_t = \lambda f_t^L, \tag{E.2}$$

and is the product of the fraction of potential sellers, f_t^L, and the probability that each potential seller is able to offload his position in a period, λ.

In simulations of the baseline model, we find that both f_t^L and $Turnover_t$ is approximately constant over time. This is because consumption smoothing by the government results in a steady debt level as the government mostly rolls over debt by issuing new debt to replace maturing debt (i.e. $b_{t+1} - (1-m)\, b_t \approx mb_t$). For example, proportional debt issuance is 0.017 on average, which is approximately equal to the debt maturity rate $m = 0.0167$. As a result, both f_t^L and $Turnover_t$ are approximately equal to their stationary values in a setting where the government only rolls over debt (i.e. $b_{t+1} = b_t$). That is, $f_t^L \approx \zeta \,/ [m + (1-m)\,(\lambda + \zeta)]$ and $Turnover_t \approx \lambda\zeta \,/ [m + (1-m)\,(\lambda + \zeta)]$. We confirm these approximations in simulations of baseline model which fully implements the exact law of motion (E.1). For example, the volatility of turnover is $\sigma\,(Turnover_t) = 7 \times 10^{-6}$.

F Welfare Cost of Business Cycles

In order to put the welfare cost of liquidity frictions in perspective, we compute how much a representative agent in Argentina would pay in order to avoid fluctuations in consumption. The exercise follows Lucas (2003). We detail our calculations in this Appendix. As in the main body of the paper, the de-trended output process is given by:

$$\begin{aligned} y_t &= e^{z_t} + \epsilon_t \\ z_t &= \rho_z z_{t-1} + \sigma_z u_t. \end{aligned}$$

We assume that the de-trended consumption process is AR(1) and , in particular, follows:

$$c_t = y_t^{\kappa} \tag{F.1}$$

We choose $\kappa = 1.1$ so that $\frac{\sigma(c)}{\sigma(y)} = 1.1$; as in the data for Argentina. The value function of the stream of consumption (F.1) solves:

$$V\,(c) = (1-\beta)u(c) + \beta \mathbb{E}_{c'|c}\left[V\left(c'\right)\right].$$

Note that the expectation is taken with respect to the conditional distribution of consumption, where consumption is defined in (F.1), and that this distribution is derived from the distribution of output. The unconditional value function is then given by:

$$V \equiv \mathbb{E}_{c'}\left[V\left(c'\right)\right]$$

where the expectation is taken with respect to the unconditional distribution of consumption equivalents. What is the welfare of a constant stream of consumption c^E? This is the measure of welfare in terms of consumption equivalent. It is given by:

$$V\left(c^E\right) = (1-\beta)u(c^E) + \beta V\left(c^E\right).$$

This implies that the certainty equivalent of stream of consumption with value $\tilde{V}$ is given by:

$$c^E = u^{-1}\left(\tilde{V}\right).$$

For the CRRA case, this is given by:

$$c^E = \left[(1-\gamma)\,\tilde{V}\right]^{\frac{1}{1-\gamma}}.$$

How much consumption is the agent willing to forgo in order to avoid shocks? In the absence of shocks, $u_t = 0$ for all t, $\bar{c} = 1$. Therefore, the cost of the business cycle is $1 - c^E$. The implementation of the calculation is as follows. First, $V(c)$ is computed numerically using our calibrated income process for Argentina (the smoothing component is ignored). Second, given $V(c)$ we obtain $V = \mathbb{E}_{c'}\left[V\left(c'\right)\right]$ using the Ergodic distribution of consumption implied by the process $c_t = y_t^{\lambda}$. Third, we obtain the consumption equivalent and therefore cost of the business cycle as $1 - c^E$. This value is 0.0040, or 0.4 percent in terms of consumption equivalents.

G Effective Spread for Argentinean Sovereign Bonds

In this section, we compute effective spreads for Argentinean bonds using daily closing prices. We use a widely adopted measure introduced in Roll (1984) and find that the mean and median effective spread for Argentinean bonds is 84.1 and 66.0 basis points. Our objective is to show that the target of 50 basis points in our baseline calibration in Tables 1 and 2, for bid-ask spreads before default, is a conservative value.

Theory: Roll (1984).[30] We provide the main ideas behind the measure by Roll (1984); we follow the treatment of Foucault et al. (2013). Suppose that the fundamental value of the asset follows a Random walk $m_t = m_{t-1} + \epsilon_t$, where ϵ_t is white noise. Bid and ask prices are given by $a_t = m_t - \frac{S}{2}$ and $b_t = m_t + \frac{S}{2}$, where S is the *spread* to be determined. The observed market price is $p_t = m_t + \frac{S}{2}d_t$ where $d_t = \{-1, 1\}$, depending on whether there is a sell or buy order. The idea is to obtain S from market prices. Under the assumption that: trade is balanced, there is zero auto-correlation or orders, and that market orders carry no news, then it can be shown that

$$Cov(p_{t+1} - p_t, p_t - p_{t-1}) = -\frac{S^2}{4}.$$

Thus, S can be computed given a time series of transaction prices $\{p_t\}_{t=0}^{\infty}$.

Data. Our data comprises daily high, low, and close prices of all the Argentinean bonds that are available in Bloomberg for the period M04-2004 to M07-2018. Because some of the bonds mature or were defaulted and then exchanged, the panel is unbalanced. There are bonds in different currencies (USD, EUR, JPY, CHF, ARS), and for different maturities. In our initial sample, there are 236,904 observations, 162 instruments, and 3737 trading dates.

Measurement. We construct the aggregate measure of liquidity for Argentina at time t as follows. We begin by constructing the liquidity measure of an individual bond i at time t, that we denote by $s_t^{R,i}$. It is given by

$$s_t^{R,i} \equiv 2\sqrt{-\max\left\{\overline{Cov(p^i_{t+1} - p^i_t, p^i_t - p^i_{t-1})}, 0\right\}},$$

where $\overline{Cov(p^i_{t+1} - p^i_t, p^i_t - p^i_{t-1})}$ is the (empirical) co-variance over a rolling window of 28 days. Following the literature (see for example Abdi and Ranaldo (2017)) we replace by zero the observations when $\overline{Cov(p^i_{t+1} - p^i_t, p^i_t - p^i_{t-1})}$ is positive. Then, we aggregate the individual measures to obtain the aggregate measure of the effective spread for Argentina at time t. Denote by $B(t)$ the set of bonds that are in the sample at date t. The aggregate

[30] There is an extensive literature that tries to infer the effective spread between buying and selling an asset from transaction prices. The objective of these studies is to obtain transaction costs from price data when bid-ask spreads are not reliable. For example, bid and ask quotes are only indicative (i.e., the intermediary is not required to fulfill the other at the quoted price) for emerging market bonds in Bloomberg. Thus, bid-ask spreads can be a noisy measure of prices. In this Appendix, we compute the measure by Roll (1984). Other popular measures in the literature of market micro-structure are Corwin and Schultz (2012) and Abdi and Ranaldo (2017). See Foucault et al. (2013) for a textbook treatment.

measure, s_t^R, is given by

$$s_t^R \equiv \sum_{i \in B(t)} \omega_t^i s_t^{R,i},$$

where we weight the individual measures $s_t^{R,i}$ by the corresponding outstanding market value of the individual bond. In particular, the weight is given by

$$\omega_t^i \equiv \frac{\text{Value Outstanding}_{i,t}^{USD}}{\sum_{i \in B(t)} \text{Value Outstanding}_{i,t}^{USD}},$$

where the variable Value Outstanding$_{i,t}^{USD}$ measures the face value value outstanding measured in US Dollars of individual bond i.

Results. Our main specification focuses on Non-Defaulted bonds in USD. For this, we drop the observations of bonds that are labeled are defaulted in Bloomberg and that are in a currency that is not the US Dollar. The mean and median are 84.1 and 66.0 basis points. The results are of a similar order of magnitude if we include bonds in all currencies, or if we use different sub-samples or measures of liquidity such as Corwin and Schultz (2012) and Abdi and Ranaldo (2017).