The Macroeconomics of Hedging Income Shares

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September 2018

PRELIMINARY – PLEASE DO NOT CIRCULATE

Abstract

Since the 1980's the US economy has experienced a decline in the labor share, falling real rates, and accumulation of safe assets in the corporate sector. To study these facts, we propose a neoclassical growth model with capital-biased technological change, a production function with non-constant income shares (CES), and financial friction for firms. We discuss theoretically how risk sharing is distorted by the combination of changing shares of income and financial frictions, and how a hoarding of safe assets by firms emerges naturally as a tool to improve risk sharing. We calibrate our model to the US economy after 1980's and show that low rates, rising capital shares, accumulation of safe assets by firms and risky assets by households, can be rationalized by persistent capital-biased shocks and limited risk sharing.

Keywords: Decreasing labor share. Risk Sharing. Asset prices.

*We would like to thank Vasco Carvalho, Sebastian Di Tella, Claudio Michelacci, Dejanir Silva, and seminar participants at the European Central Bank, EIEF, Paris School of Economics, Universidad Di Tella, Cambridge University, Universidad Carlos III, University of Cologne, CORE 2018 meetings, SED Meetings at Edinburgh, for helpful comments and suggestions. We thank Maria Tiurina for excellent research assistance. All remaining mistakes are ours.

1 Introduction

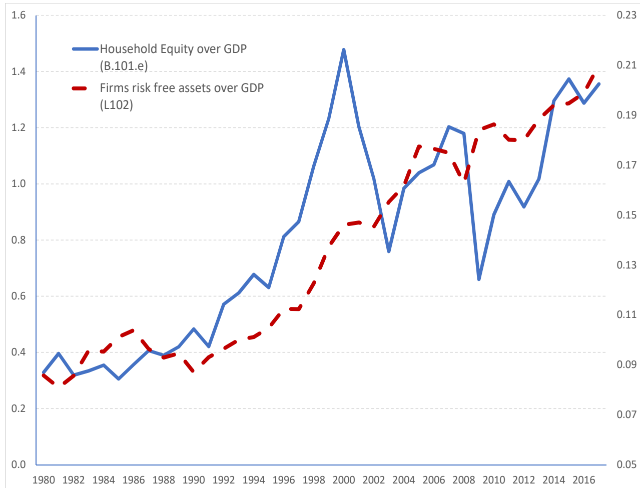
The prevalence over time of the Kaldor facts led to the dominant belief that the capital and labor income shares were, besides some small short run variations, roughly constant. This has many implications, among those is the impossibility of insurance against aggregate risk between workers and capitalists. Since aggregate shocks affect both sectors in the same way, even if it were possible, aggregate risk would be uninsurable. However, many recent studies find that the labor share seems to be varying far more than the Kaldor's prediction, maybe even having a downward trend.[1] But, if aggregate shocks have differential impacts on capitalists and workers, new possibilities of insurance arise. How does these insurance possibilities affect the financial markets? In particular, what kind of assets could be affected? Last but not least, how quantitatively important are its implications?

Since the labor share is pro-cyclical in the short run and counter-cyclical in the medium-long run, we need to take a stand on what horizon we have in mind. Because the downward trend appears to be the dominant feature, we focus on the long run. We first show theoretically that the counter-cyclical changes on the labor share (pro-cyclical capital share) can be insured between capitalist and workers by capitalist accumulating large quantities of risk free assets, lending it to the workers, and then workers using the loans to leverage and buy large quantities of risky assets (equity). Further, because, capitalists are subject to more uninsured idiosyncratic risk than workers, the increase in the capital share acts as an increase of uninsured idiosyncratic risk, increasing the demand of savings for precautionary savings, which in turn decreases the risk free interest rate. We show that this is not only a theoretical possibility but is also quantitatively relevant and analyze its normative implications.

It is interesting that all the predictions of this straightforward channel are present in different strands of literature. For instance, there is a growing literature trying to account for what now is known as the "Corporate savings glut", changing the view of corporations as net borrowers to net lenders. Our theory, has the additional implication that it must be accompanied by a "household equity glut." Indeed, if we look at the portfolio choices of both households and corporations since the 80's, we see, as we shown in Figure 1 that both the holdings of debt securities by corporations and household's equity holdings (both direct and indirect) have almost triple from 1980 to 2014. Also, the continuously falling interest rate have been widely documented. Explanations for this falling rates

[1] See Karabarbounis and Neiman (2014) and Koh et al. (2017) for two recent studies pointing out to the downward trend.

range from demographics, passing from a secular stagnation, to a sudden increase on uncertainty.[2]



US Data. Source: Flow of Funds

We build on the neoclassical growth model with constant elasticity of substitution technology (CES) and capital intensity that fluctuates persistently over time. The economy is populated by a continuum of entrepreneurs with different endowments of capital and households-workers. Entrepreneurs own the capital, rent labor from households and carry over the production. Households supply labor inelastically and potentially funds to the firms through the financial markets. There is a contracting friction for the entrepreneurs. As in DeMarzo and Fishman (2007) the returns of the entrepreneur cannot be verified as entrepreneurs can privately divert some resources for consumption. Firms would like to share risk with households and obtain funding, but they are subject to a "skin in the game" constraint. This constraint implies that the households require the entrepreneurs to keep a fraction of their investment in order to deter them from diverting funds to a private accounts. There are enough financial instruments available such that both entrepreneurs and households can hedge the aggregate risk. However, the financial friction prevents capital owners from fully insuring against idiosyncratic risk, which in turn affects their willingness to bear aggregate risk.

Notice that an important departure from the standard macro-finance literature is the CES technology rather than Cobb-Douglas. This assumption allows us to study the effects of changing labor and capital income shares over times. We assume that the economy is subject to capital-biased technological shocks, and, since we are focusing on the medium-long run, that the elasticity of substitution is bigger than one, as supported by recent

[2]See for example Karabarbounis and Neiman (2014)and Caballero et al. (2017a), Carvalho et al. (2016) and Summers (2013, 2014).

empirical studies; in particular, Koh et al. (2017) and Karabarbounis and Neiman (2014). Therefore, positive aggregate shocks and growth are correlated with a increasing capital income share.

We start by showing the optimal risk sharing. We show that if entrepreneurs are able to fully insure their idiosyncratic risk, there is efficient risk sharing of aggregate risk between capital holders and workers. For instance, if the capital income share increases along the business cycle, entrepreneurs compensate workers with contingent transfers. The opposite is true if the labor share increases. The complete markets (only for aggregate risk) assumption is an abstraction that help us to characterize the solution, but which do not see as restrictive. In reality, there are multiple types of financial assets that have payoffs correlated with realizations of aggregate shocks and do not rely and/or are constrained by individual moral hazard or commitment problems. In principle, by properly combining these assets, any individual could replicate the same insurance target as a complete set of Arrow-Debreu securities will do. In fact, it is well known, that one only needs as many (not perfectly correlated) assets as possible shocks exist to achieve the best insurance possible.[3] Because of this, we consider the straightforward and intuitive case in which at any given period there are only two possible realizations of the aggregate shock, so that a risk free asset and a risky asset suffice to implement the complete markets allocation.

When we analyze this implementation, we find that it is implemented with firms taking a long position (saving) on the risk free asset and households taking a long position on the risky asset (buying equity). Intuitively, households leverage (borrowing from firms) and buy shares on corporations to participate on the changes in the income shares. This market allocation is reminiscent of a corporate savings glut, which is complemented with a households-workers equity glut. However, precisely because markets are complete, all wealth effects are absent, thus in a stationary economy all financial positions and asset prices should be constant and independent of history.

On the other end, if the idiosyncratic risk is present (because the moral hazard friction is relevant) but the labor income share is constant, aggregate risk sharing is not longer possible, and thus, aggregate net holdings of assets become degenerated. However, the presence of idiosyncratic risk brings about wealth effects that can distort inter-temporal choices. We show that if the production function exhibits a elasticity of substitution equal

[3]The number of assets needed to implement the complete market allocation is reduced as the frequency of trades increases. For instance, as the frequency of trades becomes infinitesimal and the underlying risk is characterized by a Brownian motion (a continuous time economy), only two assets are needed; the Brownian motion can be approximated as the limit of a Bi-nomial process. See for example Merton, 1992 and Stokey, 2008.

to one (Cobb-Douglas), because it is not possible to insure aggregate risk, the risk free asset is not traded in equilibrium and the volatility of financial assets is efficient. The market incompleteness, due to the financial friction, does generate an inefficient level of output and consumption, but does not affect the reaction of the economy to aggregate shocks.

When idiosyncratic and aggregate income shares risk coexist, there are important wealth effects that distort the efficient allocation of insurance. First, we show that the presence of idiosyncratic risk generates precautionary savings needs that dampen the incentives to insure aggregate risk, reducing the financial positions that workers and capitalists would have otherwise chosen. However, the idiosyncratic risk generates an additional channel: a wealth effect. As opposed to the efficient economy, where relative wealth's are unaffected, because firms are more exposed to idiosyncratic risk than households, as the capital income share increases the aggregate demand for insurance also rises. In addition, since the main net suppliers of funds for insurance are households, the implied decline in the labor share decreases the supply of insurance. At first sight, it looks like the aggregate shock generates time varying uncertainty. However, it is important to notice that from each individual entrepreneur's perspective the uncertainty remains constant. It is the aggregate weight on the different agent's exposure to uninsured risk what changes. Thus, if positive aggregate shocks increase the capital income share, there is an increase in the aggregate demand for insurance that has two important consequences: a fall of the risk free interest rate (increase in the price of insurance) and an increase in the demand of safe assets on part of the firms. Both predictions are consistent with the important recent developments in the US economy.

As we do in the efficient benchmark, we can also implement this equilibrium with only a risk free and a risky asset. As one might have expected from the previous discussion, the tendency towards capitalists accumulating risk free assets and workers accumulating equity remains, in spite of the additional precautionary savings that significantly temper the tendency toward efficiency. But in addition, the wealth effects generate an additional channel that amplifies the reallocation of aggregate portfolios over time. All in all, the economy generates a large positive correlation between firms' long position on the risk free asset and the capital income share. On the other side, households increase their leverage, borrowing from firms, and increasing their positions on the risky asset. It is important to emphasize that the key difference between the complete markets economy and the incomplete markets is the wealth effect. In the efficient economy, the realization of aggregate shocks do not alter the split of the total wealth between households and entrepreneurs. Instead, when capitalists are exposed to idiosyncratic risk, to self insure firms hold inefficiently high levels of the risk free asset, thus when positive aggregate shocks happen, not only the capital share increases, but also the wealth distribution tilts in favor of capitalists.

Literature Review. We build on the findings of decreasing labor shares, capital-specific productivity, and increased corporate savings. In particular, Karabarbounis and Neiman (2014) explore thoroughly the declining global labor share and find evidence consistent with earlier work by Blanchard et al. (1997), Blanchard and Giavazzi (2003), Jones et al. (2003), and Bentolila et al. (1999). Examples of broader studies of trends in labor shares are Harrison (2005) and Rodriguez and Jayadev (2010). Also, our paper relates to the literature on capital-specific productivity change (Greenwood et al., 1988, Hsieh and Klenow, 2007, Greenwood et al., 1997, Fisher, 2006). The rise in Corporate savings has been documented in Sánchez et al., 2013, Chen et al., 2017 and recently discussed in Begenau and Palazzo, 2017.

Our paper is related to the literature on the amplification of shocks in the macroeconomy from the seminal works of Bernanke and Gertler (1989) and Kiyotaki et al. (1997). These two papers spurred a literature that place at the center of the stage financial frictions as amplifiers of business cycle fluctuations. Our paper relates to the recent contributions of He and Krishnamurthy (2011), Brunnermeier and Sannikov (2014), DiTella (2017) where financial frictions and heterogeneity play a key role in determining allocations (see also, Silva, 2016 and Gomez et al., 2016). We depart from some of the previous contributions by introducing labor income and one the production side a CES technology and workers that obtain labor income. This assumption allows us to study the effects of changing labor and capital income shares over asset prices and quantities. Further, we propose a discrete time time framework and a numerical algorithm to find a global solution.

Our paper related also to a recent literature that connects low risk free interest rates, risk-premia, changes in the labor shares, as well as pother. Caballero et al. (2017a) proposes an accounting framework that relates falling short term real rates, constant marginal product of capital, the decline of the labor share and a stable earnings yield from corporations. See Caballero et al., 2017b for a discussion focused on the shortage of safe assets. Carvalho et al., 2016 proposes a demographic explanation for the evolution of real rates.[4]

[4]Our paper also speaks to some of the facts that have been labeled as the secular stagnation. The key idea of secular stagnation, a notion dating back to Hansen (1939), is that the real interest rate needed to achieve full employment is negative and this casts on the economy the shadows of low growth and high unemployment. From a theoretical point of view, Eggertsson and Mehrotra (2014) was one of the first attempts to formally model secular stagnation. Since , there has been an increasing number of papers aimed

Our paper focuses on imperfect risk sharing and capital-biased technological change as the mechanism driving the hoarding of assets by corporations and implying low rates. Also, we uncover that on the other side of these increased holdings, consumers are holding a higher fraction of aggregate risk, consistent with the recent US experience.

2 Main Mechanisms

2.1 A Two Period Economy

In this section we characterize a two period model that highlights the key economic mechanisms generating the results. In Section 3, we develope a general version of this economy to perform quantitative analysis. There are two types of agents, household/workers and entrepreneurs. The economy last for two periods, $t = 1, 2$. There are two sources of uncertainty, aggregate shocks, indexed by s, and idiosyncratic production shocks, indexed by i. In this paper we will assume that s is capital-biased technological shock, and as such it will affect the capital and labor income shares. For simplicity, assume that in the two period model time is not discounted.

Consumers. Households are endowed with initial assets W_1 and can supply, with no utility cost, the labor endowment at wage ω_1. The state in period 1 is known, but households would like to insure the realization of the aggregate state in the second period. To do this, the consumer has access to a complete set of Arrow-Debreu securities, $W(s)$, contingent on state s, which can by bought or sold at prices $p(s)$. Finally, in the second period consumers receive, contingent on the realization of the aggregate shock, $\omega(s)$ as labor income. Thus, the consumer maximizes expected utility:

$$\max_{\{c_1, c_2(s), W(s)\}} u(c_1) + \mathbb{E}_s(u(c_2(s)))$$

in understanding secular stagnation, including Kocherlakota (2013), Benigno and Fornaro (2016), Schmitt-Grohé and Uribe (2012), Caballero and Farhi (2014), Eggertsson et al. (2016), and Marx et al., 2018. With the exception of Kocherlakota (2013) and Marx et al., 2018, all the previous papers study the subject including nominal wage rigidities. Instead, the labor market in our design is totally flexible, so much that households can completely insure against idiosyncratic labor income shocks, and there is no unemployment. The mechanism we derive stems from a risk taking motive due to changes in the relative share of income that goes to capital.

subject to the budget constraints

$$c_1 + \sum_s p(s)W(s) \leq W_1 + \omega_1 L$$
$$c_2(s) \leq W(s) + \omega(s).$$

Note that these two constraint hold for all s. The consumer has access to a full set of Arrow Securities to insure aggregate shocks.

Entrepreneurs. Are born with initial financial assets E_1 and capital K_1. Since they hold all the capital in the economy, they are the only agents allowed to use it for production. Individual capital is subject to idiosyncratic risk. The entrepreneurs would like to share this risk with the consumers, but they are prevented to do so because of a "skin in the game constraint" which forces them to keep a fraction of their investment in their balance sheet so that they cannot divert it to private accounts. As consumers, entrepreneurs can buy a complete set of Arrow-Debreu securities $E(s)$, but only contingent on s, the i shocks remain uninsured. The problem of the entrepreneur is:

$$\max_{\{e_1, e_2(s,i), E(s)\}} \{u(e_1) + \mathbb{E}_{s,i}(u(e_2(s,i)))\}$$

$$\text{st.;} \quad e_1 + \sum_s p(s)E(s) \leq E_1 + \pi_1$$
$$e_2(s,i) \leq E(s) + \pi(s,i), \qquad \forall s, \forall i$$

where π_1 represents a certain capital income at $t = 1$ and $\pi(s,i)$ is the $t = 2$ uncertain profit from capital.

Bankers: Technology. Entrepreneurs combine labor and capital to produce using a CES production function:

$$y(k(s,i), L) = y(s,i) = [\alpha\, (k(s,i))^{\frac{\rho-1}{\rho}} + (1-\alpha)L^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}$$

with the Cobb-Douglas as a special case when $\rho = 1$. It is straightforward to show that with this production function the static problem of maximizing profits, $\pi(s,i)$, given individual capital $k(s,i)$,

$$\pi(s,i) = r(s)k(s,i) = \alpha(s)y(s,i)$$

where $\omega(s) = (1-\alpha(s))\frac{Y(s)}{L}$ and $Y(s) = \mathbb{E}_i y(s,i)$. Also, note that:

$$\alpha(s) = \alpha \left(\frac{Y(s)}{K(s)} \right)^{\frac{1-\rho}{\rho}}. \tag{1}$$

Since the exogenous shocks affect capital directly, $k(s,i) = g_i g_s K$, the entrepreneur's budget constraint is linear in both individual capital and the idiosyncratic shock, which greatly simplifies the entrepreneur's dynamic optimization problem. Later, in Section 3.2, when we consider the infinite horizon model with depreciation rate δ, to maintain the linearity of the entrepreneur's budget constraint, we assume that the exogenous shocks also affect depreciation. Therefore we define the gross return on capital: $R(s) = (1-\delta)g_i + r(s)g_i$.

Notice from equation (1) that when $\rho = 1$, we are back to the standard Cobb-Douglas case with constant shares, i.e., $\alpha(s) = \alpha$, for all s. In our environment, instead, the capital share will change with aggregate shocks. $\alpha(s)$ would be pro-cyclical if $\rho > 1$ and counter-cyclical if $\rho < 1$. We start assuming that capital is exogenous.[5] Also, let $\Pi(s) = \mathbb{E}_i(\Pi(s,i))$ and $K(s) = \mathbb{E}_i(k(s,i)) = g_s K_1$. Therefore, g_s represents the aggregate shock and $P(s)$ its probability.

Bankers: Contracting. Since entrepreneurs are subject to idiosyncratic risk, they will try to insure it. To that end, we assume that entrepreneurs have access to risk neutral intermediaries who can provide insurance. However, entrepreneurs are subject to moral hazard problems, which puts a limit on how much idiosyncratic risk can be offloaded. To be precise, following the literature we model moral hazard as endowing entrepreneurs with the possibility of diverting resources from the firm to their private accounts at a cost $0 < 1-\psi < 1$. That is, for each unit of profits that they divert to their private accounts only ψ units transform into consumption goods (or savings). We analyze a setup, similar to DeMarzo and Fishman (2007), where a risk neutral principal provides insurance to the entrepreneurs. The contract stipulates that the entrepreneur must hand over to the financial intermediary a given proportion of her risky profits and in return receives an average of the profits of all firms. Since entrepreneurs can misreport their profits and consume (or save) a proportion ψ of the misreported profits, in Appendix C we show the optimal contract implies that the entrepreneur must retain (or being exposed) to a proportion ψ of the idiosyncratic risky. This is known in the literature as the "skin in

[5] This simplifies the proofs; in Section 3.2, when we analyze the infinite horizon economy, we introduce investment.

the game".[6] As result, we can write the exposure to the idiosyncratic risk in a simple reduced form way. Let $\tilde{g}_i$ be the productivity shock to which the firm is exposed. Then, the economy with idiosyncratic risk $\tilde{g}_i$ and possibility of insurance is equivalent to an economy in which there is no possibility of insurance of individual risk and firms are subject to idiosyncratic risk g_i satisfying:

$$g_i = (1-\psi)\mathbb{E}_i\tilde{g}_i + \psi\tilde{g}_i.$$

Markets. Finally, since we are analyzing general equilibrium, market clearing requires:

$$c_1 + e_1 = Y_1 \tag{2}$$
$$c_2(s) + \mathbb{E}_i(e_2(s,i)) = Y(s);\;\; \forall s \tag{3}$$
$$W(s) + E(s) = 0;\;\; \forall s \tag{4}$$
$$1 = L^s = L^d$$

The first constraint is market clearing in period 1. It implies that the initial asset holdings are such that $W(1) + E(1) = 0$. The second constraint is market clearing in period 2; note that the i.i.d. shocks wash out in the aggregate. The final two constraints specify assets and labor markets clearing, respectively. A *Competitive Equilibrium* is an allocation of consumption and labor $\{c_1, e_1,\, c_2(s), e_2(s,i), L\}_{s\in S, i\in \boldsymbol{I}}$, asset holdings $\{W(s), E(s)\}_{s\in S}$, asset prices $\{p(s)\}_{s\in S}$ and wages $\{\omega(s)\}_{s\in S}$ such that: given prices the consumer maximizes utility by choosing asset holdings and consumption; given prices the entrepreneur maximizes utility by choosing asset holdings, labor, and consumption.

General Equilibrium Conditions. Lets start by deriving the optimality conditions for consumers and entrepreneurs. Taking the first order conditions in both problems, we obtain:

$$p(s)u'(c_1) = Pr(s)u'(c_2(s))$$
$$p(s)u'(e_1) = Pr(s)\mathbb{E}_i[u'(e_2(s,i))]$$

[6] DeMarzo and Fishman (2007) assumes that the principal can sign long term contracts (there is commitment) and that both the principal and the agent are risk neutral. In contrast, we consider a risk averse agent who can only commits to short term contracts. For similar setups and results in continuous time see DeMarzo and Sannikov (2006). We also show in the Appendix that as long as insurance contracts cannot be history dependent, this is the best possible insurance independently of whether entrepreneurs have access or not to hidden savings. This contract is akin to an equity contract where the entrepreneur creates a company issue equity for a value amounting to a proportion $1-\psi$ of the ex-ante value of the company and retains a proportion ψ of the shares.

A key element of the above equations is that, due to the existence of complete set of securities for the aggregate state, the Euler equation holds state by state; for the aggregate state $s \in S$. The two of them together imply:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[u'(e_2(s,i))]}{u'(c_2(s))} \tag{5}$$

Note that equation (5) states that the ratio of average marginal utilities is constant in equilibrium. We now re-express the Arrow-Securities as claims on aggregate output. In particular, let $\phi(s)$ be such that:

$$\begin{aligned} A(s) &= \phi(s)Y(s) \\ E(s) &= -\phi(s)Y(s). \end{aligned}$$

Let x be worker's wealth over total wealth. Denote by x_1 this ratio at $t = 1$. Then it can be show that it holds that:

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} = \frac{\mathbb{E}_i[(\overbrace{-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i}^{E(s)+\pi(s,i)})^{-\sigma}]}{(\underbrace{\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s)}_{a(s)+w(s)L})^{-\sigma}}; \quad \forall s. \tag{6}$$

2.2 Risk Sharing with Complete Markets.

The first question we answer is: how agents share risk in this Economy when there are no contracting frictions? For that, we can either assume that the idiosyncratic shock can be fully insured, i.e. $\psi = 0$, or similarly, that there is no idiosyncratic risk, i.e. $\mathbb{V}(\tilde{g}_i) = 0$; either case results in $g_i = 1 \ \forall i$.

Proposition 1. *For each first period equilibrium wealth ratio x_1, if entrepreneurs can insure their idiosyncratic risk, the competitive equilibrium is characterized by*

$$c_1 = x^{CM}Y_1, \quad e_1 = (1-x^{CM})Y_1, \quad c_2(s) = x^{CM}Y_2(s), \quad e_2(s) = (1-x^{CM})Y_2(s) \tag{7}$$

together with the prices: $p(s)^{CM} = \Pi(s)\left(\frac{Y_1}{Y_2(s)}\right)^{\sigma}$, $\omega(s)^{CM} = (1-\alpha(s))Y_2(s)$. *Asset holdings are* $W(s) = -E(s)$, $W(s) = \phi^{CM}(s)Y(s)$ *with*

$$\phi(s)^{CM} = x^{CM} - (1-\alpha(s)); \quad \forall s \tag{8}$$

Proof. See Appendix A.2 . □

There are two points that are worth noting from Proposition 1. First, consumption shares are constant over aggregate states. This is a standard result.[7] It is useful to keep this allocation in mind because it generates the efficient reaction of the economy to aggregate shocks, and as such, it is a natural benchmark. In this equilibrium the entrepreneurs fully compensate the workers with contingent payments when the capital income share increases. This compensation is through Arrow-Debreu securities.

Second, note that if α is constant, i.e. $\rho = 1$, the capital income share does not vary over the business cycle and neither do the positions on the Arrow-Debreu securities of the agents ($\phi(s)^{CM} = \phi^{CM}\ \forall s$). Intuitively, it would be pointless to write contracts contingent on the aggregate state as both types of agents equally suffer the consequences of aggregate shocks and therefore there are fixed gains from trading financial assets. When the capital share varies, the workers and entrepreneurs are affected in different ways by the shocks and therefore trading financial assets contingent on the aggregate state can make everyone better off.

The question that remains, and the second one, is: can we implement this insurance arrangement? If we can, how? Achieving this kind of insurance arrangements may sound troublesome, but we show below that one can implement this equilibrium with only non-state contingent assets. In fact, the complete markets assumption can be relaxed if we assume that there are as many types of financial assets as aggregate shocks. In particular, in an economy with only two shocks, we will only need a risk free asset and an asset indexed by the returns of the production sector.

Implementation with two assets. Suppose there are only two aggregate shocks $s_L < s_H$, and two financial assets, a risk free bond B and a stock-market-indexed risky asset A with payoff $A\alpha(s)Y_2(s)$ for $s = L, H$. The risk free rate is denoted by R_L, and P_A denotes the price of the risky asset. $\{A^c, B^c\}$ is the portfolio allocation of the consumers and $\{A^e, B^e\}$ is the portfolio allocation of the entrepreneurs. In Appendix A.5 we show that the equivalence is given by:

$$R_L^{CM} B^{CM} = -\frac{Y_2(L)Y_2(H)(\alpha(H) - \alpha(L))(1 - x^{CM})}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{9}$$

$$A^{CM} = 1 - \frac{Y_2(H) - Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}(1 - x^{CM}) \tag{10}$$

[7] Note that here we are using the change of state variable to express the consumption share as a function of the initial ratio of marginal utilities, m_1, rather than the initial distribution of assets W_1/E_1. ext

Notice that if $\alpha(H) \neq \alpha(L)$ the household will take an active position in the risk free asset. If the position is positive or negative depends on the value of ρ, the EIS in production. If $\rho > 1$, the positive productivity shocks are associated with higher α. In this case, the household would take a negative (short) position on the riskless asset and a positive (long) position on the risky asset. Intuitively, she would borrow in the risk free asset to participate in the gains of the entrepreneurs should there be a positive shock to the capital share. This in turn means that the entrepreneurs are holding a positive amount of the risk free asset, and that this amount increases with $1 - x^{CM}$, the share of their own consumption. Alternatively, if $\alpha(H) = \alpha(L)$, i.e. we have a Cobb Douglas production function, the households do not hold risk free assets and the amount of risky assets that are traded depends on the initial distribution of assets, $A = 1 - \frac{1-x^{CM}}{\alpha}$. In particular if $E_1 = W_1 = 0$, then $A = 0$ and there is no asset trading. Instead, if $A \neq 0$ either the entrepreneurs or the households want to transfer resources across time. They do so by using the risky asset, not the risk-less asset.

Take Out and What Follows. We now want to understand how capital biased technological shocks affect portfolio allocations and prices over time. A useful setting is an economy in which there is no growth and each period there are only two possible outcomes for output (so that each period only the pairs $\{Y(L), \alpha(L)\}$, $\{\alpha(H), Y(H)\}$ can happen on the equilibrium path). Suppose we replicate this economy for more than two periods, then the portfolio allocation of the consumer (and, consequently, of the entrepreneur) is constant. Moreover, the price of the risky asset P_A is constant and therefore we have a constant risk premia. We will see later that if output is stationary but capital biased technological shocks are such that capital shares increase over each replication there will be important consequences for the behaviour of prices and quantities with the cycle.

2.3 Risk Sharing and Incomplete Markets

Now consider the case in which idiosyncratic risk is present. The first result is to show that, even when the entrepreneurs cannot fully insure this risk, efficient risk sharing can still be achieved when the income shares are constant. Instead, when the labor share varies with the cycle, this variation in addition to the presence of idiosyncratic risk distorts the allocation of risk.

Proposition 2. *Assume the entrepreneurs have to bear some idiosyncratic risk. If the capital share is constant along the business cycle, i.e. $\rho = 1$, for each first period distribution of wealth x_1, then* *equation (6) is given by*

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} = \frac{\mathbb{E}_i[(-\hat{\phi}+\alpha g_i)^{-\sigma}]}{(\hat{\phi}+(1-\alpha))^{-\sigma}}; \quad \forall s$$

and its solution, $\hat{\phi}$, is constant state by state. .

Proof. See Appendix A.3 □

The constancy of the capital and labor shares implies that, in spite of the idiosyncratic risk, the future consumption shares of the agents are also independent of the aggregate state. In addition, more importantly, this result implies that consumption shares are also constant across time. Intuitively, it is optimal for consumers and entrepreneurs not to share the idiosyncratic risk, and thus, they both bear the full effect of the aggregate risk. If consumption were to vary this would add additional variation to the consumption of both agents that is clearly sub optimal. Since the consumption share, x_1, is also constant in the complete markets equilibrium, namely x^{CM}, this implies that we can always find an initial ratio of marginal utilities (or, equivalently, an initial distribution of wealth) that will make both economies equivalent. For a given distribution of wealth, one economy would be just a proportion of the other. In other words, when the labor share is constant along the business cycle, even when markets are incomplete and there is uninsured idiosyncratic risk, the stochastic response of the economy resembles that of an efficient economy. In short, if there is no idiosyncratic risk, with or without varying labor shares the economy is efficient, and when there is idiosyncratic risk but the labor share is constant, the economy is also efficient.

One of the results in our paper is that a combination of rising capital shares, coming from a capital biased technological change, and idiosyncratic risk will generate distortions in risk sharing. These distortions will generate excessive precautionary savings and, as a consequence, lower interest rates. To gain some intuition we can perform a second order Taylor approximation of the right hand side of (6) around the complete markets solution to obtain:

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} \simeq \frac{(-\phi(s)+\alpha(s))^{-\sigma}}{(\phi(s)+(1-\alpha(s)))^{-\sigma}}\left(1+\frac{\sigma(1+\sigma)\alpha(s)^2}{(-\phi(s)^{IM}+\alpha(s))^2}\frac{\mathbb{V}ar(g_i)}{2}\right); \quad \forall s.$$

We can see that the main difference between the incomplete market and the complete market economies is given by the last term, which is multiplicative in $\mathbb{V}ar(g_i)$, the measure of idiosyncratic uncertainty. More importantly, this term is increasing in $\alpha(s)$. For a given level of idiosyncratic risk, the larger the capital share in the economy, the larger the demand for insurance. At the same time, as $\alpha(s)$ increases, there are less resources available to the workers, and therefore the supply of funds for insurance purposes also decreases. Notice that what creates the different asset positions is the fact that the economy looks like it is exposed to time varying idiosyncratic risk. The larger $\alpha(s)$, the larger the individual risk. However, the mechanism is different; from the perspective of each individual entrepreneur the idiosyncratic risk, $\mathbb{V}ar(g_i)$, remains constant, what happens is that the share of "risky income" over total income increases, and so does the difficulty to insure it. Therefore, we have:

Proposition 3. *For a given initial level of the ratio of marginal utilities, if* $\mathbb{V}ar(g_i) > 0$, *and* $\rho \neq 1$, *then:*

a. $\phi(s)^{IM} < \phi(s)^{CM}$, for all s. (Precautionary savings)

b. ϕ^{IM}, ϕ^{CM} is increasing in α. (Decreasing risk sharing)

c. $\frac{\partial \phi(\alpha)^{IM}}{\partial \alpha} < \frac{\partial \phi(\alpha)^{CM}}{\partial \alpha} = 1.$ (Increasing precautionary savings)

Proof. See Appendix A.4 □

The first part of Proposition 3 states that entrepreneurs hold some precautionary savings (recall that $\phi(s)$ is the position of consumers). The second part of Proposition 3 states that as the differnces in income shares widen so do the the positions on Arrow-Debreu securities demanded in equilibrium. This is true, independently of the existance of uninsured idiosyncratic risk. The last part states that the demand for precautionary savings increases with $\alpha(s)$. This demand is measured as the difference between the asset positions that the entrepreneur holds in the incomplete markets (IM) economy and the position that it would hold if markets were complete. This also implies that the inefficiency of the reaction of the economy to aggregate shocks increases with $\alpha(s)$.

The statements of Proposition 3 are depicted in Figure 1. In the left panel we can see that, in both the complete and incomplete markets economies, workers share the risk of the income shares. Also, in both cases, the trade of financial assets decreases with $\alpha(s)$. Finally, the positive position of the entrepreneurs (negative position of the workers) is larger in the incomplete markets economy. However, part 3) of Proposition 3 is not immediately apparent from the left panel. To clearly show this part, in the right panel we plot exactly the same values as in the left panel, but we have translated the line ϕ^{IM} to coincide with ϕ^{CM} in the first point. After correcting for this "level effect", it is evident

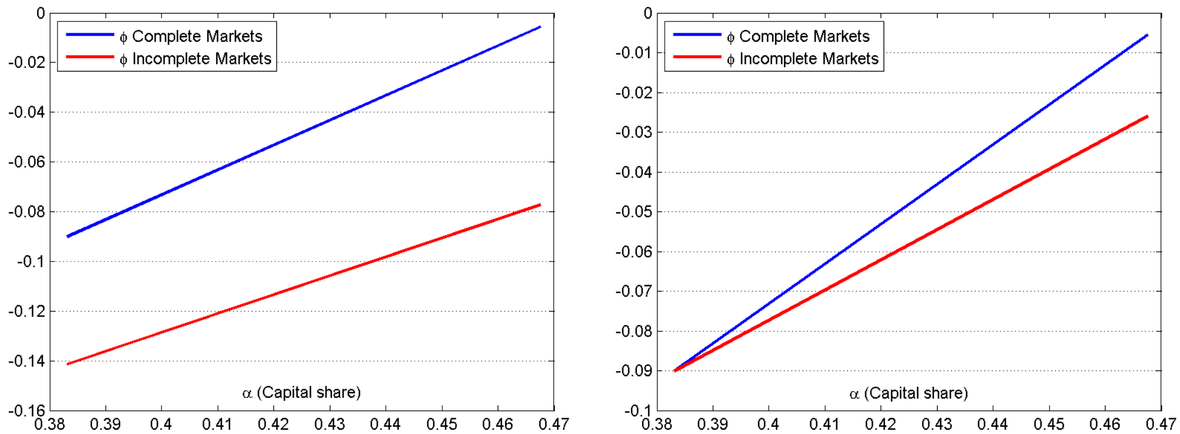


Figure 1: Financial positions: complete vs incomplete markets

how the curves move apart as $\alpha(s)$ increases. The distance between the two lines represents how the inefficient reaction of the economy, due to excess holdings of precautionary savings, increases as the labor share decreases. Proposition 3 has important implications for asset prices and the trading of financial assets.

Implementation with two assets. As for the case of complete markets, we now want to illustrate how capital biased technological change affects prices and the agents' positions in the risk free and in the risky assets. Again, we study an implementation with two assets and we show that the risk free interest rate decreases as the capital share increases and that our model generates a steep increase in the demand for safe (risk free) assets, reminiscent of the corporate saving glut. Suppose once again that there are only two shocks $s_L < s_H$, and two financial assets, a risk free bond B and a stock-market-indexed risky asset A with payoff $A\alpha(s)Y_2(s)$ for $s = L, H$. We can define $g^{CE}(\alpha, \phi)$ the function such that[8]

$$\mathbb{E}_i[(-\phi(s) + \alpha(s)g_i)^{-\sigma}] = (-\phi(s) + \alpha(s)g^{CE}(\alpha, \phi))^{-\sigma} \tag{11}$$

This function depends on α and ϕ and is such that $g^{CE}(\alpha, \phi) < 1$ if there is un-insurable idiosyncratic risk, because of the convexity of the marginal utility. Note that if there were complete markets, then $g^{CE}(\alpha, \phi) = 1, \forall \alpha, \phi$.

Before showing the implementation, it is instrumental to characterize the $\phi(s)$ in terms of the certainty equivalent $g^{CE}(\alpha, \phi)$. Performing similar manipulations on the equilibrium condition as in the complete markets equilibrium, it is possible to show that:

[8] With an abuse of terminology we will sometimes refer to g^{CE} as the "certainty equivalent" even though we are not working with utilities but with marginal utilities

$$\phi(s)^{IM} = x[\alpha(s)g^{CE}(\alpha,\phi^{IM}) + (1-\alpha(s))] - (1-\alpha(s)); \quad \forall s \tag{12}$$

This equation does not provide a closed form solution for $\phi^{IM}(s)$ but anyhow gives a clear intuition about the effect of incomplete markets. Comparing ϕ^{CM} with ϕ^{IM} we can see that, because $g^{CE}(\alpha,\phi^{IM}) < 1$ for all ϕ^{IM}. First, it confirms the result of Proposition 3, so that $\phi^{CM}(s) > \phi^{IM}(s)$, for all s. But, second, this has implications for the implementation. The incomplete markets distortion is multiplied by the worker's wealth ratio, x. Thus, the smaller x, the smaller the distortive effect. In the limit, as $x \to 0$, it must be that. $\phi^{IM} \to \phi^{CM}$. Also, the distortion decreases the amount of AD securities that are traded in equilibrium, which shows that the presence of idiosyncratic risk diminishes the quantitative relevance, on average, of changes of α on the trading of financial assets.

To analyze the implementation with two assets we can write the analogous of equations (9) and (10), writing assets positions in terms of the capital share and the certainty equivalent, α and $g^{CE}(\alpha,\phi)$, as:

$$R_L B = R_L^{CM} B^{CM} - x_1 \frac{Y_2(L)Y_2(H)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \left[g^{CE}(H) - g^{CE}(L)\right] \tag{13}$$

$$A = A^{CM} + x_1 \left[\left(g^{CE}(L) - 1\right) + \frac{\alpha(H)\alpha(L)\left(g^{CE}(H) - g^{CE}(L)\right)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}\right] \tag{14}$$

We have purposely written equations (13) and(14), as the efficient allocation plus/minus a distortion, to emphasize the impact of the idiosyncratic risk. First notice that if α is constant, it must be the case that $g^{CE}(H)$=$g^{CE}(L)$. Thus, the distortive term affecting the holdings of the risk free asset vanishes; and since we have already shown that $R_L^{CM}B^{CM}$= 0, it must also be that $R_L B = 0$. Again, as in the complete markets economy, the risk free bond is not used in equilibrium. If it is necessary to transfer resources across periods, this is done using the risky asset. But now, the capitalists need to accumulate some savings to hedge the idiosyncratic risk. They do so buying the risky asset, which leaves less for the workers. Hence, the term $x_1\left(g^{CE}(L) - 1\right) < 0$ reducing A.

What about the economy with varying shares? Recall that we are considering an economy with ρ >1, therefore the capital income share must be pro-cyclical, i.e., $\alpha(H) > \alpha(L)$, which in turn implies $g^{CE}(H) < g^{CE}(L)$. Looking at the distortive term, now we have that $R_L B$ is equal to $R_L^{CM}B^{CM} < 0$ plus a positive term. That is, workers borrow less on (capitalist accumulate less of) the risk free asset. Similarly, the distortive effect is also negative on A. The presence of idiosyncratic risk, further reduces the workers holdings of risky assets. In short, capitalists hoard on both assets, hindering the possibility of insuring aggregate risk.

It is interesting, that the distortion to the holdings of the risky asset stems from two sources. The first source, captured by the term $\left(g^{CE}(L)-1\right)<0$, arises just because of the existence of uninsured idiosyncratic risk, and it remains there even when α is constant. The second source, captured by the term $\frac{\alpha(H)\alpha(L)\left[g^{CE}(H)-g^{CE}(L)\right]}{\alpha(H)Y_2(H)-\alpha(L)Y_2(L)}<0$, arises because of the presence of "time varying" uncertainty. The inefficiency due to uninsure idiosyncratic risk interacts with the stochastic income shares amplifying the distortions.

Finally, notice that the distortions are multiplied by the workers' wealth ratio, x. In particular, as x decreases, so it does the extend of the distortion. In the limit, when $x=0$ (workers have zero net worth) the asset's positions coincide with the efficient ones. This also gives us a hint about the impact of the wealth effects. Since the wealth effects tend, on average, to reduce x over time, we should expect the aggregate asset's holdings converging to those in the complete markets allocation. But because the positions would start as a compressed version of the efficient, it would look like and continuous widening of the aggregate holdings. In other words, one should expect and continuous increase in the firms savings on the risk free asset, accompanied by a continuous increase in the workers equity holdings.

3 Infinite horizon economy

In this section we relax the assumption that the economy lasts for only two periods and we allow for $t\in\mathbb{N}$. Also we allow for any arbitrary number of aggregate states $s\in[s^1,s^2,...,s^N]$. The probability of each state is as before: $Prob(s'|s)=\Pi(s'|s)$. To simplify notation in what follows we characterize the solutions in a recursive fashion. In the two period economy there was no investment and given that after the second period there was no choice to be made, keeping track of the exogenous aggregate shock was enough. However, we also showed that the initial distribution of wealth was a determinant of allocations. In the infinite horizon economy the distribution of wealth will be changing along the business cycle. Thus, we will need to keep track of it, along with the effective stock of capital, to determine the equilibrium. The redefined state space is $s=\{g_sK,x\}$, where x is the ratio of the consumer's wealth to the total wealth in the economy. We formally show in Section 3.3 that these two states variable are enough to characterize the equilibrium. Since both K and x are endogenous variables, the transition function $\Pi(s'|s)$ is an equilibrium object. However, when solving the individual problems, subsection 3.1 and subsection 3.2, the composition of s and how its transition is determined are irrelevant, because they are taken as given by each individual.

3.1 Consumer

In the infinite horizon economy the consumer-worker solves:

$$\begin{aligned} V^c(a,s) &= \max_{\{c(s),a(s'|s)\}} \left\{ u(c(s)) + \beta \mathbb{E}_{s'}[V^c(a(s'|s),s')|s] \right\} \\ & st. \quad c(s) + \sum_{s'} p(s'|s)a(s'|s) \leq a(s) + \omega(s), \end{aligned}$$

where $\omega(s)$ is the wage in state s and $a(s'|s)$ are the Arrow-Debreu securities bought by the consumer in state s, that pay next period contingent on the realization of state s'. The initial financial wealth $a_1 \equiv a(s_0)$ is given. The first order conditions for consumption and financial decisions implies

$$p(s'|s)u'(c(s)) = \beta \Pi(s'|s)u'(c(s')); \quad \forall s'$$

Denote by $\zeta(s)$ the savings rate. We show in Appendix B.1 that the solution is characterized by:

$$c(s) = (1 - \zeta(s))(a + \omega(s) + h(s)) \tag{15}$$

$$a(s'|s) = \phi^c(s'|s)\zeta(s)[a + \omega(s) + h(s)] - \omega(s') - h(s') \tag{16}$$

where $h(s) = \sum_{s'|s} p(s'|s)[\omega(s') + h(s')]$ is the consumer's present value of future incomes, or human wealth. The function $\phi^c(s'|s)$, to be determined, pins down the portfolio allocation. Note that using the budget constraint it must be true that:

$$a'(s) \equiv \sum_{s'|s} p(s'|s)a(s'|s) = \zeta(s)[a + \omega(s) + h(s)] - h(s)$$

The latter implies $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$. In the next sections we will use the consumer's total wealth, $W^c(s) \equiv a + \omega(s) + h(s)$, to characterize the solution. Thus, $\zeta(s)$ is the saving rate out of wealth and $1 - \zeta(s)$ is the implied consumption rate. As we show in Appendix B.1 using the guessed policy functions the consumer's problem solution is fully characterized by:

$$\phi^c(s'|s) = \left[\frac{\beta \Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1 - \zeta(s))}{(1 - \zeta(s'))\zeta(s)}; \quad \forall s, s' \tag{17}$$

Using the condition $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$ and (17) we obtain:

$$(1-\zeta(s))^{-1} = 1 + \sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma}(1-\zeta(s'))^{-1}\right]; \;\; \forall s \tag{18}$$

Taking prices, $p(s'|s)$, and the low of motion of s as given, the last is a recursive equation, linear in $(1-\zeta(s))^{-1}$, which solves for the savings rates. Once $\zeta(s)$ has been found, equation (17) solves for the state contingent assets holdings.

3.2 Entrepreneur

In this section we show that despite being subject to idiosyncratic risk the consumption and saving rates of the entrepreneurs are very simple and akin to those of the consumers.[9] In particular, due to the homothetic preferences, savings rate are linear in total wealth, and thus total savings are independent of the distribution of wealth. In other words, there will be aggregation: knowing the average net worth is enough to forecast future aggregate capital. As in the two period model, with the natural extension to an infinity horizon, the entrepreneur solves:

$$\begin{aligned} V^e(E,k;s,i) \;\; &= \max_{\{e(s,i),E(s'|s),k'\}} \left\{u(e(s,i)) + \beta\mathbb{E}_{s',i'}[V^e(E(s'|s),k';s',i')|s]\right\} \\ & st. \quad e(s,i) + k' + \sum_{s'|s} p(s'|s)E(s'|s) \leq E + R(s)kg_i \end{aligned}$$

where $R(s)$ is the average gross return on capital, $E(s'|s)$ are Arrow-Debreu securities bought by the entrepreneur in state s, contingent on the realization of state s' the following period.[10] Notice that the return on capital depends only on the aggregate state, because the production function is of the CES type. The initial financial wealth $E(s_0)$ is given. Finally, as in the two period economy, g_i is the idiosyncratic shock to which the entrepreneur is subject to. We maintain the assumption that g_i is *i.i.d.* over time. The first

[9] See Angeletos (2007) for a similar result.

[10] Notice that $R(s)$ is the gross return on capital, which shouldn't be confused with the net $r(s)$, and as such includes any potential depreciation of capital. In equilibrium, it would be true that $R(s) = (1-\delta)g_s + r(s)$, with $r(s) = \frac{\partial y(K,L,S)}{\partial K}$. Therefore, productivity shocks, both g_s and g_i, also affect capital depreciation. This assumption is necessary to obtain the linearity of the budget constraint respect to individual holdings of capital. This assumption is widely used in the literature. See for example Brunnermeier and Sannikov (2014) and Di Tella (2014).

order conditions for capital and Arrow-Debreu securities imply:

$$p(s'|s)u'(e(s,i)) = \beta\Pi(s'|s)\mathbb{E}_i[u'(e(s',i))]; \quad \forall s, s' \tag{19}$$

$$u'(e(s,i)) = \beta\mathbb{E}_{s',i}[u'(e(s',i))R(s')g_i|s]; \quad \forall s \tag{20}$$

As in the last subsection, 3.1, we guess and then verify (see Appendix B.2) that the solution is characterized by:

$$e(s,i) = (1-\vartheta(s))W^e(s,i,k) \tag{21}$$

$$k'(s,i) = \nu(s)\vartheta(s)W^e(s,i,k) \tag{22}$$

$$E(s'|s,i) = \phi^e(s'|s)E_1(s,i) \tag{23}$$

where $\vartheta(s)$ is the savings rate, and $\nu(s)$ is the portion of savings invested in capital; also, entrepreneur's total wealth is:

$$W^e(s,j,k) = E + R(s)g_jk$$

In what follows we will refer to $\nu(s)$ as the investment rate. Using the budget constraint we have that total savings, $E_1(s,i)$, must satisfy: $E_1(s,i) \equiv \vartheta(s)(1-\nu(s))W^e(s,i,k)$. Therefore, it must also be true that $\sum_{s'|s} p(s'|s)\phi^e(s'|s) = 1$. The law of motion of individual wealth is:

$$W^e(s',i',k') = \vartheta(s)o(s',i;\phi^e,\nu)W^e(s,i,k) \tag{24}$$

where

$$o(s',i;\phi^e,\nu) \equiv [(1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')g_i]$$

is the ex-post growth rate of wealth. Using both Euler equations for the entrepreneur, equations (19) and (20) we obtain that the portfolio allocation, ϕ^e and $\nu(s)$, are determined by:

$$\mathbb{E}_{s',i|s}\left[((1-\vartheta(s'))o(s',i;\phi^e,\nu))^{-\sigma}\left(R(s')g_i - \frac{1}{\sum_{s'|s} p(s'|s)}\right)\right] = 0 \tag{25}$$

$$\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma} = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma}\frac{(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))}; \ \forall s, s' \tag{26}$$

First, note as long as $\vartheta(s)$ is independent of wealth and gi is i.i.d., the investment rate is also independent of individual wealth and of the current idiosyncratic shock. Second, note that equation pins down $\phi^e(s'|s)$ which is also independent of wealth. Comparing (17) and (26) we see that the consumer's saving rate is affected by $\phi^c(s'|s)$ while for entrepreneurs the equivalent term is $(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma})^{-1/\sigma}$, which in turn is affected by both risk aversion and the exposure to idiosyncratic risk. The fact that equation (26) involves an expectation while equation (17) does not, is what it generates the different behavior between workers and entrepreneurs. In absence of idiosyncratic risk both agents would react equally to aggregate shocks. Another way of to see the role of id. risk in pining down the savings rate for the This can be seen in a more clear way when comparing the saving rates, which satisfy:

$$(1-\vartheta(s))^{-1} = 1 + m(s)^{-1} \sum_{s'|s} \left[(\beta\Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma} (1-\vartheta(s'))^{-1} \right]; \forall s \tag{27}$$

where $m(s) = \sum_{s'|s} p(s'|s) \left(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}\right)^{-1/\sigma}$. Notice that the only difference between (18) and (27) is $m(s)$. If $m(s) = 1, \forall s$, consumers and entrepreneurs would choose the same savings rates. Indeed, as we show in Appendix B.2, that is the case when the moral hazard friction vanishes. However, when the moral hazard prevents the full insurance of idiosyncratic risk, in general $m(s) > 1; \forall s$. As result, in equilibrium for any price function $p(s)$, it must be true that $\vartheta(s) > \zeta(s)$: on average the entrepreneurs' wealth grows faster than the consumer's wealth. This creates a downward drift on the consumers to entrepreneurs wealth ratio: x. As we showed in the two period model, this wealth effect has very important quantitative implications generating large changes in the financial asset's positions.[11]

3.3 Equilibrium

For these allocations to be feasible, they must satisfy the assets' and goods' market clearing conditions, pinning down the equilibrium prices $p(s'|s)$, and $\Pi(s'|s)$ must be consistent with the laws of motion generated by individual decisions. The assets' and goods'

[11]The drift also implies that in the limit the workers end up with zero wealth, while the entrepreneurs hold all the wealth in the economy. This may seem an odd implication of the model. This is a standard result, though, and the literature has alternative strategies to guarantee that there is a well defined stationary distribution of wealth. For example, introducing difference in discount factors, in particular entrepreneurs discount the future more heavily (lower β). See for instance DiTella (2017)), Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2011). Alternatively, we could assume that entrepreneurs become workers with some exogenous probability, keeping their wealth, and are replaced by workers. In this paper the distribution of wealth will be moving for two reasons: expected drift and the persistence of the relative shocks to capital $g_s g_i$, and does not affect our qualitative results.

market clearing conditions are:

$$a(s'|s) + E(s'|s) = 0; \quad \forall s, s' \tag{28}$$

$$c(s) + e(s) + K'(s) = y(s); \quad \forall s \tag{29}$$

where $e(s) = \int_i e(s,i,k,E)$, $K'(s) = \int_i K'(s,i,k,E)$, $y(s) = \int_i y(s,i,k,E)$ and $E(s'|s) = \int_i E(s'|s,i,k,E)$. Where we have avoided the dependency of the allocations on individual wealth because, as we show in the previous section, the savings and consumption rates are independent of it. However, for the aggregation we take it into account. Using equations for consumption and investment for the consumer and the entrepreneur, (15), (16), (21), (22) and (23), the market clearing conditions (28) and (29), can be written as:

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)\vartheta(s)(1-\nu(s))(1-x) = \frac{w(s') + h(s')}{W^T(s)}; \quad \forall s, s' \tag{30}$$

$$(1-\zeta(s))x + (1-\vartheta(s)(1-\nu(s))(1-x) = \frac{y(s)}{W^T(s)}; \ \forall s \tag{31}$$

where $W^T(s) = W^c(s) + W^e(s)$ and $x = W^c(s)/W^T(s)$. If instead of a CES, we were using a Cobb-Douglas production function, we could show that x would be independent of K and so would be $\iota(s', s)$. Thus, the state space would not need to include K. By Walras' Law, one of the market clearing conditions is redundant, while the other determines the equilibrium prices $p(s'|s)$.

To find $\Pi(s'|s)$, recall that the aggregate state includes endogenous variables, we need to characterize the endogenous laws of motion. First, recall that $k'(s,j) = \nu(s)\vartheta(s)W^e(s,j,k)$. Aggregating we obtain:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s) \tag{32}$$

Also, it is possible to show that the low of motion of the wealth ratio satisfies:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)}{\iota(s',s) + \nu(s)\vartheta(s)(1-x)R(s')} x(s) \tag{33}$$

where $\iota(s',s) = \frac{w(s')+h(s')}{W^T(s)}$. Notice that even though the process for x is not stationary, it is still Markovian. Thus, it is possible to compute its transition probabilities. As result, (32) and (33) together with the exogenous probability distribution over g_s determine the transition probabilities $\Pi(s'|s)$.

An important issue that arises from equation (33) is the possibility of multiple equilibria. The source of potential multiplicity is that s' contains x' itself. In general, $R(s')$

Parameter	Description	Value
γ	Risk aversion	2
β	Discount Factor's	0.95
ρ	Elasticity of Substitution	1.25
α	Capital Share	0.275
δ	Depreciation	0.06
$g_{s,h}, g_{s,l}$	Aggregate Shocks to Capital	1.02, 0.98
$p_{s,h}$	Probability of g_h	0.5
$g_{i,h}, g_{i,m}, g_{i,l}$	Id. Shocks to Capital	0.3, 1, 1.1
$p_{i,h}, p_{i,m}, p_{i,l}$	Id. Shocks to Capital	1/3, 1/3, 1/3
ϕ	Exposure to Id Risk	0.2

Table 1: Baseline Calibration

depends only on $g_{s'}K'$, the same is true for $\iota(s', s)$. However, $\phi^c(s'|s)$ will depend on x', i.e., agents would buy AD securities contingent on the expected realization of the distribution of wealth. Thus, the realized distribution, say $\tilde{x}$, must satisfy:

$$\tilde{x} = \frac{\phi^c(\{g_{s'}K', \tilde{x}\}|s)\zeta(s)}{\iota(g_{s'}K', s) + \nu(s)\vartheta(s)(1-x)R(g_{s'}K')}x$$

Without additional knowledge about the shape of $\phi^c(.|s)$, it is not possible to state if the above equation has more than one solution. We address this issue in the numerical implementation. We want to emphasize that this is not a problem that arises only in our environment, but is also present on many studies for economies with financial frictions. This setup just makes it more transparent.

3.4 Numerical calibration.

Regarding the preferences we use standard parameters in the literature. We set the discount factor $\beta = 0.95$ and a degree of risk aversion $\sigma = 2$, which are standard values in the real business cycles literature; see for example Cooley (1995). These values are consistent with risk free interest rate of 5% and the value of risk aversion of standard for business cycle fluctuations.[12]

Regarding the parameters for the production function, we need to find values for ρ, α, δ. For the elasticity of substitution, it has been documented that the labor share appears to be pro-cyclical in the short run, while is counter cyclical in the medium-long run;

[12] In this paper, we would like to understand long trends on quantities, and the risk free interest rate; the objective is not to match the equity premium puzzle. A large literature has explored deviations from expected utility (see for example Ju and Miao, 2012), long run risk (see for example Bansal and Yaron, 2004 and Hansen et al., 2008), and disaster risk (see for example Barro, 2009 and Gourio, 2012), as possible explanations of the premium between equities and safe bonds.

see for instance León-Ledesma and Satchi (2018).[13] A pro-cyclical labor share would imply an elasticity of substitution of $\rho < 1$, while the pro-cyclical labor share requires $\rho > 1$. Our focus is on the medium-long run and thus we use the estimates of Koh et al. (2016) and Karabarbounis and Neiman (2014), who analyze long term movements. The former estimates an elasticity of substitution between capital and labor $\rho = 1.15$ while the latter finds $\rho = 1.25$. Our baseline calibration is based on the estimation of Karabarbounis and Neiman (2014) and then we consider the alternative value found by Koh et al. (2016). Furthermore, since with a CES technology the average capital income share depends on the capital output ratio, we calibrate α and δ to target these two moments. Thus, we set $\alpha = 0.275$ in order to target, on average, a capital income share of around 0.3, and $\delta = 0.06$ to target a capital-output ratio of around 3.

Regarding the aggregate shock we consider only two values: $g_H = 1.02$ and $g_L = 0.98$ and we assign probability 1/2 to each realization. The *i.i.d.* structure of the shock is simplifies the state space; if this were not to be the case, we would to add another state variable. The fact that we allow for only two possible realizations is to be able to construct the straightforward mapping from the Arrow-Debreu securities economy to the economy with only two assets: a risk free and a risky asset. Adding more realizations would have minimal quantitative effects and would make this mapping less clear. Notice that the variance of the assumed process is $\mathbb{V}(g_s) = p(1-p)(g_H - g_L)^2 = \frac{1}{4}0.04^2 = 0.0004$, which is in line with the medium-long term variation of the GDP in the U.S. economy. As described in Section what matters for the entrepreneur is the residual risk $\psi^2\mathbb{V}(g_i)$ at which she is exposed. Also, because we are assuming that workers are not subject to idiosyncratic risk, this risk must be interpreted in relative terms. There is ample evidence that firms-entrepreneurs are more exposed to idiosyncratic risk than workers. Following He and Krishnamurthy (2011), we set $\psi = 0.2$ to match the 20% share of profits that hedge funds charge. The id. risk takes three values, $g_{i,h}, g_{i,m}, g_{i,l}$, that are given by 0.3750, 1.25, 1.3750 with equal probability. This amount to a total id risk of the variance of the idiosyncratic shock to $\mathbb{V}(g_i) = 0.1979$ and thus total id risk is given by to target $\psi^2\mathbb{V}(g_i) = 0.0079$.

3.5 Results

Here we present some of the main results of our paper. As we mentioned in sections 2 and 3, one of the main implications of the model is the implication for changes in the aggregate portfolio allocation in the economy. In figure 2 we show the implied worker's

[13] For more information about the implications of CES technology for the business cycle see for example Cantore et al. (2014) and Cantore et al. (2015).

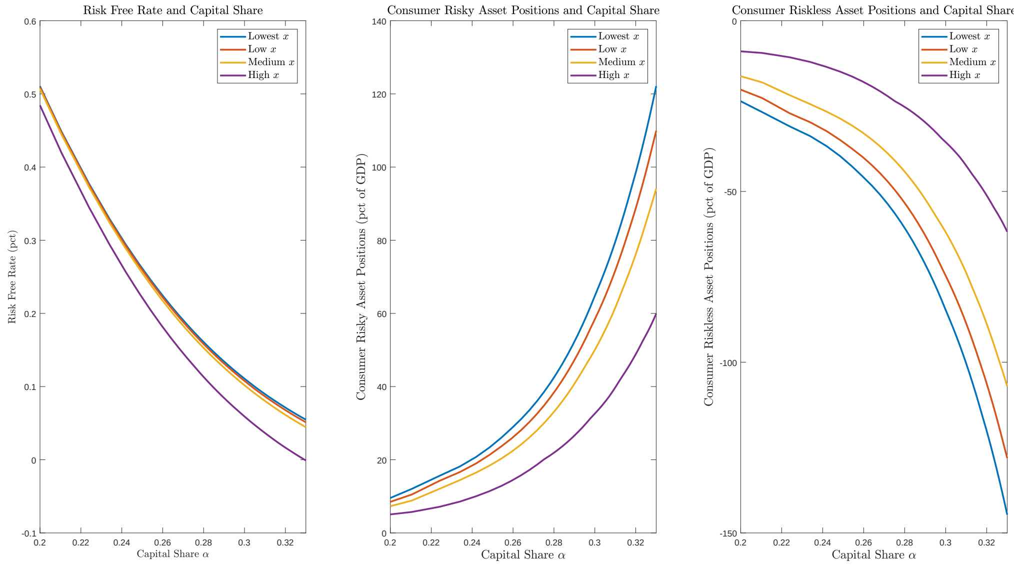


Figure 2: This figure plots the risk free rate, risky and risk-less asset positions for consumers as a function of the labor share. The values of x lowest, low, medium and high are given by 0.0153, 0.2136, 0.4424 ,0.8542.

holdings of risky and risk-less assets. To construct Figure 2 we simply use the equilibrium aggregate laws of motion generated by the model for the main calibration. Since the model generates $\phi^c(s'|g_s K, x)$, using the implied capital for each value of $g_s K$, we can construct $\phi^c(s'|\alpha, x)$. Then using similar transformations as in section 3, and given that we are considering only two possible aggregate shocks, we can map $\phi^c(s'|\alpha, x)$ into holdings of risky assets $A(\alpha, x)$ and risk-less assets $R(s)B(\alpha, x)$.

First, in the middle panel of Figure 2 we plot the risky asset positions, $A(\alpha, x)$, for different values of x, as a proportion of $Y(s)$. As we can see, larger values of α imply larger holdings of A. As we show in the other panels, the relationship remains for alternative values of x. The smaller x, the larger the implied positions.

Second, in the right panel of Figure 2, we can see the other implication of the model anticipated in the two period model, the pattern observed for $A(\alpha, x)$ is mirrored for $B(\alpha, x)$, with a negative sign. That is, the model predicts exploding changes in the financial positions of households, that can duplicate when moving from values of α around 0.3 to values of α around 0.35. Because of market clearing, the financial positions of the corporate sector are the negative of those corresponding to the households. Thus, if households are borrowing (leveraging) to buy equity, it must be the case that the corporate sector is increasing the issuance of equity and accumulating risk-free assets. This last phenomenon

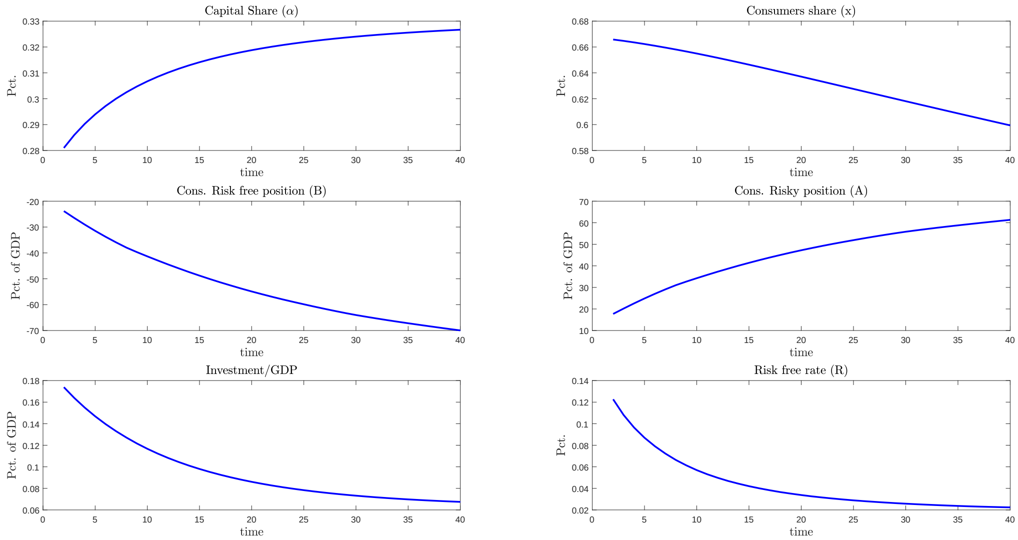


Figure 3: Time path: this figure displays key quantities for our calibrated model after a sequence of capital biased shocks.

is known as the corporate savings glut. [14]

The next question, after analyzing the policies, is: which are the implied paths for the this economy? To answer this question. We pick and arbitrary path for the capital share in which a succession of positive shocks to g_s generates an increase in the capital share from 0.28 to 0.33 as observed in the data. The results are shown in Figures 3. The simulated paths confirm the results we obtained from the policy functions, but more importantly, show how far the mechanism can take us quantitatively. In particular, a change in the capital share from 0.28 to 0.33 almost triplicates the percentage of risky assets held by the consumers, and implies a drop of the real risk free rate from 12 percent a year to 2 percent a year.

[14] What we add to this discussion is the implication that the accumulation of risk-free assets by part of corporations, have to be accompanied by the increasing holdings of risky assets by households. Is this true in the data? As a first attempt to answer this question we constructed, using the Flow of Funds for the U.S. economy, the holdings (direct and indirect) of equity by households. Figure 1 shows that indeed there has been a large increase in the holdings of equity by households from 0.4GDP to 1.4GDP, almost tripling its value. In the same figure we also plot, dashed line, all the debt instruments, not related to their main activity, hold by corporations. We also see a steady increase in, almost tripling its value from around 0.07GDP to 0.21GDP (see right vertical axes).

4 Conclusions

Kaldor facts led to the prevailing belief that the capital and labor income shares were, besides some small short-run variations, roughly constant. One of the implications of constant income shares is that it is impossible for workers and capitalists to insure each other. With constant income shares, aggregate fluctuations affect both sectors in the same way, and therefore aggregate variation just main the same distribution of wealth. Recent studies, however, have shown that the labor share moves both in the short and medium long run.

Motivated by these deviations from the Kaldor facts, in this paper we study how varying income shares affect sharing, and which are the predictions over allocations. In particular, we study a standard growth model with financial frictions. We started the discussion with a two-period version of our model. The main result is to show how the combination of limited risk sharing and time-varying shares distorts the allocation of risk, and which assets are used for insurance. Even from the two-period version of the model, it is clear that to share risk, firms want to hoard safe assets, driving interest rates down, and households on the contrary invest in risky assets. Not only that, we show how time-varying shares of different groups in the economy crucially distort their ability to absorb risk from other sectors and diverts the asset allocation of other sectors. To explore our channel quantitatively, we then calibrate our model to match long run moments in the US data. We show that the recent US experience of low rates, rising capital shares, accumulation of safe assets by firms and risky assets by households, after 1980's, can be rationalized by persistent capital-biased shocks and limited risk sharing.

The focus of this paper is the medium long-run. However, our model also would lend itself naturally to study how income shares exacerbate or mitigate fluctuations. Also, our model is suitable for studying question about inequality and asset pricing. In particular, ours is a two-factor asset pricing model in which, the capital share and the relative wealth of financial intermediaries are factors pricing the "cross-section" of assets. Both, business cycles and asset prices, are topics for further research.

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A Proofs two period model

A.1 Capital share in the CES production function

The firms maximizes $\pi(s,i) = \left[\alpha\left(g_i g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - \omega L$, which implies $L^d(s,i) = \alpha^{\frac{\rho}{\rho-1}}\left[\left(\frac{\omega}{1-\alpha}\right)^{\rho-1} - (1-\alpha)\right]^{\frac{\rho}{1-\rho}} g_i g_s k$. From the labor market clearing condition $1 = L^s = L^d(s) = \mathbb{E}(L^d(s,i))$ we can get the wage

$$\omega(s) = (1-\alpha)\left[\alpha\left(g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)\right]^{\frac{1}{\rho-1}}.$$

Moreover recall that

$$\alpha(s,i) = \frac{\partial y(s,i)}{\partial k}\frac{k}{y(s,i)} = \frac{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)(L)^{\frac{\rho-1}{\rho}}}$$

so $\alpha(s) = \mathbb{E}_i(\alpha(s,i))$ is given by

$$\alpha(s) = \frac{\alpha(g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

and, given that $Y(s) = \mathbb{E}(y(s,i))$, in the same way, the labor share is

$$(1-\alpha(s)) = \frac{\partial Y}{\partial L}\frac{L}{Y} = \frac{(1-\alpha)}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}.$$

Then

$$\omega(s) = (1-\alpha(s))Y(s)$$

Now given that we have the wage, we can find $L^d(s,i)$. We find that $L^d(s,i) = g_i$ and therefore

$$\pi(s,i) = \alpha(s)Y(s)g_i.$$

A.2 Proof of Proposition 1

Proof. Using the first order conditions from the consumer problems and the market clearing conditions (2) and (3) we can obtain consumption's as a function of aggregate endowments. This implies that prices are given by: $p(s)^{CM} = \Pi(s)\left(\frac{Y_1}{Y_2(s)}\right)^{\sigma}$. Using the prices and the FOCs we can see that $\eta_2(s) = \eta_1\ \forall s$. Then, the allocations in (7) together with the prices and wages constitute a competitive equilibrium with the consumption shares given by $\eta_1 = \eta^{CM} = \frac{x_1^{\frac{1}{\sigma}}}{1+x_1^{\frac{1}{\sigma}}}$. Given $\eta_2(s) = \eta^{CM}\ \forall s$, we use that in this economy we can write the profits of the entrepreneurs and the wages as $\pi(s) = \alpha(s)Y(s)$ and $\omega(s) = (1-\alpha(s))Y(s)$, where $Y(s) = \mathbb{E}_i[y(s,i)] = g_s K_1$ and use equation

$$\frac{u'(c_1)}{u'(e_1)} = \frac{(-\phi(s)+\alpha(s))^{-\sigma}}{(\phi(s)+(1-\alpha(s)))^{-\sigma}}; \quad \forall s$$

to solve for $\phi(s)$. Using the market clearing conditions (3) we can express $\phi(s)$ as in (8). □

A.3 Proof of Proposition 2

Proof. The capital share is constant along the business cycle if $\rho = 1$, i.e. the production function is Cobb-Douglas. In this case

$$\begin{aligned}\left(\frac{1-x_1}{x_1}\right)^{-\sigma} &= \frac{\mathbb{E}_i[(-\phi(s)Y_2(s)+\alpha Y_2(s)g_i)^{-\sigma}]}{(\phi(s)Y_2(s)+(1-\alpha)Y_2(s))^{-\sigma}} \\ &= \frac{\mathbb{E}_i[(-\phi(s)+\alpha g_i)^{-\sigma}]}{(\phi(s)+(1-\alpha))^{-\sigma}}\end{aligned}$$

and thus, $\hat{\phi}$, is constant and solves

$$\left(\frac{1-x_1}{x_1}\right) = \frac{\mathbb{E}_i[(-\hat{\phi}+\alpha g_i)^{-\sigma}]}{(\hat{\phi}+(1-\alpha))^{-\sigma}}; \quad \forall s.$$

□

A.4 Proof of Proposition 3

A.5 Derivation Equations (9) and (10)

In equilibrium $A^c + A^e = 0$ and $B^c + B^e = 0$. Consider the problem of the consumer. Note that there is a one-to-one mapping between the second period's payoffs of the AD securities $\phi(s)$ and of the portfolio with two assets:

$$\phi(L)Y_2(L) = RB^c + A^c\alpha(L)Y_2(L)$$
$$\phi(H)Y_2(H) = RB^c + A^c\alpha(H)Y_2(H).$$

The latter implies positions and prices given by

$$A^c = \frac{\phi(H)Y_2(H) - \phi(L)Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{34}$$

$$RB^c = \frac{Y_2(L)Y_2(H)(\phi(L)\alpha(H) - \alpha(L)\phi(H))}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{35}$$

$$R = \frac{1}{\sum_s p(s)} \tag{36}$$

$$P_A = \sum_s p(s)\alpha(s)Y_2(s). \tag{37}$$

Where $p(s)$ is the price of the AD securities in state s, then both the consumer and the entrepreneur are optimizing. Notice that, in this case

$$p(s) = Pr(s)\left(\frac{Y_1}{Y_2(s)}\right)^{\gamma},$$

for all s, so the price of the AD security in state s only depends on the (exogenous) growth rate of total output Y, and so does the risk free rate R. Furthermore, the output growth is stationary, the interest rate is constant through time and in particular does not depend on the capital share.

B Proofs infinite horizon economy

B.1 Proof of consumer's solution

Using the consumer's first order condition and the assumed utility function, it is straightforward to show that:

$$c(s') = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} c(s) = \tilde{\beta}(s', s)c(s); \quad \forall s'$$

Then, using the guesses for consumption and savings we obtain:

$$(1-\zeta(s'))(a(s')+\omega(s')+h(s')) = \tilde{\beta}(s'|s)(1-\zeta(s))(a(s)+\omega(s)+h(s))$$
$$(1-\zeta(s'))\phi^c(s'|s)\zeta(s)W^c(s) = \tilde{\beta}(s'|s)(1-\zeta(s))W^c(s)$$

Thus, the individual consumption ratio solves:

$$(1-\zeta(s'))\phi^c(s'|s)\zeta(s) = \tilde{\beta}(s',s)(1-\zeta(s)); \quad \forall s,s' \tag{38}$$

Using (38) it is immediate to arrive to equation (17)

$$\phi^c(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\rho}\frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)}; \quad \forall s,s'$$

To solve for the consumption rates we use the present value of (17). Multiplying the last by $p(s'|s)$ and adding over s' we obtain:

$$1 = \sum_{s'}\phi^c(s'|s)p(s'|s) = \sum_{s'}\left[\frac{(\beta\Pi(s'|s))^{1/\rho}p(s'|s)^{1-1/\rho}}{(1-\zeta(s'))}\right]\frac{(1-\zeta(s))}{\zeta(s)}$$
$$(1-\zeta(s))^{-1} = 1+\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\rho}p(s'|s)^{1-1/\rho}(1-\zeta(s'))^{-1}\right]$$

The last is equation (18).

B.2 Proof of entrepreneur's solution

Using the proposed solutions, the implied law of motion of wealth is:

$$W^e(s',i',k') = E(s')+R(s')g_{i'}k'$$
$$W^e(s',i',k') = \vartheta(s)(1-\nu(s))\phi^e(s'|s)W^e(s,i,k)+\nu(s)\vartheta(s)R(s')g_{i'}W^e(s,i,k)$$
$$W^e(s',i',k') = \vartheta(s)[(1-\nu(s))\phi^e(s'|s)+\nu(s)R(s')g_{i'}]W^e(s,i,k)$$

The last equation is (24). Then, using both Euler equations:

$$\mathbb{E}_{s',i|s}[u'(e(s',i))R(s')g_i] = \frac{\mathbb{E}_{s',i|s}\left[u'(e(s',i))\right]}{\sum_{s'|s}p(s'|s)}$$

Now use the guessed solution for consumption, (24) and the CRRA preferences to write:

$$\mathbb{E}_{s',i|s}\left[((1-\vartheta(s'))[(1-\nu(s))\phi^e(s'|s)+\nu(s)R(s')g_i])^{-\rho}\left(R(s')g_i-\frac{1}{\sum_{s'|s}p(s'|s)}\right)\right]=0$$

Introducing the definition of $o(s', i; \phi^e, \nu)$ in the last we obtain (25). To solve for the saving rates, we use the state by state first order condition to obtain:

$$\begin{aligned}
u'(e(s)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'(e(s',i))];\ \ \forall s,s' \\
u'((1-\vartheta(s))h^e(s,i,k)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'((1-\vartheta(s'))h^e(s',i,k'))] \\
u'(1-\vartheta(s)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'((1-\vartheta(s'))\vartheta(s)o(s',i))] \\
(1-\vartheta(s))^{-\rho} &= \beta\frac{\Pi(s'|s)}{p(s'|s)}[(1-\vartheta(s'))\vartheta(s)]^{-\rho}\mathbb{E}_i o(s',i;\phi^e)^{-\rho}
\end{aligned}$$

Similar manipulations as in the consumer's problem deliver:

$$(1-\vartheta(s'))\vartheta(s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\rho}\right)^{-1/\rho}=\tilde{\beta}(s',s)(1-\vartheta(s));\ \ \forall s,s'$$

Given the solution for $\nu(s)$, the last equation, (27) in the text, pins down $\phi^e(s'|s)$, which is also independent of wealth. Comparing (18) and (27) we see that the consumption for consumers is affected by $\phi^c(s'|s)$ while for entrepreneurs the equivalent term is $(\mathbb{E}_i o(s',i;\phi^e)^{-\rho})^{-1/\rho}$.

We can perform similar manipulations to the consumer's problem. Multiplying (27) by $p(s',s)$, and adding over s' we obtain:

$$\begin{aligned}
m(s)=\sum_{s'}\left(\mathbb{E}_i o(s',i;\phi^e)^{-\rho}\right)^{-1/\rho}p(s'|s) &= \sum_{s'}\left[\frac{(\beta\Pi(s'|s))^{1/\rho}p(s'|s)^{1-1/\rho}}{(1-\vartheta(s'))}\right]\frac{(1-\vartheta(s))}{\vartheta(s)} \\
(1-\vartheta(s))^{-1} &= 1+m(s)^{-1}\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\rho}p(s'|s)^{1-1/\rho}(1-\vartheta(s'))^{-1}\right]
\end{aligned}$$

The last equation is (27) in the text.

B.3 Proofs of equilibrium conditions.

Let $W^e(s)=\int_{i,k,E}W^e(s,i,k,E)$, then the aggregate capital follows:

$$K'(s)=\int_{i,k,E}k'(s,i,k,E)=\nu(s)\vartheta(s)\int_{i,k,E}W^e(s,i,k,E)=\nu(s)\vartheta(s)W^e(s)$$

Operating with the asset's market clearing.

$$a(s'|s) + E(s'|s) = 0; \quad \forall s, s'$$

$$\phi(s'|s)\zeta(s)W^c(s) + \phi^e(s'|s)\vartheta(s)(1-\nu(s))W^e(s) = \omega(s') + h(s')$$

Define $x = W^c(s)/W^T(s)$, then

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)\vartheta(s)(1-\nu(s))(1-x) = \frac{\omega(s') + h(s')}{W^T(s)}; \quad \forall s, s'$$

Which is equation (30) Operating with the goods market clearing:

$$\begin{aligned} c(s) + e(s) + k'(s) = &\quad y(s); \quad \forall s \\ (1-\zeta(s))W^c(s) + (1-\vartheta(s))W^e(s) + \vartheta(s)\nu(s)W^e(s) = &\quad y(s); \quad \forall s \\ (1-\zeta(s))x + (1-\vartheta(s)(1-\nu(s))(1-x) = &\quad \tfrac{y(s)}{W^T(s)}; \ \forall s \end{aligned}$$

Which delivers (31). The next period distribution of wealth, x' is:

$$\begin{aligned} \frac{x(s')}{1-x(s')} &= \frac{W^c(s')}{W^e(s')} = \\ = \frac{\phi^c(s'|s)\zeta(s)W^c(s)}{\mathbb{E}_i o(s',i,s)\vartheta(s)W^e(s)} &= \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i,s)\vartheta(s)(1-x)} \end{aligned}$$

Which can be written as:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i,s)\vartheta(s)(1-x) + \phi^c(s'|s)\zeta(s)x}$$

Using market clearing (30), this equation can also be written as:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)}{\iota(s',s) + \nu(s)\vartheta(s)(1-x)r(s')}x(s)$$

Which is (33) in Section 3.3. For the law of motion of $g_s k$ recall that: $k'(s,i,E) = \nu(s)\vartheta(s)W^e(s,i,k,E)$ and: $E(s',i,s,E) = \phi^e(s',s)\vartheta(s)(1-\nu(s))W^e(s,i,k,E)$. Therefore in every state

$$\frac{E(s',i,s,k,E)}{k'(s,i,k,E)} = \phi^e(s',s)\frac{(1-\nu(s))}{\nu(s)}$$

Which is independent of i. Thus, assuming that $E(s_0)$ is also proportional to k

$$k'(s,i,k,E) = \nu(s)\vartheta(s)W^e(s,i,k,E)$$

As a result we can write:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s)$$

Delivering (32) in Section 3.3.

C Optimal Contract

Suppose a 1 period model. There is a risk neutral principal which can provide insurance to the firms. Suppose there are three possible shocks $g_L < g_M < g_H$, with probability γ_i (three shocks can be generalized to any number, with only 2 shocks may not be general).

The firm can enter the insurance contract with the financial intermediary at the beginning of the period, before knowing the realization of g_i. The firm's profits are $\pi k g_i$, the linearity in k and g_i of the CES model is shown in the appendix. Thus, in absence of insurance, the entrepreneur's utility is:

$$\mathbb{E}_i[u(e_i)] = \mathbb{E}_i[u(\pi k g_i)]$$

The principal (financial intermediary) could sign a contract and offer insurance to the entrepreneur. First, suppose that g_i is observable. In this case the contract is very simple. The principal "buy" all the proceeds of the production at a price J. Then, after the shock is realized the entrepreneur hands over to the profits to the principal. Because the principal must break even it must be that $\mathbb{E}_i(\pi k g_i) - J = 0$. Thus, the utility of the entrepreneur is:

$$\mathbb{E}_i[u(e_i^O)] = E_i[u(J)] = u(\mathbb{E}_i[\pi k g_i]) > \mathbb{E}_i[u(\pi k g_i)] = \mathbb{E}_i[u(e_i)]$$

However, the entrepreneur is subject to moral hazard problems: g_i is not observable and must be incentive compatible for him to revel the true realization of g_i. The entrepreneur can report an alternative value of g_i, say $g_{i'}$, and keep the difference for himself. But, transforming these "stolen" profits into consumption is not for free. Each unit of stolen profit transforms into consumption at the rate $0 \leq \phi \leq 1$. Thus, when the entrepreneur steals profits obtains an additional consumption of only $\phi \pi k(g_i - g_{i'})$.

Because of this, the contract must be incentive compatible. Now the principal must give additional payments d_i, contingent on the realization of g_i, to make the contract incentive compatible. Since the entrepreneur will not lie on equilibrium, his consumption is $e_i^c = J + d_i$, while because the principal must break even the contract must satisfy: $\mathbb{E}_i(\pi k g_i - d_i) - J = 0$.

As a result, the optimal contract solves:

$$\max_{\{J,d_i\}} \mathbb{E}_i u(J + d_i)$$
$$\text{st.} \quad \phi\pi k(g_i - g_{i'}) + d_{i'} + J \leq J + d_i; \quad \forall i, i'$$
$$\mathbb{E}_i(\pi k g_i - d_i) - J = 0$$

The first set of constraints are the incentive compatibility, or truth telling, constraints. First notice that only the adjacent constraints matter (single crossing). To see this, consider that the entrepreneur would never lie when observes the low shock. So, only the following can be binding:

$$\phi\pi k(g_H - g_M) + d_M \leq d_H$$
$$\phi\pi k(g_M - g_L) + d_L \leq d_M$$
$$\phi\pi k(g_H - g_L) + d_L \leq d_H$$

Adding the first two inequalities:

$$\phi\pi k(g_H - g_M) + d_M + \phi\pi k(g_M - g_L) + d_L \leq \quad d_H + d_M$$
$$\phi\pi k(g_H - g_L) + d_L \leq \quad d_H$$

Thus, the third constraint is irrelevant, in general is a version of the single crossing property. This can be generalized to any arbitrary number of idiosyncratic shocks. Rewriting the problem we have:

$$\max_{\{J,d_i\}} \sum_i \gamma_i u(J + d_i)$$
$$\text{st.} \quad \phi\pi k(g_H - g_M) + d_M \leq d_H$$
$$\phi\pi k(g_M - g_L) + d_L \leq d_M$$
$$\sum_i \gamma_i(\pi k g_i - d_i) - J = 0$$

Let λ be the multiplier in the break even constraint and μ_i the multiplier in each incentive compatibility. Taking first order conditions:

$$\begin{aligned}
\sum_i \gamma_i u'(J+d_i) &= \lambda \\
\gamma_L u'(J+d_L) &= \gamma_L \lambda + \mu_M \\
\gamma_M u'(J+d_M) &= \gamma_M \lambda + \mu_H - \mu_M \\
\gamma_H u'(J+d_H) &= \gamma_H \lambda - \mu_H
\end{aligned}$$

It is clear that $\mu_L = \mu_H = 0$ cannot be a solution because it violates the IC constraints. Now, suppose $\mu_M = 0$, while $\mu_H > 0$. Then it must be that $d_L = d_M$. If $d_L > d_M$, the IC constraint implies

$$\phi \pi k(g_M - g_L) + d_L - d_M < 0$$

Which is a contradiction. If $d_L < d_M$ a small increase in d_L accompanied by a small reduction on d_M, to keep the break even constraint satisfied, generates a change of welfare of:

$$\gamma_L d_L[u'(e_L) - u'(e_M)] > 0$$

which is true because $u''(.) < 0$ and $e_L < e_M$, thus increasing welfare. A similar argument can be used to show that $\mu_M > 0$ and $\mu_H = 0$ is not possible either. A result, because μ_M and μ_H are both strictly positive, we must have:

$$\begin{aligned}
\phi \pi k(g_H - g_M) &= d_H - d_M \\
\phi \pi k(g_M - g_L) &= d_M - d_L
\end{aligned}$$

It is easy to see that $d_i = \phi \pi k g_i$, together with $J = (1-\phi)\mathbb{E}_i(\pi k g_i)$, is a solution for all the equations. And since the problem has a unique solution, it must be the solution. This

contract can be interpreted as an equity contract. Each entrepreneur sells a share $1-\phi$ of his firm to the intermediary and uses the proceeds to buy an indexed stock market financial instrument. This completely smooths out a proportion $(1-\phi)$ of the idiosyncratic risk. However, to prevent stealing not all the shares can be sold, the entrepreneur must retain a proportion ϕ of his shares, which is his "skin in the game." This is the best insurance possible with only short term contracts. Note that here we assume that there was no aggregate risk. This result would not be affected by it, since it would affect all the IC constraints proportionally. It would only change the pricing of J. I'll do this next.

D Efficiency: Planner's Problem with Exogenous Capital

We start characterizing the Planner's problem when history cannot be used to provide insurance. Lets set $E_1 = 0$. This at the end is irrelevant since only picks a point in the Pareto frontier. Here what we do is to maximize the entrepreneur's utility subject to satisfy a given level of utility to the workers: $\bar{u}$. Thus, moving $\bar{u}$ from the minimum level to the maximum we can characterize the whole Pareto Frontier. Also, in order to characterize the optimal plan is is convenient to write the individual profits in the reduced form: $\pi(s)kg_i = \alpha(s)Y(s)g_i$. The planner solves:

$$\max_{\{J_1, d_{1,i}, J(s), d_{2,i}(s)\}} \Big\{ \mathbb{E}_i[u(J_1 + d_{1,i})] + \mathbb{E}_{i,s}[u(J(s) + d_{2,i}(s))] \Big\}$$

$$\text{st.} \quad \phi\alpha(s)Y(s)[g_{2,i} - g_{2,i'}] + d_{2,i'}(s) + J(s) \leq J(s) + d_{2,i}(s); \quad \forall i, i' \text{ and } \forall s$$

$$\alpha Y_1[g_{1,i} - g_{1,i'}] + d_{1,i'} + J_1 \leq J_1 + d_{1,i}; \quad \forall i, i'$$

$$u\left(\mathbb{E}_i[\alpha Y_1 g_{1,i} - d_{1,i}] - J_1 + (1-\alpha)Y_1\right) + ...$$

$$... + \sum_s \Pi(s) u\Big(\mathbb{E}_i[\alpha(s)Y(s)g_{2,i} - d_{2,i}(s)] - J(s) + (1-\alpha(s))Y(s)\Big) \geq \bar{u}$$

Notice that in the worker's utility the labor share cancels out when $\mathbb{E}_i g_{t,i} = 1, \ \forall t$. Thus the problem reduces to:

$$\max_{\{J_1, d_{1,i}, J(s), d_{2,i}(s)\}} \Big\{ \mathbb{E}_i[u(J_1 + d_{1,i})] + \mathbb{E}_{i,s}[u(J(s) + d_{2,i}(s))] \Big\}$$

$$\text{st.} \quad \phi\alpha(s)Y(s)[g_{2,i} - g_{2,i'}] + d_{2,i'}(s) + J(s) \leq J(s) + d_{2,i}(s); \quad \forall i, i' \text{ and } \forall s$$

$$\alpha Y_1[g_{1,i} - g_{1,i'}] + d_{1,i'} + J_1 \leq J_1 + d_{1,i}; \quad \forall i, i'$$

$$u\left(-\mathbb{E}_i[d_{1,i}] - J_1 + Y_1\right) + \sum_s \Pi(s) u\Big(- \mathbb{E}_i[d_{2,i}(s)] - J(s) + Y(s)\Big) \geq \bar{u}$$

The solution for the IC constraints is still the same as before, therefore replacing the optimal d_i's we are left with just the inter temporal smoothing problem:

$$\max_{\{J_1, J(s)\}} \Big\{ \mathbb{E}_i[u(J_1 + \phi\alpha Y_1 g_{1,i})] + \sum_s \Pi(s)\mathbb{E}_i[u(J(s) + \phi\alpha(s)Y(s)g_{2,i})] \Big\}$$

$$\text{st.} \quad u\left(-\phi\alpha Y_1 - J_1 + Y_1\right) + \sum_s \Pi(s) u\Big(- \phi\alpha(s)Y(s) - J(s) + Y(s)\Big) \geq \bar{u}$$

Let λ be the Lagrange multiplier in the constraint, the foc's are:

$$\mathbb{E}_i[u'(J_1 + \phi\alpha Y_1 g_{1,i})] = \lambda u'\left(-\phi\alpha Y_1 - J_1 + Y_1\right)$$

$$\Pi(s)\mathbb{E}_i[u'(J(s) + \phi\alpha(s)Y(s)g_{2,i})] = \lambda\Pi(s)u'\Big(-\phi\alpha(s)Y(s) - J(s) + Y(s)\Big);\ \ \forall s$$

Which canceling λ becomes:

$$\frac{\mathbb{E}_i[u'(J_1 + \phi\alpha Y_1 g_{1,i})]}{u'\left(-\phi\alpha Y_1 - J_1 + Y_1\right)} = \frac{\mathbb{E}_i[u'(J(s) + \phi\alpha(s)Y(s)g_{2,i})]}{u'\left(-\phi\alpha(s)Y(s) - J(s) + Y(s)\right)} \quad \Rightarrow \quad \frac{\mathbb{E}_i[u'(e_i)]}{u'\left(c_1\right)} = \frac{\mathbb{E}_i[u'(e_{2,i}(s))]}{u'\left(c_2(s)\right)}$$

Which is exactly the same condition as in the competitive equilibrium. Thus, the CE is constraint efficient. The only possibility for the equilibrium to be inefficient is that the introduction of endogenous capital creates a distortion on the optimal investment or that in some way we can introduce history dependency in the contracts.

E Numerical Appendix

This section briefly describes the Numerical Algorithm to solve the model.

a. Using $g_s k$, compute once and for all $Y(s)$, $r(s)$ and $w(s)$

b. Guess $\Pi_0(s'|s)$, $p_0(s'|s)$ and $\nu_0(s)$, then compute $T(s)$ and $h(s)$.

c. Use (18) to solve for $\zeta(s)$, then (17) to get: $\phi(s'|s)$

d. From (31) obtain $\vartheta(s)$ and then use (27) to recover $\mathbb{E}_i[o(s', i; \phi^e)^{-\sigma}]$, which can be inverted to obtain: $\phi^e(s', s)$. (See next slide)

e. Use (25) to compute a new $\nu(s)$. This finishes the solution given the guess.

f. Update guesses. For this use (30) to obtain new prices: $p_1(s'|s)$.

g. Using (33) and (32) compute a new law of motion of s: $\Pi_1(s'|s)$.

h. If $\Pi_1 = \Pi$ and $p_1 = p$, stop otherwise update and start again in step 2 with:

$$\Pi_0 = 0.5\Pi_0 + 0.5\Pi_1; \quad p_0 = 0.5p_0 + 0.5p_1; \quad \nu_0(s) = 0.5\nu(s) + 0.5\nu_0(s).$$