Illiquidity in Sovereign Debt Markets*

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Abstract

We study debt and default policy for sovereign countries when credit and liquidity risk are jointly determined. To account for both types of risks we focus on an economy with incomplete markets, limited commitment, and search frictions in the secondary market for sovereign bonds. We quantify the role of liquidity on sovereign spreads, debt capacity, and welfare, by performing quantitative exercises when our model is calibrated to match key features of the Argentinean default. We find that the liquidity premium is a substantial component of spreads, increases during bad times and reductions in secondary market frictions improve welfare.

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1 Introduction

Sovereign countries borrow to smooth shocks to income and tax revenue. One friction that prevents the smoothing of expenses over time and states of nature is the inability for governments to commit to future debt and default policies. This lack of commitment implies that the government will repay its debts only if it is convenient to do so and will dilute debt holders whenever it sees fit. To compensate investors for bearing these risks the sovereign pays a credit risk premium that reduces the available resources for domestic consumption and can substantially increase borrowing costs during bad times. The sovereign debt literature has helped us to understand how a lack of commitment shapes the outcomes of sovereign countries from a positive point of view and what policies are desirable from a normative point of view.\(^1\)

The recent European debt crisis, however, has underscored that decentralized markets impose additional frictions that prevent smoothing for sovereign countries. Their bonds are traded mostly in over the counter markets, where trading is infrequent, so if an investor holds a large position in a sovereign bond it might take time to find a counterparty willing to trade at a fair price. For this reason, investors need to be compensated not only for the risk of default or dilution but also due to illiquidity, introducing, in addition, a liquidity risk premium. This liquidity premium further reduces available resources and constraints policies of the sovereign. So far, the literature has been silent about this feature of sovereign borrowing. Our objective in this paper is to fill this gap by answering the following questions: How does credit and liquidity premium interact? What portion of total spreads can be explained by credit and how much by liquidity? What would be the welfare gains of reducing frictions in the secondary market?

Our paper contributes to the literature on sovereign borrowing in two ways. First, we propose a tractable model of sovereign borrowing in which credit and liquidity premia jointly determine borrowing and default decisions. Second, in a quantitative exploration focusing on one of the most studied cases of sovereign default, Argentina’s default in 2001, we show that the liquidity premium is a substantial component of total spreads. In particular, through the lens of our model, the welfare gains from the elimination of liquidity frictions are, quantitatively, of the same order of magnitude as the gains from eliminating business cycle fluctuations.

We start our paper by constructing a model of sovereign debt where debt and default policy take into account credit and liquidity risk. We focus on a small open economy that

\(^1\)See for example Arellano (2008), Aguiar and Gopinath (2006), and Aguiar and Amador (2013) for a review.
borrows from international investors to smooth income shocks following the quantitative literature of sovereign debt that builds on Eaton and Gersovitz (1981). A benevolent government designs debt and default policies to maximize the utility of the households by issuing non-contingent debt. The government cannot commit to future debt and default policies and might default in some states of nature. The distinctive feature of our model, in comparison to the previous literature, is the introduction of frictions in the secondary market for sovereign bonds, following the literature on over-the-counter markets as in Duffie et al. (2005). In our model, investors buy bonds in the primary market and can receive idiosyncratic liquidity shocks. If that occurs, they will bear a cost for holding the asset and therefore become natural sellers of it. Due to search frictions in the secondary market, it will take time for them to find a counterparty with whom to transact.

One of the main features of the model we propose is that default and liquidity risk will be jointly determined. On the one hand, the presence of search frictions in the secondary market introduces a liquidity premium that affects prices in the primary market, thereby affecting debt and default policies, which in turn changes the credit risk premium. On the other hand, as credit risk premium increases, the probability of default increases, and because investors foresee worse liquidity conditions in the future, liquidity conditions will also deteriorate. Therefore, in our model, default and liquidity risk premium are jointly determined. This joint determination is important because it will enable us to decompose total spreads into liquidity and credit components and to study the effects on welfare of reducing liquidity frictions in the secondary market.

After building a model of sovereign borrowing where both credit and liquidity premia constrain the choices of the government, we perform quantitative exercises to assess how much of total spreads are due liquidity frictions and what would be the welfare gains of eliminating these frictions. To do so, we calibrate the model to match key features of Argentina’s default in 2001. In particular, to match debt levels, mean and volatility of spreads, and bid-ask spreads, moments on the data.

Our first quantitative finding is that liquidity premium is a substantial component of total spreads. In particular, we find that during good times, low debt and high output, around 30 percent of total spreads can be explained by liquidity frictions in the case of Argentina’s 2001 default. During bad times, high debt and low output, when credit risk premium is high, the percentage of total spreads is lower. The intuition of these results is as follows. Bid-ask spreads measure the loss of an investor conditional on receiving a liquidity shock. Thus, the compensation to investors due to liquidity risk will be a function of bid-ask spreads. During good times, the investors demand a low credit risk premium, because the event of default is unlikely, but the liquidity risk is bounded away
from zero. During bad times, as the probability of default increases, investors demand a higher credit risk premium. This higher probability of default, also, will increase the liquidity risk premium because the liquidity of defaulted bonds is lower. However, credit risk premium will increase faster than the liquidity risk premium, and that is why that the liquidity premium will decrease as a fraction of total spreads, during bad times.

Our second quantitative finding is that the welfare gains from eliminating liquidity risk are substantial. To explore the size of these welfare gains, we perform an exercise following Lucas (2003). We find that the welfare loss induced by secondary market frictions is 0.17 percent in consumption equivalent terms. To put this number in perspective, given the volatility of consumption for Argentina in the period of study, a representative agent would pay 0.24 percent of consumption equivalent to eliminate fluctuations.

We think that the distinction between credit and liquidity risk is important for the design of debt policies for three reasons. First, in the long run, the policies to mitigate lack of commitment differ from those to mitigate frictions in the secondary market. For example, Hatchondo and Martinez (2015) and Chatterjee and Eyigungor (2015) show that fiscal rules and covenants on debt improve welfare in models when the government lacks commitment. However, policies in the long run that would decrease the liquidity premium could be the introduction of a centralized exchange for sovereign bond trading or increasing transparency in the secondary market, as reported in Edwards et al. (2004). Second, the policies during a short-term crisis might also be different. For example, a government could use resources to repay debt or to bail-out financial institutions that hold government debt and are in distress. An alternative policy, focusing on the secondary market, would be providing liquidity to intermediaries. Finally, we think that liquidity premium has implications for debt management. By issuing debt in different currencies and maturities the government caters to investors and completes the market. However, this increase in the number of assets might imply low liquidity of each one of these bonds, which in turn increases the cost of debt for the government.

Our paper connects with different strands of the literature. First, our model builds on the setting of the quantitative models of sovereign debt as in Aguiar and Gopinath (2006) and Arellano (2008); these two papers, extend the Eaton and Gersovitz (1981) framework of endogenous default to study business cycles in economies with a risk of default. These early quantitative implementations study economies with short-term debt and no recovery on default. Long term debt was introduced by Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). Chatterjee and Eyigungor (2012) introduce randomization to guarantee convergence of the numerical algorithm and show the existence of an equilibrium pricing function. Endogenous recovery of defaulted debt was intro-
duced by Yue (2010). In terms of modeling, in this paper the sovereign borrows by issuing long term bonds and after a default investors recover a fraction of their investment. The first ingredient crucial to obtain a positive liquidity premia, and the second one is necessary to guarantee that bonds have a positive value during default. We model long term debt following Chatterjee and Eyigungor (2012). To keep the model numerically tractable we introduce a reduced form recovery after default and we abstract from the bargaining process; it is worth noting that our reduced form recovery after default resembles the endogenous bargaining outcome in a setup as in Yue (2010). Our paper differs from the previous literature by introducing liquidity frictions in the market for sovereign bonds.

Second, our paper relates to the literature on over the counter markets. To model secondary market frictions we build on the setting of over-the-counter markets first studied by Duffie et al. (2005). In this paper, investors hold one unit of an asset and can receive non-diversifiable liquidity shocks. Once they receive a shock, they search for a counter-party, and meet them randomly. This framework was extended by Lagos and Rocheteau (2009) to allow for arbitrary asset holdings for investors. Our paper structures the debt market as in Duffie et al. (2005) but to keep the model numerically tractable we follow He and Milbradt (2013) and we do not keep track of the asset holdings of high and low valuation investors.

Third, our paper is closely related to models of corporate borrowing. In particular, we build the framework of He and Milbradt (2013), which extends the models of corporate default as in Leland and Toft (1996) by introducing an over-the-counter market as in Duffie et al. (2005). This paper uncovers a joint determination of liquidity and credit risk. Building on this framework Chen et al. (Forthcoming) and decompose spreads into a liquidity and credit component over the business cycle. Our paper differs from He and Milbradt (2013) and Chen et al. (Forthcoming) in two dimensions. First, there is a crucial qualitative difference between the sovereign and corporate settings. The value of default

\[\text{\footnotesize There has been extensive literature on search on asset markets following Duffie et al. (2005). Some recent examples are: Lagos et al. (2011) which studies crises in over-the-counter markets; Afonso and Lagos (2012) which studies high frequency trading in the market for federal funds; Atkenson et al. (2013) studies the decisions of financial intermediaries to enter and exit an over-the-counter market. A recent literature studies the role of search frictions on asset markets over macroeconomic outcomes, such as output, employment and asset prices. For example, Cui and Radde (2016) studies an economy with financial frictions in which private debt is illiquid and traded by intermediaries in an OTC market. Kozlowski (2017) develops a joint theory of corporate investment and maturity choice. Both decisions are affected by the liquidity premia of corporate bonds, that are issued to finance the investment, generated by frictions in the secondary market. Gutkowski (2017) studies, through the lens of a New Keynesian Model, how an exogenous inability to sell sovereign bonds used as collateral impacts on output and employment. On the empirical side, using a VAR identification, Gazzani and Vicondoa (2016) finds that shocks to liquidity, proxied by shocks to bid ask spreads, have effects over macroeconomic outcomes such as unemployment and over indicators of confidence.}\]
in our model is endogenously determined whereas in He and Milbradt (2013) and Chen et al. (Forthcoming) this value is fixed, does not depend on future liquidity conditions, and is zero in most cases. Second, in our setup total debt of the sovereign country is changing over the cycle. In both He and Milbradt (2013) and Chen et al. (Forthcoming) the capital structure fixed. In our paper, the fact that total debt varies over the cycle implies that the response of debt and default policies to liquidity frictions is to decrease debt levels and to default in more states of nature. The former case cannot occur with a fixed capital structure.

Finally, our paper relates to a growing empirical literature showing that liquidity is an important factor explaining sovereign spreads. Pelizzon et al. (2013) study market micro-structure using tick by tick data and document the strong non-linear relationship between changes in Italian sovereign risk and liquidity in the secondary bond market. Bai et al. (2012) find that liquidity risk explains most of the spread variations before the European sovereign debt crisis and credit risk explains most of these variations in the onset of the crisis. Ashcraft and Duffie (2007) find evidence of trading frictions in the pricing of overnight loans in the federal funds market. Fleming (2002) finds evidence of liquidity effects in treasury markets. Pelizzon et al. (2016) finds that, in the context of the European sovereign debt crisis using data for Italian bonds, credit risk drives liquidity premia. Our paper builds on these empirical findings, that document sizable liquidity frictions in the market for sovereign bonds, and complements these empirical studies by quantifying the size of the liquidity premium over the business cycle and assessing the welfare losses due to liquidity frictions.

2 Model

In this section, we present a sovereign default model with trading frictions in the secondary market for sovereign bonds. We first describe the setting: section 2.1 describes the macroeconomic environment, section 2.2 describes the secondary bond market, and 2.3 describes the timing of the model. We then then characterize the decisions of the

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3The evidence showing that liquidity is a factor explaining the spread of corporate bonds is more established. Longstaff et al. (2005) use data of credit default swaps to measure the size of the default and non-default component of credit spreads. They find that most of the spread is due to default risk and that the nondefault component is explained mostly by measures of bond illiquidity. Bao et al. (2011) show that there is a tight link between illiquidity and bond prices. Edwards et al. (2007) study transaction costs in over-the-counter (OTC) markets and find that transaction costs decrease significantly with transparency, trade size, and bond rating, and increase with maturity. Friewald et al. (2012) find that liquidity effects account for approximately 14 percent of the explained market-wide corporate yield spread changes. Chen et al. (2007) also find that liquidity is priced into corporate debt for a wide range of liquidity measures after controlling for common bond-specific, firm-specific, and macroeconomic variables.
government (section 2.4), bond prices (sections 2.5 and 2.6), as well as the equilibrium (section 2.7). Finally, section 2.8 discusses the choice of ingredients in the model.

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, \ldots\}$. The small open economy receives a stochastic stream of income denoted by $y_t$. Income follows a first order Markov process $\mathbb{P}(y_{t+1} = y' \mid y_t = y)$. The government is benevolent and wants to maximize the utility of the household. To do this, it trades bonds in the international bond market to smooth the household’s consumption. The household evaluates consumption streams, $c_t$, according to:

$$
(1 - \beta) E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],
$$

with time-preference $\beta \in (0, 1)$ and utility function $u(\cdot)$.

The sovereign issues long-term debt when it is not in default. As in Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012), each unit of debt matures with probability $m$ each period. Non-maturing bonds pay coupon $z$. This memoryless formulation of debt maturity structure means that the face value of outstanding debt is the only relevant state variable for the obligations of the government. The government can issue bonds at a price $q_t$ in the primary bond market. In equilibrium, the price of debt depends on current income, $y_t$, and the next period’s bond position, $b_{t+1}$ (our convention is that $b_{t+1} > 0$ denotes debt). The budget constraint for the economy is given by:

$$
c_t = y_t - [m + (1 - m) z] b_t + q_t [b_{t+1} - (1 - m) b_t],
$$

(2.1)

where $mb_t$ is the repayment of principal for maturing debt, $(1 - m)zb_t$ is the total coupon payment for non-maturing debt, and $q_t [b_{t+1} - (1 - m) b_t]$ is the proceed from newly issued debt.

There is limited enforcement of debt so the government can default at its convenience. There are two consequences of default. First, the government loses access to the international credit market and goes into autarky. Second, output is lower during default and is given by $y_t - \phi(y_t)$. That is, there is also a direct output cost of default, $\phi(y_t)$, which is a standard assumption in the literature. The government can regain access to the international credit market, which occurs with probability $\theta$ each period. A fraction of defaulted debt is written off when the government regains access to credit markets. In particular,
the new face value of outstanding debt is $\mathcal{R}(b_t) = \min \{\bar{b}, b_t\}$, where $b_t$ is the amount of defaulted debt and $\bar{b}$ is a maximum recovery value.\(^4\) That is, the fraction of recovered debt is $\mathcal{R}(b_t) / b_t = \min \{\bar{b} / b_t, 1\}$. Note that this fraction converges to zero as the amount of defaulted debt goes to infinity.

2.2 Primary and Secondary Bond Markets

There are two bond markets: the primary market and the secondary market. The government issues debt in the primary bond market. International investors can initially purchase newly issued debt in the primary market. Subsequent trading of bonds occurs in the secondary market. As in Duffie et al. (2005) and He and Milbradt (2013), trading in the secondary bond market is subject to search frictions.

Each international investor is assumed to be small and can hold at most a single unit of government debt. They are risk-neutral and can be either constrained or unconstrained. Unconstrained investors price bonds by discounting future payoffs at the rate risk-free rate, $r$. Unconstrained investors can become constrained if they receive a liquidity shock. Liquidity shocks are idiosyncratic in nature and have a per period probability $\zeta$ of occurrence. Constrained investors also discount payoffs at the risk-free rate, but are additionally subject to per period holding costs $h_c > 0$.\(^5\) As a result, unconstrained investors have a high bond valuation, $q^H$, while constrained investors have a low bond valuation, $q^L$ (the exact expressions for $q^H$ and $q^L$ are given in sections 2.5 and 2.6). Therefore, unconstrained investors are the natural buyers of bonds in both primary and secondary markets, while constrained investors are the natural sellers of bonds in secondary markets.

Unconstrained investors try to offload their bond positions in the secondary market. As in He and Milbradt (2013), secondary market trading is intermediated. The per period contact probability between constrained investors and intermediaries is $\lambda$.\(^6\) Constrained

\(^4\)Models of sovereign debt renegotiation such as Yue (2010) and Bai and Zhang (2012) endogenize recovery as the equilibrium outcome of a Nash bargaining game between the government and its creditors. These models produce recovery values of the form $\mathcal{R}(\bar{b}(y_t), b_t) = \min \{\bar{b}(y_t), b_t\}$ with state-dependant maximum recovery values, $\bar{b}(y_t)$. In our simple specification, the maximum recovery value does not vary over states.

\(^5\)We interpret liquidity events as investor-specific events that prompt an immediate need to sell (e.g. in order to meet an expenditure). Holding costs represent the utility loss associated with delayed transactions.

\(^6\)In equilibrium, there is no need for unconstrained investors to contact intermediaries: unconstrained investors already have high valuations so there are no gains from trade.
investors and intermediaries bargain upon making contact. The total surplus is:

$$S = A - q^L,$$

where \(q^L\) is the low valuation of the constrained investor, and \(A\) denotes the ask price at which the intermediary can then offload the bond. Following He and Milbradt (2013), we assume that there is a large mass of competitive unconstrained investors waiting on the sideline that intermediaries could contact with immediate effect. This simplifying assumption means that we do not have to keep track of dealer inventory as intermediaries could instantaneously offload bonds to high valuation investors. Since the large mass of high valuation investors on the sideline act competitively, intermediaries can offload their positions at high valuations so that:

$$A = q^H.$$

The total surplus, \(S = q^H - q^L\), is then split according to a Nash bargaining rule with the bargaining power of the constrained low valuation investors being \(\alpha \in [0, 1]\). This implies that the price at which constrained investors sell to intermediaries upon contact is:

$$q^S = q^L + \alpha(q^H - q^L).$$

The dollar bid-ask spread is the difference between intermediaries’ selling price and buying price: \(q^H - q^S = (1 - \alpha)(q^H - q^L)\). The proportional bid-ask spread:

$$ba = \frac{(1 - \alpha)(q^H - q^L)}{\frac{1}{2}(q^H + q^L) + \frac{\alpha}{2}(q^H - q^L)},$$

is simply the dollar bid-ask spread normalized by the mid price \(\frac{1}{2}(q^H + q^S)\).

We let that the bargaining power of constrained low valuation investors depend on whether or not the government is in default, with \(\alpha_D\) (\(\alpha_{ND}\)) denoting investors’ bargaining power when trading bonds that are (not) in default. We assume:

$$\alpha_D < \alpha_{ND}$$

so that investors’ have lower bargaining power when trading sovereign bonds that are in default. This implies that bid-ask spreads that are higher during default episodes. As we discuss in subsection 2.8, there is wide evidence that bid-ask spreads are higher during default for US corporate Bonds (see, e.g., Edwards et al. (2007) and He and Milbradt (2013)). To the best of our knowledge, we are unaware of cross country studies of bid-ask
spreads for sovereign bonds that are in default.

2.3 Timing

The timing for the government is as follows. First, consider the case in which the government has credit access (i.e. is not in default) and begins period $t$ with an amount $b_t$ of outstanding debt. Income $y_t$ is then realized. The government then decides whether or not to default $d_t \in \{0, 1\}$. If the government chooses not to default ($d_t = 0$), principal payments for maturing debt, $mb_t$, and coupon payments for non-maturing debt, $(1 - m) zb_t$, are made. The government can then issue new debt in the primary market. An issuance with face value $b_{t+1} - (1 - m)b_t$ leads to outstanding debt with face value $b_{t+1}$ at the start of the next period. As previously mentioned, unconstrained investors are the natural buyers of new bond issues so that bonds are always issued at the high valuation $q_t^H$. Finally, consumption takes place and is given by $c_t = y_t - [m + (1 - m) z] b_t + q_t^H [b_{t+1} - (1 - m)b_t]$.

Next, consider the case in which the government is already in default or chose to default in the current period ($d_t = 1$). In this case, $b_t$ is the amount of debt that is in default. The government is in autarky and does not take any actions. Consumption is simply equal to income adjusted for costs of default: $c_t = y_t - \phi(y_t)$. Nature determines whether or not the government regains credit access between the end of period $t$ and the start of the next period $t + 1$. The probability of regaining credit access is $\theta$, and in that event, the government reaccesses the debt market with an outstanding debt of $R(b_t) = \min \{ b_t, b_{t+1} \}$ at the start of the next period. Otherwise, the government remains in autarky.

The timing for investors are as follows. Investors are assessed holding costs $h_c$ for the period if they begin period $t$ being constrained. Secondary market trading for outstanding bonds occur once per period immediately before the government issues new bonds in the primary market. As mentioned in section 2.2, only constrained bond holders attempt to sell in the secondary market. With probability $\lambda$, a constrained investor meets an intermediary and offloads his bond position. The transaction price is $q^S_{ND} (y_t, b_{t+1})$ when the government is not in default, and $q^S_D (y_t, b_t)$ when the government is in default. Constrained investors who fail to contact intermediaries remain constrained going into the next period $t + 1$.

An investor who is unconstrained at the beginning of period $t$ is not subject to holding costs for the period. However, such an investor can become constrained for the start of the next period $t + 1$ if he receives a liquidity shock during period $t$. Liquidity shocks occur with probability $\zeta$ and take place after the conclusion of trading in the secondary
Figure 2.1: This figure summarizes the timing before and after default in period $t$. The government enters the period with bonds $b_t$. Then, income $y_t$ is realized and the government chooses whether or not to default, $d_t$. Constrained investors are subject to holding costs $h_c$. The upper branch depicts the sequence of events in the absence of default: secondary market trading of outstanding bonds occur, unconstrained investors receive liquidity shocks. First, liquidity constrained investors can sell their debt positions if they meet an intermediary. Then, the liquidity shock is realized. Then, the government issues a face value of debt $b_{t+1} - (1 - m)b_t$, facing a price $q_{ND}^H(y_t, b_{t+1})$. Finally, consumption is realized. The lower branch depicts what happens in the case that the government defaults. First, liquidity constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Note that the primary market is closed while the government is in autarky. Then, the government will re-access the debt market next period with probability $\theta$. Finally, consumption is equal to $c^d(y_t) = y_t - \phi(y_t)$.

market. This means that a newly constrained investor in period $t$ is unable to immediately offload his position in the same period. In addition, liquidity shocks occur prior to new bond issuances in the primary market. This implies that the unconstrained investors who purchased newly issued bonds during period $t$ will still be unconstrained at the beginning of period $t + 1$. Figure 2.3 summarizes the timing within each period.

2.4 The Government’s Decision Problem

The government maximizes household welfare taking bond prices as given. This infinite horizon decision problem can be cast as a recursive dynamic programming problem. We focus on a Markov equilibrium with income, $y$, as the exogenous state variable and debt, $b$, is the endogenous state variable. The value for a government with an option to default,
\( V_{ND} \), is the larger of the value of defaulting, \( V_D \), and the value of repayment, \( V_C \),

\[
V_{ND} (y, b) = \max_{d \in \{0,1\}} d V_D (y, b) + (1 - d) V_C (y, b).
\]

The solution to this problem gives the government’s default policy:

\[
d = D (y, b) = 1 \{ V_D (y, b) > V_C (y, b) \}.
\] (2.2)

That is, the government defaults whenever the value of defaulting is higher than the value of repayment.

The value of defaulting is:

\[
V_D (y, b) = (1 - \beta) u(y - \phi(y)) + \beta E_y | y \left[ \theta V_{ND} (y', R(b)) + (1 - \theta) V_D (y', b') \right],
\]

where the flow utility is determined by household consumption in default, \( y - \phi(y) \), while the continuation value takes into account the possibility of regaining credit market access with debt level \( R(b) \).

The value of repaying:

\[
V_C (y, b) = \max_{b'} \left\{ (1 - \beta) u(c) + \beta E_y | y \left[ V_{ND} (y', b') \right] \right\} (2.3)
\]

is subject to two constraints. The first constraint is the standard budget constraint:

\[
c = y - [m + (1 - m)z] b + q^H_{ND} (y, b') [b' - (1 - m) b],
\]

where \( q^H_{ND} (y, b') \) is the bond issuance price schedule corresponding to the high valuation of unconstrained investors during periods in which the government is not in default. In addition, as in Chatterjee and Eyigunog (2015), the government faces an upper bound in the ex ante one period ahead expected default probability:

\[
\delta (y, b') \equiv E_y [d (y', b')] \leq \delta
\] (2.4)

whenever there is a positive debt issuance, \( b' - (1 - m) b > 0 \). As explained in Chatterjee and Eyigunog (2015), in long-term debt models with positive recovery, the government has incentives to dilute existing bond holders by issuing large amounts of debt just prior to default. Since the liability of the government upon regaining credit access is at most \( \bar{b} \), the government will then issue an infinite amount of debt just prior to default in order to fully dilute existing bond holders. Constraint (2.4) rules out such counterfactual behav-
ior.\textsuperscript{7} The solution to the repayment problem gives the debt policy of the government:

\begin{equation}
\begin{align*}
b' &= B (y, b) .
\end{align*}
\end{equation}

### 2.5 Debt Valuation Before Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is not in default. Let $y$ be current income and suppose that $y'$ is the post-issuance face value of outstanding debt. The value of one unit of debt for an unconstrained investor with a high valuation is:

\begin{equation}
\begin{align*}
q_{ND}^H (y, b') &= E_{y' | y} \left\{ (1 - d (y', b')) m + (1 - m) \left[ z + \zeta q_{ND}^L (y', b'') + (1 - \zeta) q_{ND}^H (y', b'') \right] \right. \\
& \left. + d (y', b') \frac{\zeta q_{D}^L (y', b') + (1 - \zeta) q_{D}^H (y', b')}{1 + \rho} \right\} ,
\end{align*}
\end{equation}

which reflects the state contingent payoffs of the bond. An investor receive principal $m$ and coupon $(1 - m) z$ in the absence of default during the next period, $d (y', b') = 0$. In this case, the continuation value of the $1 - m$ non-maturing fraction of the bond depends on next period’s optimal debt policy, $b'' = B (y', b')$, and the realization of the idiosyncratic liquidity shock. A liquidity shock arrives with probability $\zeta$, in which case the investor obtain a low continuation value, $q_{ND}^L (y', b'')$. Otherwise, the investor remains unconstrained and assigns a high continuation value, $q_{ND}^H (y', b'')$, to the bond. An investor does not receive any cashflow in the event of a default, $d (y', b') = 0$. In this case, the government defaults on $b'$ units of debt, and the per unit price of defaulted debt is, respectively, $q_{D}^H (y', b')$ and $q_{D}^L (y', b')$ for investors who do not receive and receive liquidity shocks. The value of defaulted bonds will be described shortly, in the next section 2.6.

The price of debt for a constrained investor with a low valuation is:

\begin{equation}
\begin{align*}
q_{ND}^L (y, b') &= E_{y' | y} \left\{ (1 - d (y', b')) \frac{-h_c + m + (1 - m) \left[ z + (1 - \lambda) q_{ND}^L (y', b'') + \lambda q_{ND}^S (y', b'') \right]}{1 + r} \\
& \left. + d (y', b') \frac{-h_c + (1 - \lambda) q_{D}^L (y', b') + \lambda q_{D}^S (y', b')}{1 + r} \right\} .
\end{align*}
\end{equation}

The valuation of a constrained investor is similar to that of an unconstrained investor,

\textsuperscript{7}As noted in Chatterjee and Eyigungor (2015), sovereign bonds issued in financial centers (e.g. New York) have to be underwritten by some investment bank. Reputational concerns may prevent these from issuing bonds with very high probabilities of immediate default.
Figure 2.2: This figure details the bond market if the sovereign is not in default and does not default in period $t$. It starts by issuing debt $b_{t+1}$. The high valuation investors buy this debt in the primary market. After that, with probability $\lambda$ the low valuation investors will meet an intermediary. They will sell their bonds at the price $q_{ND}^{S}(y_t, b_{t+1})$. After selling their bonds, they exit the market. The low valuation investors that do not meet an intermediary will try to sell their bonds next period. Then, with probability $\zeta$, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond next period in the secondary market. Both the high and low valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

but reflects the following differences. First, a constrained investor is assessed holding costs $h_c$ which lower the effective value of a bond. Second, the continuation value for a constrained investor depends on trading outcomes in the secondary market. As described in section 2.2, a constrained investor sells with probability $\lambda$. The selling price (in the next period) is:

$$q_{ND}^{S} (y', b'') = \alpha_{ND} q_{ND}^{H} (y', b'') + (1 - \alpha_{ND}) q_{ND}^{L} (y', b'')$$

in the absence of default, and $q_{ND}^{S}(y', b')$ in the event of a default.

### 2.6 Valuations of Debt: After Default

In this section, we characterize the valuations of constrained and unconstrained investors during periods in which the government is in default. Let the current income be $y$ and $b$ be the amount of debt in default. In this case, the value of one unit of debt for unconstrained
high valuation investors is:

\[
q^H_D(y, b) = \frac{1 - \theta}{1 + r} \mathbb{E}_{y'|y} \left[ \zeta q^H_D(y', b) + (1 - \zeta)q^L_D(y', b) \right] + \theta \frac{\mathcal{R}(b)}{b} q^H_{\text{ND}}(y, \mathcal{R}(b)). \tag{2.8}
\]

The government regains credit access next period with probability \( \theta \), in which case \( \mathcal{R}(b) / b \) is the fraction recovered by each unit of default debt. The value of recovered bonds, \( q^H_{\text{ND}}(y, \mathcal{R}(b)) \), is given by (2.6) and reflects the new value of total outstanding debt, \( \mathcal{R}(b) \). Otherwise, investors receive no payments if default is not resolved and the continuation value reflects the probability \( \zeta \) of receiving a liquidity shock.

Similarly, the per unit value of debt for constrained low valuation investors is:

\[
q^L_D(y, b) = \frac{1 - \theta}{1 + r} \mathbb{E}_{y'|y} \left[ -h_c + \lambda q^S_D(y', b) + (1 - \lambda)q^L_D(y', b) \right] + \theta \frac{\mathcal{R}(b)}{b} q^L_{\text{ND}}(y, \mathcal{R}(b)). \tag{2.9}
\]

This valuation is analogous to that of unconstrained investors (2.8), but accounts for holding costs and trading frictions in the secondary market. Constrained investors sell during the next period with probability \( \lambda \). The selling price for a defaulted bond is:

\[
q^S_D(y, b) = \alpha_D q^H_D(y, b) + (1 - \alpha_D)q^L_D(y, b),
\]

and reflects a lower investor bargaining power during default: \( \alpha_D < \alpha_{ND} \).

2.7 Equilibrium

We focus on a Markov equilibrium with state variables \( y \) and \( b \). An equilibrium consists of a set of policy functions for consumption \( C(y, b) \), default \( D(y, b) \), and debt \( B(y, b) \), as well as bond valuations \( q^H_{\text{ND}}(y, b') \), \( q^L_{\text{ND}}(y, b') \), \( q^H_D(y, b) \) and \( q^L_D(y, b) \) such that: (1) the policies solve the government’s problem (2.3) taking bond valuations as given, and (2) the bond valuations satisfy equations (2.6) (2.7), (2.8) and (2.9).

2.8 Discussion

We now discuss our modeling assumptions. Our goal is to provide a parsimonious framework to study debt and default policy in a setting where default and liquidity risk are jointly determined. We generate bid-ask spreads by introducing search frictions in the secondary market as in Duffie et al. (2005). Indeed, decentralized bond market trading is an important feature of reality. Having long term debt is then necessary in order to generate realistic levels of bid-ask spreads. There is no need to trade, and no bid-ask
Figure 2.3: The figure details the bond market if the government is in default or defaults in period $t$. There is no debt issue or debt service. The sovereign has an outstanding balance of debt $b_t$. The low valuation investors will meet an intermediary with probability $\lambda$. They will sell their bonds at the price $q_S(y_t, b_t)$. After they sell the bond they exit the market. The low valuation investors that do not meet an intermediary will try to sell next period. Then, with probability $\zeta$, the high valuation investors will receive a liquidity shock. They will have the opportunity to sell next period in the secondary market. Finally, with probability $\theta$, the government resolves the default and re-accesses the next period with a face value of debt $b_t \times R(b_t)$.

spreads as a result, if outstanding debt matures every period: investors can simply wait and receive principal payouts from the government. The combination of long term debt and secondary market search frictions leads to a difference in valuation between liquidity constrained and unconstrained investors prior to default, $q^H_{ND} - q^L_{ND}$. There is substantial evidence of trading frictions in the secondary market for sovereign bonds (see, e.g., Pelizzon et al. (2013) for recent evidence). Long-term debt is now a standard feature of models of sovereign debt following the contributions of Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), and Arellano and Ramanarayanan (2012).

We also introduce positive debt recovery, $R(b) > 0$. Positive recoveries are an important feature of the data (see, e.g., Cruces and Trebesch (2013)) and is necessary in order to generate realistic behaviour of bid-ask spreads. This is because the absence of recovery, $R(b) = 0$, implies no future cashflows and therefore zero valuations for both constrained and unconstrained investors during default, $q^H_D = q^L_D = 0$. In turn, this then implies that
the bid-ask spread is zero during default. More importantly, it would also imply that bid-
ask spreads approach zero as default probabilities approach one, which is at odds with a
widening of bid-ask spreads during sovereign crises documented in Pelizzon et al. (2013).

We assume a decrease in the bargaining power of investors after a sovereign default,
\( \alpha_D < \alpha_{ND} \), in order to capture the positive comovement between bid-ask spreads and
sovereign default probabilities. Under this assumption, bid-ask spreads are higher dur-
ding default. As a result, bid-ask spreads will also increase leading up to a default as
is consistent with evidence from Pelizzon et al. (2013). There is substantial evidence of
higher bid-ask spreads for defaulted corporate bonds (see, e.g., Edwards et al. (2007) and
He and Milbradt (2013)). To the best of our knowledge, we are unaware of similar studies
that report bid-ask spreads for a large sample of defaulted sovereign bonds.\(^8\)

In Appendix B we explicitly illustrate these points with a simple jump to default
model in which debt policies are fixed and there is an exogenous default probability.

3 Results

We numerically solve a discretized version of the model. As is discussed in Chatterjee
and Eyigungor (2012) grid-based methods have poor convergence properties when there
is long-term debt. To overcome this problem we follow their prescription and compute a
“slightly” perturbed version of the model described in this section. To ensure numerical
accuracy, we choose a very dense grid with 200 points for the persistent component of
output and 450 points for debt. We implement the model in CUDA and numerically
compute the model on a Tesla K80 GPU. Appendix A discusses the details.

3.1 Calibration

We calibrate the model developed in Section 2 to account for the main features of Ar-
genita’s default in 2001. We focus on Argentina over the period of 1993:I to 2001:IV
for three reasons. First, this facilitate comparison to prior studies in the sovereign debt
literature (see, for example, Hatchondo and Martinez (2009), Arellano (2008) and Chatter-
jee and Eyigungor (2012)) and in this way, we can be transparent about what our paper
brings to the table. Second, this sample satisfies our model’s main assumptions: (1) our
model is real and Argentina had a fixed exchange rate vis-a-vis the dollar during this pe-
riod, and (2) Argentina’s bonds were traded in an illiquid secondary market during this

\(^8\)Using Bloomberg data, we frequently observe stale prices for defaulted Argentine bonds that can be
constant for weeks. This is indicative of lower trading.
period. We calibrate and simulate the model at a monthly frequency because liquidity is inherently a short run phenomenon. However, we report results at a quarterly frequency to facilitate comparison to previous studies.\footnote{To be precise about this conversion, the quarterly debt to output ratio in our paper is the stock of debt at the end of the quarter divided by the sum of monthly output within the quarter. This implies that the average debt to quarterly output will be a third of average debt to monthly output.}

**Functional Forms and Stochastic Processes.** As is standard in the literature, we specify the household utility to be CRRA $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We set the endowment process to be:

$$y_t = e^{zt} + \epsilon_t,$$
$$z_t = \rho z_{t-1} + \sigma z u_t,$$

where $z_t$ is a discretized AR(1) process with persistence $\rho_z$, volatility $\sigma_z$, and normally distributed innovations $u_t \sim N(0, 1)$. We also add a small amount of noise $\epsilon_t \sim \text{trunc } N(0, \sigma^2_\epsilon)$ that is continuously distributed. As shown in Chatterjee and Eyigungor (2012), this is necessary to achieve numerical convergence. We set output loss during default to:

$$\phi(y) = \max \left\{ 0, d_y y + d_{yy} y^2 \right\}.$$  

This loss function is proposed by Chatterjee and Eyigungor (2012) and nests several cases in the literature. When $d_y < 0$ and $d_{yy} > 0$, the cost is zero for the range $0 \leq y \leq -\frac{d_y}{d_{yy}}$ and rises more than proportionally with output for $y > -\frac{d_y}{d_{yy}}$. Alternatively, when $d_y > 0$ and $d_{yy} = 0$ the cost is a linear function of output.\footnote{The case studied in Arellano (2008) features consumption in default that is given by the mean output if the output is over the mean and equal to output if the output is less than the mean. This implies a cost function $\phi^A(y) = \max \{ y - \mathbb{E}(y), 0 \}$, which closely resembles the case of $d_y > 0$ and $d_{yy} = 0$.} As is explained in Chatterjee and Eyigungor (2012), the convexity of output costs is necessary to match the volatility of sovereign spreads.

**Apriori set parameters.** We set risk aversion to $\gamma = 2$ which is standard in the sovereign debt literature. The parameters for output are estimated from (linearly) detrended and seasonally adjusted data for Argentina for the quarterly sample from 1980:I to 2001:IV available from Neumeyer and Perri (2005). After estimating a AR (1) model for output at a quarterly frequency, we obtain monthly values $\rho_z = 0.983$, $\sigma_z = 0.0151$ and we fix $\sigma_\epsilon = 0.004$.\footnote{The conversion is as follows. Total monthly volatility is given by $\sqrt{0.004^2 + 0.0151^2} = 0.0156$. Then, total monthly volatility and the monthly auto correlation are converted to a quarterly frequency.} We set the risk free rate to $r = 0.0033$ per month so that the quarterly risk
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Sovereign’s discount rate</td>
<td>0.9841</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Sovereign’s risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of output</td>
<td>0.983</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Volatility of output</td>
<td>0.0156</td>
</tr>
<tr>
<td>$m$</td>
<td>Rate at which debt matures</td>
<td>0.0167</td>
</tr>
<tr>
<td>$z$</td>
<td>Coupon rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$1 - \theta$</td>
<td>Probability of reentry</td>
<td>0.0128</td>
</tr>
<tr>
<td>$d_y$</td>
<td>Output costs for default</td>
<td>-0.264</td>
</tr>
<tr>
<td>$d_{yy}$</td>
<td>Output costs for default</td>
<td>0.337</td>
</tr>
<tr>
<td>$r$</td>
<td>Discount rate of international investors</td>
<td>0.0033</td>
</tr>
<tr>
<td>$h_c$</td>
<td>Holding costs for constrained investors</td>
<td>0.0014</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Probability of getting a liquidity shock</td>
<td>0.139</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Probability of meeting a market maker</td>
<td>0.865</td>
</tr>
<tr>
<td>$\alpha_{ND, a_D}$</td>
<td>Bargaining power of market maker</td>
<td>0.875, 1</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>Maximum recovery rate for sovereign bonds</td>
<td>0.83</td>
</tr>
<tr>
<td>$\tilde{\delta}$</td>
<td>Max. Default Probability</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Table 1: Baseline**

The free rate is 1%, which is standard. We set $m = 1/60$ to match an average debt maturity of 5 years based on values reported in Chatterjee and Eyigungor (2012) and Broner et al. (2013). We set the coupon rate to $z = 0.01$ so that the annualized coupon rate is 12 percent, which is close to the 11 percent value weighted coupon rate for Argentina reported in Chatterjee and Eyigungor (2012). We fix the re-entry probability at $1 - \theta = 0.0128$ following Chatterjee and Eyigungor (2012). This implies an average exclusion period of 6.5 years.\(^{12}\) We set the maximum one month ahead default probability to $\tilde{\delta} = 0.75$. Finally, we fix $\lambda = 0.8647$ so that the average time that it takes for a constrained investor to offload his position to an intermediary is two weeks, as in Chen et al. (Forthcoming).

**Calibrated parameters.** We choose time preference $\beta = 0.9841$ to target an average debt to quarterly output ratio of 100 percent based on the mean debt to quarterly output ratio for Argentina for the period 1993:I and 2001:IV.\(^{13}\) As our model features recovery, we set the maximal debt recovery to be $\bar{b} = 0.83$ in order to capture a mean recovery of $E \left[ \min \{ b_{def}, b_{def} \} \right] / b_{def} = 0.3$ in the model. This is based on a realized recovery rate of 30% for the 2001 Argentine default. In addition, we choose the default cost parameters $d_y = -0.264$ implies an autocorrelation of $0.983^3 = 0.95$ and an output volatility of $0.0156 \times \sqrt{3} = 0.027$. The latter values are the ones recovered in the data.

\(^{12}\)Beim and Calomiris (2001), report that for the 1982 default episode, Argentina spent until 1993 in a default state. For the 2001 default episode, Benjamin and Wright (2009) report that Argentina was in default starting in 2001 until 2005 when it settled with most of its bondholders.

\(^{13}\)Chatterjee and Eyigungor (2012) target a debt to quarterly output ratio of 70 percent as their model does not feature recovery. We target the full 100 percent as our model features recovery.
Table 2: Model moments.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>CE (2012), Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Debt to Gdp</td>
<td>1.0</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Expected Recovery</td>
<td>0.30</td>
<td>0.297</td>
<td>0</td>
</tr>
<tr>
<td>Mean Sovereign Spread</td>
<td>0.0815</td>
<td>0.0815</td>
<td>0.0815</td>
</tr>
<tr>
<td>Vol. Sovereign Spread</td>
<td>0.0443</td>
<td>0.0437</td>
<td>0.0443</td>
</tr>
<tr>
<td>Mean Bid-Ask Spread, ND</td>
<td>0.0050</td>
<td>0.0049</td>
<td>-</td>
</tr>
<tr>
<td>Mean Bid-Ask Spread, D</td>
<td>0.0500</td>
<td>0.0503</td>
<td>-</td>
</tr>
<tr>
<td>Mean Turover</td>
<td>0.12</td>
<td>0.12</td>
<td>-</td>
</tr>
</tbody>
</table>

and $d_{yy} = 0.337$ to match the first two moments for the quarterly behavior of (annualized) sovereign spreads. Following Chatterjee and Eyigungor (2012), this involves targeting mean spreads of 0.0815 along with a quarterly volatility of 0.0443. \(^{14}\) We calibrate parameters relating to secondary market frictions as follows. We first normalize the bargaining power of market makers during periods of default to $a_D = 1$. We then choose a holding cost of $h_c = 0.0014$ in order to target a mean proportional bid-ask of 500 basis points during default. Due to data limitations, we base this target on the bid-ask spreads of defaulted US bonds which are documented to range between 200 basis points during normal times Edwards et al. (2007) and 620 basis points during recessions Chen et al. (Forthcoming). We then set the bargaining power of market makers outside of default to be $a_{ND} = 0.875$ in order to target a mean proportional bid-ask of 50 basis points outside of default. For European bonds, Pelizzon et al. (2013) find a median bid-ask spread of 43 basis points in their sample, which increased to 125 basis points during the period June 2011 to November 2012. For non-investment grade US corporate bonds, Chen et al. (Forthcoming) report bid ask spreads of 50 basis points during normal times and 218 during bad times. Finally, we set the probability of receiving a liquidity shock each period to $\zeta = 0.139$ in order to match an average turnover of 12 percent per month based on the average turnover rate for US corporate bonds (see Bao et al. (2011)). \(^{15}\) This value implies that, on average, an unconstrained investor becomes constrained every 7 months.

Calibration Results. The final parameter values and the results from our baseline calibration are reported in Tables 1 and 3.1 respectively. The last column of Table 3.1 lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model closely matches all targeted moments. It generates a mean debt to GDP of 100 percent

\(^{14}\)The annualized sovereign spread is given by $cs(y, b') = (1 + r^H(y, b'))^{12} - (1 + r)^{12}$ where the yield to maturity is given by $r^H(y, b') = \frac{m + (1 - m)z}{q^H(y, b') - m}$.

\(^{15}\)To the best of our knowledge, turnover data for Argentine bonds is unavailable for our period of interest. Turnover, or the fraction of outstanding bonds being traded each period, is $\lambda \zeta / (\lambda + \zeta)$ on average in the model.
and an average recovery rate of 29.7 percent. The average sovereign spread is 0.0815 and the volatility is 0.0437. The average bid-ask spread is, respectively, 50 and 503 basis points before and after default. Finally, the mean turnover rate is 12 percent.

Coming Next. In the next sections, we use our calibrated model to perform a series of quantitative exercises. In subsection 3.2, we report the pricing functions in the primary market and the bid-ask spreads. In subsections 3.3 and 3.4 we provide a structural decomposition of total spreads into a liquidity and default components. In subsection 3.5 we quantify the welfare implications of liquidity frictions. In subsection 3.6 we use our calibrated model to impute the liquidity component of Argentine spreads in the lead up to Argentina’s 2001 default. Finally, subsection 3.7 reports business cycles statistics of the model.

3.2 Bond Prices, Bid-Ask Spreads, Feedback

Figure 3.1 plots the model-implied bond prices and bid-ask spreads as a function debt choice $b'$. The plots are for high and low values of output $y$, which correspond to values at plus and minus one standard deviation of the unconditional distribution for output, respectively. Panel A plots bond prices in the primary market $q^H(y, b')$ during the credit
access regime. Bond prices are always strictly positive as a consequence of having a positive recovery following default. Standard comparative statics apply: bond prices are increasing in output and decreasing in debt choice, as is usually the case in quantitative models of sovereign debt. Note that since the model is calibrated at a monthly frequency, the debt to quarterly output ratio is approximately $b'/3$.

Panel B plots proportional bid-ask spreads, \( \frac{(q^H - q^S)}{\frac{1}{2} (q^H + q^S)} \), during periods in which the government is not in default. The bid-ask spread is approximately 50 basis points for low levels of debt choice (e.g. $b' \leq 2.5$) regardless of income levels. Bid-ask spreads increase as the likelihood of default increases. This occurs when output is low or debt choice is high. For example, the bid-ask spread reaches 500 basis points when debt choice is $b' = 3$ and output is low, and subsequently continues to increase for larger choices of debt. The logic is as follows. First, recall that bid-ask spreads are larger during periods of default due to a decrease in investor bargaining power when trading defaulted bonds ($\alpha_D < \alpha_{ND}$). This can be seen in Panel C which plots proportional bid-ask spreads during periods of default as a function of the amount of debt in default. Since prices are forward looking, the increase in bid-ask spreads as the probability of default increases reflects the increased likelihood of encountering the worser liquidity conditions associated with trading defaulted bonds.

Panel C also illustrates that the proportional bid-ask spread of defaulted bonds is also increasing in the amount of debt in default. The reasoning is as follows. To first order, the dollar bid-ask spread during default, $q^H_D - q^S_D$, is proportional to holding costs. Since holding costs are constant, the proportional bid-ask spread increases as the value of bonds decrease. The latter is true if the amount of debt in default is high and the fractional recovery per unit of defaulted debt, $\mathcal{R}(b)/b$, is low.

In addition, Panel C also shows that, to first order, there is no difference in the proportional bid-ask of defaulted bonds across different current output states. From equations (2.8) and (2.9), we see that the value of defaulted bonds depend on the current output state only through its influence on the value of recovered debt upon the government reentering international credit markets. In our calibration, periods of autarky last 6.5 years on average. Over such a horizon, the influence of current output on the eventual recovered debt value is weak. As a result, the proportional bid-ask spread for defaulted bonds depend mainly on the amount of debt in default.

In combination, Panels B and C of 3.1 demonstrate a liquidity-default feedback loop first highlighted by He and Milbradt (2013) in the context of corporate bonds. To see this feedback loop, consider the net proceeds from debt rollover, the difference in the value of newly issued bonds and the repayment of principal of maturing debt, $q^H_{ND,t} \left[ b_{t+1} - (1 - m) b_t \right] -$
3.3 Sovereign Spread Decomposition

In this subsection we present a decomposition of total spreads into a liquidity and credit component. Our main result is that liquidity premia can be a substantial component of total spreads. As a first step towards a decomposition, we start by defining total spreads, \( cs(y, b') \), as a function of government policies. Consider a government with debt and default policies given by \( b' = \tilde{B}(y, b) \) and \( d = \tilde{D}(y, b) \), respectively. Let \( q_{ND}^H(y, b') \big{|}_{(B,D,\xi)} \) be the corresponding value of this government’s debt to an unconstrained investor who
receives liquidity shocks with probability $\zeta$ when the government follows policies $\tilde{B}, \tilde{D}$. That is, $q_{ND}^H(y, b') \big|_{(\tilde{B}, \tilde{D}, \zeta)}$ solves equation (2.6) under debt policy $\tilde{B}$ and default policy $\tilde{D}$. Since our calibration is monthly, the corresponding annualized sovereign spread is then given by:

$$
\text{cs} \left( y, b' \right) \big|_{(\tilde{B}, \tilde{D}, \zeta)} \equiv \left( 1 + r^H(y, b') \big|_{(\tilde{B}, \tilde{D}, \zeta)} \right)^{12} - (1 + r)^{12},
$$

(3.1)

where $r^H(y, b') \big|_{(\tilde{B}, \tilde{D}, \zeta)}$ is given by $r^H(y, b') \big|_{(\tilde{B}, \tilde{D}, \zeta)} = \left[ m + (1 - m)z \right] / \left( q_{ND}^H(y, b') \big|_{(\tilde{B}, \tilde{D}, \zeta)} \right) - m$.

Aided by the definition of spreads as a function of policies, given by equation (3.1), we will now introduce a decomposition of total spreads as the sum of a credit and a liquidity component.\(^{16}\) Denote by $B, D$ the default and debt policies of the baseline calibration (i.e. the parameters listed in Table 1). The default component of the sovereign spread is defined as:

$$
\text{cs}_{\text{DEF}}(y, b') \equiv \text{cs}(y, b') \big|_{(B, D, 0)}.
$$

(3.2)

Thus, the default component of total spreads, $\text{cs}_{\text{DEF}}(y, b')$, is computed with a price that takes into account the default and debt policies of the baseline but the investor pricing these policies faces a zero probability of a liquidity shock. More precisely, the policies $(B, D)$ are the responses of a government that faces prices $q_{ND}^H(y, b') \big|_{(B, D, \zeta)}$ in the primary market. However, the bond price associated with $\text{cs}_{\text{DEF}}(y, b')$ is given by $q_{ND}^H(y, b') \big|_{(B, D, \zeta=0)}$.\(^{17}\) The liquidity component is then defined as the residual:

$$
\text{cs}_{\text{LIQ}}(y, b') \equiv \text{cs}(y, b') \big|_{(B, D, \zeta)} - \text{cs}(y, b') \big|_{(B, D, \zeta=0)}
$$

and accounts for the portion of the total sovereign spread that is not explained by the default component. These two definitions amount to decomposing sovereign spreads as:

$$
\text{cs}(y, b') = \text{cs}_{\text{DEF}}(y, b') + \text{cs}_{\text{LIQ}}(y, b').
$$

(3.3)

Now we delve into the numerical results. What portion of total annualized spreads is explained by each type of risk? The above decomposition is depicted in Figure 3.2. Panels A and B plot respectively, total spreads, credit risk premium, and liquidity premium for

\(^{16}\)The decomposition is analogous to the decomposition provided in He and Milbradt (2013) in the context of corporate bond spreads. As it will be clear in the next subsection, one important difference is that our decomposition takes into account the endogenous response of debt policy, while the decomposition from He and Milbradt (2013) is for a fixed debt policy.

\(^{17}\)The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account while choosing debt and default policies), individual investors are heterogeneous and in particular there may be some investors without liquidity concerns who discount at the risk free rate.
two levels of output; the values of $z$ are chosen to be $\pm 2$ standard deviations of the unconditional distribution for $z$. There are two features of this decomposition that are worth noting. First, panels A and B show that the liquidity component is increasing as the debt choice increases. This increase reflects a higher default probability combined with higher bid-ask spreads during default. Second, note also that as a percentage of total spreads, the liquidity component is sizable. In particular, when default risk is low (i.e. when output is high and/or debt levels are low) the liquidity component is predominant; one the other hand, as the overall default risk increases, the liquidity component as a fraction of total spreads becomes smaller, but still first order. For example, in panel B we can observe that the fraction of the total sovereign spreads attributable to liquidity is around 45 percent for a debt choice of 1 and a high level of output. On the other hand, close to default, for example when output is low and debt choice is around 2, is around 25 percent of total spreads. These magnitudes are in line with CDS-basis based calculations in Longstaff et al. (2005) and structural decomposition's in He and Milbradt (2013) for Corporate Bonds.

What is driving a large liquidity premium? We close this subsection with an explanation of where the liquidity premium is coming from. Suppose that the model is the same as the one developed in Section 2 but debt if fixed and the default probability is exogenous and given by $p_d$. In this case, from equation (2.6), the valuation of the unconstrained investor is given by

$$q_{ND}^H = \frac{(1 - p_d) \left( m + (1 - m) \left( z + \zeta q_{ND}^L + (1 - \zeta) q_{ND}^H \right) \right) + p_d \left( \zeta q_{D}^L + (1 - \zeta) q_{D}^H \right)}{1 + r_U}.$$

Define the liquidity premium of bonds in the primary market, $\ell_{ND}^H$, as the discount in addition to $r_U$ that an investor that only faces credit risk needs to impute in order to match the price $q_{ND}^H$, that is:

$$q_{ND}^H = \frac{1}{1 + r_U + \ell_{ND}^H} \left[ (1 - p_d) \left( m + (1 - m) \left( z + q_{ND}^H \right) \right) + p_d q_{D}^H \right].$$

Rewriting the previous expression we obtain that the liquidity premium is a combination of bid ask spreads

$$\ell_{ND} = (1 - p_d) (1 - m) \zeta q_{ND}^H - q_{ND}^L + p_d \zeta q_{D}^H - q_{D}^L.$$

Two observations. First, the value of the liquidity premium is pinned down by bid-ask

---

18In Appendix B we present this jump to default model in detail.
spreads. So, as long as the calibration has bid-ask spreads that are in line with the data, the liquidity premia that we obtain is disciplined by the friction we observe empirically. Second, there will be a positive co-movement between liquidity and credit premia as long as bid-ask spreads during the default are larger than bid-ask spreads before default. During bad times, liquidity premia will increase.

### 3.4 Liquidity, Policies, and Spreads

We now take a closer look into the two components of spreads, default and liquidity. We would like to answer quantitatively the following two questions. First, which are the determinants of the credit and liquidity components of sovereign spreads? Second, how much of the default premium is explained by liquidity frictions? How much of the liquidity premium is explained by default risk?

**A Closer look into the Default Risk Premium.** We start by analyzing the default component. In order to so, we further decompose $cs_{DEF}$ in two components:

$$cs_{DEF}(y, b') = cs_{DEF,DEF}(y, b') + cs_{LIQ\rightarrow DEF}(y, b').$$  \hspace{1cm} (3.4)

Denote by $B_0, D_0$ the debt and default policies for the baseline calibration when the probability of receiving a liquidity shock, $\zeta$, is equal to zero.\(^{19}\) These two terms are defined

\(^{19}\)More precisely, we solve for the optimal policies when the parameters are the ones Table 3.1, with $\zeta = 0$ instead of the baseline of $\zeta = 0.139$.

---

Figure 3.3: **Sovereign spread decomposition.** This figure plots the different components of the default credit spread for a fixed value of $y$ equal to 1.
as:

\[ cs_{\text{DEF}, \text{DEF}}(y, b') \equiv cs(y, b') \mid (B_0, D_0, \zeta = 0); \tag{3.5} \]
\[ cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \equiv cs(y, b') \mid (B, D_0, \zeta = 0) - cs(y, b') \mid (B_0, D_0, \zeta = 0). \tag{3.6} \]

The first term, equation (3.5), is the pure default component. This is the spread that the sovereign would pay if there were no liquidity frictions. That is, the investors do not receive liquidity shocks, and the policies are the ones of the baseline calibration with no liquidity shocks. The second term, equation (3.6), is the liquidity induced component of default spreads. This is the additional spread to the pure default component that is caused by a change in policies due to liquidity frictions. However, the change in spreads is only through a change in policies. Thus, we fix the probability of a liquidity shock at zero; i.e. \( \zeta = 0 \).

Depending on the response of debt and default policies when liquidity shocks are switched on, from \((B_0, D_0)\) to \((B, D)\), the component \( cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \) might be positive or negative. For example, fixing default policies, if due to liquidity frictions the government takes less debt, \( cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \) will be negative. Alternatively, fixing debt policies, if due to liquidity frictions the government defaults in more states of nature, then \( cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \) will be positive.\(^{20}\) To quantify the response of debt and default policies, it will be useful to to further decompose \( cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \), in two terms:

\[ cs_{\text{LIQ} \rightarrow \text{DEF}}(y, b') \equiv cs_{\text{LIQ} \rightarrow \text{DEF}, \text{Debt}} + cs_{\text{LIQ} \rightarrow \text{DEF}, \text{Def}}. \]

These two terms are defined as:

\[ cs_{\text{LIQ} \rightarrow \text{DEF}, \text{Debt}} \equiv cs \mid (B, D_0, \zeta = 0) - cs \mid (B_0, D_0, \zeta = 0) \tag{3.7} \]
\[ cs_{\text{LIQ} \rightarrow \text{DEF}, \text{Def}} \equiv cs \mid (B, D_\zeta = 0) - cs \mid (B, D_0, \zeta = 0). \tag{3.8} \]

The first term, equation (3.7), is the change in spreads due to changes in default policies, while keeping default policy fixed. This terms measures the response of spreads to changes in the frictions in the secondary market that go though debt policy. The second

\(^{20}\)The intuition is as follows. Suppose that the default policy remains fixed when the friction \( \zeta \) changes. On the one hand, an increase in the liquidity frictions might increase interest rates in the primary market, driving debt issuance down, and this might, in turn, imply a decrease of total spreads. On the other hand, this increase in the liquidity friction might induce the country to borrow even more, to sustain consumption, which in turn will exacerbate the increase in spreads caused by the liquidity friction. The same can occur with default policies: fixing debt policies, a change in the liquidity friction might induce the sovereign to default in a higher (lower) number of states of nature, which in turn imply higher (lower) spreads.
term, equation (3.8), is the change in spreads due to changes in default policies, keeping debt policy fixed, and measures the response in spreads due to default policy.

So, do liquidity frictions increase or decrease default risk? The overall response and each one of the components $c_{DEF, DEF}, c_{LIQ \rightarrow DEF}, c_{LIQ \rightarrow DEF, Debt}, c_{LIQ \rightarrow DEF, Def}$ are depicted in Figure 3.3 Panel B. Note that as we mentioned, in our calibration the component $c_{LIQ \rightarrow DEF, Debt}$ is negative. This is because worse liquidity conditions, given default policies, will imply a more precautionary debt policy. Also, note that the component $c_{LIQ \rightarrow DEF, Def}$ is always positive.

**A Closer look into the Liquidity Risk Premium.** We can further decompose the liquidity premium in two terms. That is:

$$c_{LIQ}(y, b') = c_{DEF \rightarrow LIQ}(y, b') + c_{LIQ, LIQ}(y, b').$$

(3.9)

Each one of the terms are defined as follows:

$$c_{LIQ, LIQ}(y, b') \equiv c(y, b') \mid_{(B_0, D=0, \zeta)}$$

(3.10)

$$c_{DEF \rightarrow LIQ}(y, b') \equiv c(y, b') \mid_{(B, D, \zeta)} - c(y, b') \mid_{(B, D, 0)} - c(y, b') \mid_{(B_0, D=0, \zeta)}.$$

(3.11)

The first component, equation (3.10), measures the spread when the investors can receive liquidity shocks but the government never defaults on debt. The second term, equation (3.11), measures how much in addition to the spread only due to liquidity shocks the government needs to pay in order to compensate for default risk. This term is a residual that measures the portion of the liquidity component that is not explained by the pure liquidity component, and this, by default risk. The Panel C of Figure 3.3 illustrates these two components. As one would imagine, the pure liquidity component does not vary with debt levels because this component is not sensitive to the default probabilities, which are affected by the output realizations. Note, however, the default induced liquidity component is increasing as debt increases and a default is more likely. This increase in the liquidity premia, as a consequence of higher default risk, just illustrates what was clear from the jump to default model: that default probabilities drive the variations in the liquidity premia.

**Relationship to Literature.** The previous two decomposition’s highlight one of the key difference’s of the sovereign setting with respect to the decomposition for the corporate setting in He and Milbradt (2013) and Chen et al. (Forthcoming). Note that from (3.4) and
(3.9), we can define total spreads as:

\[
s(y, b') \equiv cs_{DEF,DEF}(y, b') + cs_{LIQ\rightarrow DEF}(y, b') + cs_{DEF\rightarrow LIQ}(y, b') + cs_{LIQ,LIQ}(y, b').
\]  

(3.12)

This is precisely the decomposition of sovereign spreads studied in He and Milbradt (2013) and Chen et al. (Forthcoming). The main difference is that these papers feature a fixed debt policy and the outside option of default does not depend on future policies.\(^{21}\) Thus, in that setup, it can be shown that all four terms in (3.12) are positive. The reason is that the response of spreads to liquidity frictions that comes through a change in debt policies, \(cs_{LIQ\rightarrow DEF,Debt}\), is equal to zero. In addition, in both of these paper, the response to liquidity spreads that comes through default, \(cs_{LIQ\rightarrow DEF,Def}\), is an increase in spreads, because the corporation defaults in more states of nature, unambiguously, as a response of higher liquidity shocks.

### 3.5 Liquidity, Debt Capacity, Welfare

In this section, we examine implications of secondary market illiquidity for household welfare. We define welfare as the certainty equivalent consumption:

\[
c(h_c) \equiv u^{-1}\left(\mathbb{E}\left[V^C(y, b = 0; h_c)\right]\right)
\]

obtained by a sovereign with no debt initially \((b = 0)\) operating in an economy in which constrained international investors are subject to holding costs \(h_c\). We vary the illiquidity of secondary bond markets, as measured by bid-ask spreads, by varying the severity of liquidity shocks for international investors and consider the resulting welfare impli-
3.6 Case Study: Argentina’s Default in 2001

In this subsection, we conduct and event study of Argentina’s default in December of 2001. For the policy functions of the baseline calibration, depicted in Table 1, we feed in the process of output shocks that Argentina received in the 1990’s and its initial level of

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22The welfare cost of business cycles is substantially higher once deviations from the the CRRA framework in Lucas (2003) are introduced. See, for example Krusell et al. (2009), Barlevy (2004) and Alvarez and Jermann (2004). In that case, the welfare losses from liquidity frictions will be smaller.
First, note that our model can replicate the most salient features of the series of spreads. The first spike is the Tequila crisis in Mexico in 1996, where there is a sharp increase in the spreads coming from a recession. In addition, we can see that in 2001 the model can correctly account for the spike in spreads and default. Note that even though the recession started in 1998:III, spreads continued to be below 800 basis points until the beginning of 2001.

Second, in Panel C we can observe that that the liquidity premia is a larger portion of total spreads during good times, when total spreads are low. On the other hand, when the spreads started to spike in 1999, the liquidity component started to be a smaller portion of total spreads. In fact, in our calibration it could explain 20 percent given of the total spreads, during the period of the crisis.

### 3.7 Business Cycle Properties

The model’s business cycle properties are summarized in Table 4. The second column lists the empirical moments in the data, while the last column lists the results from Chatterjee and Eyigungor (2012) for comparison. Our calibrated model performs well. As in the data, consumption is as volatile as output and nearly perfectly correlated with output. The volatility of the current account relative to output volatility is 0.09 in the model which is close to its empirical counterpart of 0.17. The model does a good job of capturing counter-cyclical sovereign credit risk, with a correlation of -0.65 between the sovereign spread and output. In addition, there is a negative correlation of -0.50 between the current account and output. Finally, debt service (as a fraction of output) of 7.9 percent and the default frequency of 6.6 percent. Overall, the business cycle properties of the calibrated model is similar to that of the Chatterjee and Eyigungor (2012) model.

### 4 Conclusion

We studied debt policy of emerging economies taking into account credit and liquidity risk. To account for credit risk, we followed the quantitative literature of sovereign debt in studying an incomplete markets model with limited commitment and exogenous costs of default. To account for liquidity risk, we introduced search frictions in the market for sovereign bonds. By introducing liquidity risk in an otherwise standard model of
sovereign debt, default and liquidity risk are now jointly determined. To quantify the role of liquidity on sovereign spreads, debt capacity, and welfare, we performed quantitative exercises when our model is calibrated to match key features of the Argentinean default. We find that liquidity premia can be a substantial component of spreads, that given reasonable friction in the secondary market this risk premia increases during bad times, and that reductions in secondary market frictions would imply increases in welfare. The model can also account for key features of observed sovereign defaults and matches business cycle fluctuations in the data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
<th>CE (2012), Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.09</td>
<td>1.15</td>
<td>1.11</td>
</tr>
<tr>
<td>$\sigma\left(\frac{NX_y}{y}\right)/\sigma(y)$</td>
<td>0.17</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>$\text{corr}(c, y)$</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{corr}\left(\frac{NX_y}{y}, y\right)$</td>
<td>-0.88</td>
<td>-0.497</td>
<td>-0.44</td>
</tr>
<tr>
<td>$\text{corr}(r_{-rf}, y)$</td>
<td>-0.79</td>
<td>-0.65</td>
<td>-0.65</td>
</tr>
<tr>
<td>Debt service</td>
<td>0.053</td>
<td>0.079</td>
<td>0.055</td>
</tr>
<tr>
<td>Default frequency</td>
<td>0.125</td>
<td>0.066</td>
<td>0.068</td>
</tr>
</tbody>
</table>

Table 4: **Business cycle properties.**
References


_ and _, “Debt dilution and sovereign default risk,” Available at SSRN, 2015.


Appendix A: Numerical Method

It is well-known that numerical convergence is often a problem in discrete time sovereign debt models with long-term debt. To get around this problem, we adopt the randomization methods introduced in Chatterjee and Eyigungor (2012). This involves altering total output to be: \( y_t + \epsilon_t \), where \( \epsilon_t \sim \text{trunc } N(0, \sigma^2_{\epsilon}) \) is continuously distributed. As shown in Chatterjee and Eyigungor (2012), the noise component \( \epsilon_t \) guarantees the existence of a solution of the pricing function equation. The government’s repayment problem (2.3) is altered as follows:

\[
V^C(y, b, \epsilon) = \max_{y'} \left\{ (1 - \beta) u(c) + \beta \mathbb{E}_{y' \mid y} \left[ V^{ND}(y', b') \right] \right\},
\]

where the budget constraint is now given by:

\[
c = y + \epsilon - b \left[ m + (1 - m) z \right] + q^{H}_{ND}(y, b') \left[ b' - (1 - m) b \right]
\]

which contains the randomization component \( \epsilon \). Debt choice is denoted as \( b'(b, y, \epsilon) \). We impose that \( \epsilon_t \equiv 0 \) during the autarky regime so that the expression for the value to defaulting remains the same:

\[
V^D(y, b) = (1 - \beta) u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[ \theta V^{ND}(y', R(b)) + (1 - \theta) V^D(y, b) \right].
\]

The default decision is given by:

\[
d(y, b, \epsilon) = 1\{V^C(y, b, \epsilon) \geq V^D(y, b)\},
\]

and depends on the randomization component. The continuation value is adjusted:

\[
V^{ND}(y, b) = \mathbb{E}_{\epsilon} \left[ \max \left\{ V^D(y, b), V^C(y, b, \epsilon) \right\} \right].
\]
in order to take into account the randomization component. Finally, bond prices are also adjusted so as to take into account the additional randomization variable:

\[
q^H_{ND} (y, b') = E_{y', e'} | y \left\{ \frac{1-d(y', b', e')}{1+r} \left[ m + (1-m) \left[ z + \zeta q^H_{ND} (y', b', (b', y', e')) + (1-\zeta) q^H_{ND} (y', b', (b', y', e')) \right] \right] \right\} \\
q^L_{ND} (b', y) = E_{y', e'} | y \left\{ \frac{1-d(y', b', e')}{1+r} \left[ -h_c + m + (1-m) \left[ z + (1-\lambda) q^L_{ND} (y', b' (b', y', e')) + \lambda q^L_{ND} (y', b' (b', y', e')) \right] \right] \right\} \\
q^H_D (y, b) = \frac{1-\theta}{1+r} E_{y'|y} \left[ \zeta q^H_D (y', b) + (1-\zeta) q^L_D (y', b) \right] + \frac{\theta \mathcal{R}(b)}{b} q^H_{ND} (y, \mathcal{R}(b)) \\
q^L_D (y, b) = \frac{1-\theta}{1+r} E_{y'|y} \left[ -h_c + \lambda q^L_D (y', b') + (1-\lambda) q^L_D (y', b) \right] + \frac{\theta \mathcal{R}(b)}{b} q^L_{ND} (y, \mathcal{R}(b)) \\
q^S_{ND} (y, b) = (1-\alpha_{ND}) q^S_{ND} (y, b) + \alpha_{ND} q^H_{ND} (y, b) \\
q^S_D (y, b) = (1-\alpha_{D}) q^L_D (y, b) + \alpha_{D} q^H_D (y, b)
\]

The rest of the numerical scheme is standard and follows the routine outlined in Chatterjee and Eyigungor (2012). We summarize the scheme in 4 steps:

a. Start by discretizing the state space. This involves choosing grids \( \{y_i\}_{i=1}^{N_y} \) and \( \{b_j\}_{j=1}^{N_b} \) for output and debt. The grid points and transition probabilities for output is chosen in accordance with the Tauchen (1986) method and encompasses ±3 standard deviations of the unconditional distribution for output. In the baseline model the number of states for output is chosen to be \( N_y = 200 \). The grid points for debt values are uniformly distributed over the range \([0, b_{max}]\) with the upper limit \( b_{max} \) chosen large enough so as never to be binding in simulations. The baseline calibration has \( b_{max} = 6.0 \) and \( N_b = 450 \).

b. Next perform value function iteration. Given bond prices, update value functions \( V^C \) and \( V^D \). The debt and default policies \( b'(\cdot) \) and \( d(\cdot) \) are constructed using the algorithm outlined in Chatterjee and Eyigungor (2012). Where necessary, linear interpolation is used to obtain terms involving \( \mathcal{R}(b) \).

c. Given debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.
Appendix B: Jump to Default

In this Appendix, we spell out a particular case of our model in which default probabilities are exogenous. The idea is to introduce a clear definition of the liquidity premium, show how bid-ask spreads map into liquidity premium as a function of default risk, and to clarify the role of the features of the model in the results. Assume an unconditional constant default probability \( p^{LR} \) each period. Then the pricing equations yield a system of 4 equations and 4 unknowns \( \bar{q}^{H}_{ND}, \bar{q}^{L}_{ND}, \bar{q}^{H}_{D}, \bar{q}^{L}_{D} \). The system is given by

\[
\begin{align*}
\bar{q}^{H}_{ND} &= \frac{1}{1+r} \left[ (1 - p^{LR}) \left( m + (1 - m) \left( z + \zeta q^{L}_{ND} + (1 - \zeta) q^{H}_{ND} \right) \right) \\
&\quad + p^{LR} \left( \zeta q^{L}_{D} + (1 - \zeta) q^{H}_{D} \right) \right], \\
\bar{q}^{L}_{ND} &= \frac{1}{1+r} \left[ (1 - p^{LR}) \left( -h_{c} + m + (1 - m) \left( z + \lambda q^{H}_{ND} + (1 - \lambda) q^{L}_{ND} \right) \right) \\
&\quad + p^{LR} \left( -h_{c} + \lambda q^{H}_{D} + (1 - \lambda) q^{L}_{D} \right) \right], \\
\bar{q}^{H}_{D} &= \frac{1 - \theta}{1+r} \left( \zeta q^{L}_{D} + (1 - \zeta) q^{H}_{D} \right) + \theta f \bar{q}^{H}_{ND}, \\
\bar{q}^{L}_{D} &= \frac{1 - \theta}{1+r} \left( -h_{c} + \lambda q^{H}_{D} + (1 - \lambda) q^{L}_{D} \right) + \theta f \bar{q}^{L}_{ND},
\end{align*}
\]

where \( f \) denotes the fraction recovered. The solution to this system yields four value functions that depend on the unconditional default probability, \( p^{LR} \), and the parameters of the model. Fix the unconditional default probability \( p^{LR} \) and suppose there is a short run departure. In particular, the current default probability is \( p_{d} \). After a default, the default probability will remain fixed at the long run probability default \( p^{LR} \).\(^{23}\) Then,

\(^{23}\)Note that this process captures the idea of mean reversion in the hazard rates, that is common in the literature of credit risk modeling (see for example Longstaff et al. (2005)). Formally, we can think about a irreducible Markov chain with 2 states and transition matrix \( P \). The \( p^{LR} \) will be defined by the invariant distribution \( \Pi \).
current prices are given by

\[
q_{ND}^H = \frac{1}{1 + r} \left[ (1 - p_d) \left( m + (1 - m) \left( z + \zeta q_{ND}^L + (1 - \zeta) q_{ND}^H \right) \right) 
+ p_d \left( \zeta q_D^L + (1 - \zeta) q_D^H \right) \right] 
\]

\[
q_{ND}^L = \frac{1}{1 + r} \left[ (1 - p_d) \left( -h_c + m + (1 - m) \left( z + \lambda q_{ND}^H + (1 - \lambda) q_{ND}^L \right) \right) 
+ p_d \left( -h_c + \lambda q_D^H + (1 - \lambda) q_D^L \right) \right] 
\]

\[
q_D^H = \frac{1 - \theta}{1 + r} \left( \zeta q_D^L + (1 - \zeta) q_D^H \right) + \theta f q_{ND}^H(p^{LR}) 
\]

\[
q_D^L = \frac{1 - \theta}{1 + r} \left( -h_c + \lambda q_D^H + (1 - \lambda) q_D^L \right) + \theta f q_{ND}^L(p^{LR}). 
\]

Note that difference between the first system of four equations and the next one is that in the first one \( q_{ND}^H(p^{LR}), q_{ND}^L(p^{LR}) \) are taken as given. This will permit us to take limits on \( p_d \) on \([0, 1]\) and still have a well-defined system of equations.

**Bid Ask Spreads and Liquidity Premia.** How the observable frictions in the secondary market, bid-ask spreads, map into liquidity premia? The liquidity premium, \( \ell_{ND} \), is defined as

\[
\ell_{ND} = \left( 1 - p_d \right) \left( m + (1 - m) \left( z + q_{ND}^H \right) \right) + p_d q_D^H \]

where \( q_{ND}^H \) is given by (.1). We can then rewrite bond prices using the endogenous liquidity component as follows

\[
\ell_{ND} = \left( 1 - p_d \right) (1 - m) \zeta q_{ND}^H - q_{ND}^L + p_d \zeta \frac{q_{ND}^H - q_D^L}{q_D^H}. 
\]

Thus, \( \ell_{ND} \) is defined as the additional spread that is needed to explain a price \( q_{ND}^H \) if the investor has no concerns for liquidity. Two observations about this liquidity component. First, it is the expected valuation loss after taking into account a loss upon a having to sell and the probability of incurring a liquidity event and having to sell. Second, it depends on distance to default (or \( p_d \)); this is the case because the loss upon having to sell will be higher during a default episode.

**Understanding the model.** The jump to default model is also useful to highlight why each one of the pieces in the model is needed: long-term debt and frictions on the secondary market; positive recovery \( (R(b) > 0) \); worse liquidity conditions during a de-
fault ($\alpha_D < \alpha_{ND}$). First, note that long-term debt and frictions on the secondary market are needed in order to generate a bid-ask spread. That is, there has to be a friction on the secondary market, $\lambda \in (0,1), \alpha \in (0,1)$ and $m < 1$. Second, recovery is needed for the bonds to have a positive price during default. If this is not the case, bid-ask spreads are not defined. Third, worse liquidity conditions during default are needed to generate higher bid-ask spreads during default.

The next figure illustrates how the liquidity premium is changing with the default probabilities. We fix the parameters to: $z = 0.03$, $1 - \theta = 0.0385$, $r = 0.01$, $\zeta = 0.25$, $\alpha_{ND} = 0.8$, $\alpha_D = 0$, $R(b)/b = 0.3$. In addition we fix $p^{LR} = 0.03$.

In the first panel, we show the case in which there is long-term debt and frictions in the secondary market, but there is no recovery. This will imply that as we approach a default episode, the liquidity premium will actually decrease (in absolute value). The panel in the middle includes the case when there is a positive recovery value for the bonds. In this case there will be a positive bid ask spread, but it could well be the case that the bid ask spread during default is lower as is illustrated in the figure. This will imply that as we approach default, again, the liquidity premium will decrease. Finally, the panel on the right hand side illustrates the case in which $\alpha_D < \alpha_{ND}$. This implies that as we approach a default episode the liquidity premium will increase.