The Macroeconomics of Hedging Income Shares

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Abstract

Since the 1980's the US economy has experienced a decline in the labor share, falling real rates, and accumulation of safe assets in the corporate sector. To study these facts, we propose a neoclassical growth model with capital-biased technological change, a production function with non-constant income shares (CES), and financial friction for firms. We discuss theoretically how risk sharing is distorted by the combination of changing shares of income and financial frictions, and how a hoarding of safe assets by firms emerges naturally as a tool to improve risk sharing. We calibrate our model to the US economy after 1980's and show that low rates, rising capital shares, accumulation of safe assets by firms and risky assets by households, can be rationalized by persistent capital-biased shocks and limited risk sharing.

Keywords: Decreasing labor share. Risk Sharing. Asset prices.

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1 Introduction

The prevalence over time of the Kaldor facts led to the dominant belief that the capital and labor income shares were, besides some small short run variations, roughly constant. This has many implications, among those is the impossibility of insurance against aggregate risk between workers and capitalists. Since aggregate shocks affect both sectors in the same way, even if it were possible, aggregate risk would be uninsurable. However, many recent studies find that the labor share seems to be varying far more than the Kaldor's prediction, maybe even having a downward trend.[1] But, if aggregate shocks have differential impacts on capitalists and workers, new possibilities of insurance arise. How do these insurance possibilities affect the financial markets? In particular, what kind of assets could be affected? Last but not least, how quantitatively important are their implications?

Since the labor share is pro-cyclical in the short run and counter-cyclical in the medium-long run, we need to take a stand on what horizon we have in mind. Because the downward trend appears to be the dominant feature, we focus on the long run. We first show theoretically that the counter-cyclical changes on the labor share (pro-cyclical capital share) can be insured between capitalist and workers by capitalist accumulating large quantities of risk free assets, lending it to the workers, and then workers using the loans to leverage and buy large quantities of risky assets (equity). Further, since capitalists are subject to more uninsured idiosyncratic risk than workers, the increase in the capital share amplifies this risk, strenghtening the demand for precautionary savings, which in turn dampens the risk free interest rate. We show that this is not only a theoretical possibility but it is also quantitatively relevant and analyze its normative implications.

All the predictions of this straightforward channel are present in different strands of literature. For instance, there is a growing literature trying to account for what is known as the "Corporate savings glut", changing the view of corporations as net borrowers to net lenders. Our theory has the additional implication that it must be accompanied by a "household equity glut", that is, according to our model, household equity holdings must rise in the medium-long run; this aspect seems to be overlooked in the literature of financial frictions. Instead, the continuously falling interest rate has been widely documented. These stylized facts motivate our work.

We build on the neoclassical growth model with constant elasticity of substitution technology (CES) and capital intensity that fluctuates persistently over time. The economy is populated by a continuum of entrepreneurs with different endowments of capi-

[1] See Karabarbounis and Neiman (2014) and Koh et al. (2017) for two recent studies pointing out to the downward trend.

tal and households-workers. Entrepreneurs own the capital, rent labor from households and carry over the production. Households supply labor inelastically and potentially funds to the firms through the financial markets. There is a contracting friction for the entrepreneurs. As in DeMarzo and Fishman (2007) the returns of the entrepreneur cannot be verified as entrepreneurs can privately divert some resources for consumption. Firms would like to share risk with households and obtain funding, but they are subject to a "skin in the game" constraint. This constraint implies that the households require the entrepreneurs to keep a fraction of their investment in order to deter them from diverting funds to private accounts. There are enough financial instruments available such that both entrepreneurs and households can hedge the aggregate risk. However, the financial friction prevents capital owners from fully insuring against idiosyncratic risk, which in turn affects their willingness to bear aggregate risk.

Notice that an important departure from the standard macro-finance literature is the CES technology rather than Cobb-Douglas. This assumption allows us to study the effects of changing labor and capital income shares over times. We assume that the economy is subject to capital-biased technological shocks, and, since we are focusing on the medium-long run, that the elasticity of substitution is bigger than one, as supported by recent empirical studies; in particular, Koh et al. (2017) and Karabarbounis and Neiman (2014). Therefore, positive aggregate shocks and growth are correlated with an increasing capital income share.

We start by characterizing the optimal risk sharing. We show that if entrepreneurs are able to fully insure their idiosyncratic risk, there is efficient risk sharing of aggregate risk between capital holders and workers. For instance, if the capital income share increases along the business cycle, entrepreneurs compensate workers with contingent transfers. The opposite is true if the labor share increases. The complete markets (only for aggregate risk) assumption is an abstraction that help us to characterize the solution, but which do not see as restrictive. In reality, there are multiple types of financial assets that have payoffs correlated with realizations of aggregate shocks and do not rely and/or are constrained by individual moral hazard or commitment problems. In principle, by properly combining these assets, any individual could replicate the same insurance target as a complete set of Arrow-Debreu securities will do. In fact, it is well known, that one only needs as many (not perfectly correlated) assets as possible shocks exist to achieve the best insurance possible.[2] Because of this, we consider the straightforward and intuitive case

[2]The number of assets needed to implement the complete market allocation is reduced as the frequency of trades increases. For instance, as the frequency of trades becomes infinitesimal and the underlying risk is characterized by a Brownian motion (a continuous time economy), only two assets are needed; the Brownian motion can be approximated as the limit of a Binomial process. See for example Merton, 1992 and

in which at any given period there are only two possible realizations of the aggregate shock, so that a risk free asset and a risky asset suffice to implement the complete markets allocation.

When we analyze this case, we find that it is implemented with firms taking a long position (saving) on the risk free asset and households taking a long position on the risky asset (buying equity). Intuitively, households leverage (borrowing from firms) and buy shares on corporations to participate on the changes in the income shares. This market allocation is reminiscent of a corporate savings glut, which is complemented with a households-workers equity glut. However, precisely because markets are complete, all wealth effects are absent, thus in a stationary economy all financial positions and asset prices should be constant and independent of history.

On the other end, if the idiosyncratic risk is present (because the moral hazard friction is relevant) but the labor income share is constant, aggregate risk sharing is no longer possible, and thus, aggregate net holdings of assets become degenerate. However, the presence of idiosyncratic risk brings about wealth effects that can distort inter-temporal choices. We show that if the production function exhibits a elasticity of substitution equal to one (Cobb-Douglas), because it is not possible to insure aggregate risk, the risk free asset is not traded in equilibrium and the volatility of financial assets is efficient. The market incompleteness, due to the financial friction, does generate an inefficient level of output and consumption, but does not affect the reaction of the economy to aggregate shocks.

When idiosyncratic and aggregate income shares risk coexist, there are important wealth effects that distort the efficient allocation of insurance. First, we show that the presence of idiosyncratic risk generates precautionary savings needs that dampen the incentives to insure aggregate risk, reducing the financial positions that workers and capitalists would have otherwise chosen. However, the idiosyncratic risk generates an additional channel: a wealth effect. As opposed to the efficient economy, where relative wealths are unaffected, because firms are more exposed to idiosyncratic risk than households, as the capital income share increases the aggregate demand for insurance also rises. In addition, since the main net suppliers of funds for insurance are households, the implied decline in the labor share decreases the supply of insurance. At first sight, it looks like the aggregate shock generates time varying uncertainty. However, it is important to notice that from each individual entrepreneur's perspective the uncertainty remains constant. It is the aggregate weight on the different agent's exposure to uninsured risk that changes. Thus, if positive aggregate shocks increase the capital income share, there is

Stokey, 2008.

an increase in the aggregate demand for insurance that has two important consequences: a fall of the risk free interest rate (increase in the price of insurance) and an increase in the demand of safe assets on part of the firms. Both predictions are consistent with the important recent developments in the US economy.

As we do in the efficient benchmark, we can also implement this equilibrium with only a risk free and a risky asset. As one might have expected from the previous discussion, the tendency towards capitalists accumulating risk free assets and workers accumulating equity remains, in spite of the additional precautionary savings that significantly temper the tendency toward efficiency. But the wealth effects generate an additional channel that amplifies the reallocation of aggregate portfolios over time. All in all, the economy generates a large positive correlation between firms' long position on the risk free asset and the capital income share. On the other side, households increase their leverage, borrowing from firms, and increasing their positions on the risky asset. It is important to emphasize that the key difference between the complete markets economy and the incomplete markets is the wealth effect. In the efficient economy, the realization of aggregate shocks do not alter the split of the total wealth between households and entrepreneurs. Instead, when capitalists are exposed to idiosyncratic risk, to self insure firms hold inefficiently high levels of the risk free asset, thus when positive aggregate shocks happen, not only the capital share increases, but also the wealth distribution tilts in favor of capitalists.

Literature Review. We build on the findings on decreasing labor shares, capital-specific productivity, and increased corporate savings. In particular, Karabarbounis and Neiman (2014) explore thoroughly the declining global labor share and find evidence consistent with earlier work by Blanchard et al. (1997), Blanchard and Giavazzi (2003), Jones et al. (2003), and Bentolila et al. (1999). Examples of broader studies of trends in labor shares are Harrison (2005) and Rodriguez and Jayadev (2010). Also, our paper relates to the literature on capital-specific productivity change (Greenwood et al., 1988, Hsieh and Klenow, 2007, Greenwood et al., 1997, Fisher, 2006). The rise in corporate savings has been documented in Sánchez et al., 2013, Chen et al., 2017 and recently discussed in Begenau and Palazzo, 2017.

Our paper is related to the literature on the amplification of shocks in the macroeconomy from the seminal works of Bernanke and Gertler (1989) and Kiyotaki et al. (1997). These two papers spurred a literature that place at the center of the stage financial frictions as amplifiers of business cycle fluctuations. Our paper relates to the recent contributions of He and Krishnamurthy (2011), Brunnermeier and Sannikov (2014), DiTella (2017) where financial frictions and heterogeneity play a key role in determining allocations (see also, Silva, 2016 and Gomez et al., 2016). We depart from some of the previous contributions by introducing labor income and on the production side a CES technology and workers that obtain labor income. This assumption allows us to study the effects of changing labor and capital income shares over asset prices and quantities. Further, we propose a discrete time framework and a numerical algorithm to find a global solution.

Our paper is related also to a recent literature that connects low risk free interest rates, risk-premia, changes in the labor shares. Caballero et al. (2017a) proposes an accounting framework that relates falling short term real rates, constant marginal product of capital, the decline of the labor share and a stable earnings yield from corporations. Caballero et al., 2017b focuses on the shortage of safe assets. Carvalho et al., 2016a proposes a demographic explanation for the evolution of real rates.[3] Our paper focuses on imperfect risk sharing and capital-biased technological change as the mechanism driving the hoarding of assets by corporations and implying low rates. Also, we uncover that on the other side of these increased holdings, consumers are holding a higher fraction of aggregate risk, consistent with the recent US experience.

2 Hedging Income Shares

In this section we study a two period model to discuss the implications of changes in income shares over asset prices and quantities and to highlight the key economic mechanism generating the results of the quantitative analysis of Section 3.

2.1 Environment

There are two types of agents, households and entrepreneurs. The economy lasts for two periods, $t = 1, 2$. There are two sources of uncertainty. Aggregate shocks, indexed by $s \in \mathbb{S}$ which occur with probability $\Pi(s)$, and idiosyncratic production shocks, indexed

[3] Our paper also speaks to some of the facts that have been labeled as the secular stagnation. The key idea of secular stagnation, a notion dating back to Hansen (1939), is that the real interest rate needed to achieve full employment is negative and this casts on the economy the shadows of low growth and high unemployment. From a theoretical point of view, Eggertsson and Mehrotra (2014) was one of the first attempts to formally model secular stagnation. Since then, there has been an increasing number of papers aimed at understanding secular stagnation, including Kocherlakota (2013), Benigno and Fornaro (2016), Schmitt-Grohé and Uribe (2012), Caballero and Farhi (2014), Eggertsson et al. (2016), and Marx et al., 2018. With the exception of Kocherlakota (2013) and Marx et al., 2018, all the previous papers study the subject including nominal wage rigidities. Instead, the labor market in our design is totally flexible, so much that households can completely insure against idiosyncratic labor income shocks, and there is no unemployment. The mechanism we derive stems from a risk taking motive due to changes in the relative share of income that goes to capital.

by $i \in \mathbb{I}$. Both are shocks to effective capital. In this paper we will assume that s is capital-biased technological shock, and as such it will affect the capital and labor income shares. In this section, for simplicity, we assume that there is no discounting.

Consumers. Households are endowed with initial assets W_1 and can supply one unit of labor at no utility cost. Labor income in period one is certain and given by ω_1, which denotes the wage rate. Households would like to insure the realization of the aggregate state in the second period. In this period consumers receive $\omega(s)$ as labor income, which is contingent on the realization of the aggregate shock. To insure against aggregate shocks the consumer has access to a complete set of Arrow-Debreu securities, that we denote by $W(s)$, are contingent on state s, and she can be buy and sell at prices $p(s)$. The consumer maximizes expected utility:

$$\max_{\{c_1, c_2(s), W(s)\}} u(c_1) + \mathbb{E}_s(u(c_2(s)))$$

subject to the budget constraints

$$c_1 + \sum_s p(s) W_2(s) \leq W_1 + \omega_1 \tag{1}$$

$$c_2(s) \leq W_2(s) + \omega_2(s). \tag{2}$$

The consumer can use initial assets W_1 and the first period income ω_1 to consume and buy Arrow securities to insure aggregate shocks. In the second period consumption is given by the realization of income and the payoff of the assets acquired in the first period.

Entrepreneurs. Entrepreneurs are endowed with initial financial assets E_1 and capital K_1. There is no investment.[4] Capital yields a certain return in period one, which we denote as π_1. Entrepreneurs are the only agents that are allowed to use capital for production. The returns to capital are a function of the aggregate and an idiosyncratic shock, and are denoted as $\pi_2(s, i)$. The entrepreneurs would like to share idiosyncratic risk with the consumers, but they are prevented to do so due to a financial friction: they can divert the returns of capital to a private account. Entrepreneurs can buy a complete set of Arrow-Debreu securities $E(s)$, which are contingent on s but are not contingent to i. The problem of the entrepreneur is:

$$\max_{\{e_1, E_2(s)\}_{s \in \mathrm{S}}} u(e_1) + \mathbb{E}_{s,i}(u(e_2(s, i)))$$

[4] This simplifies the exposition for this section and does not alter any qualitative result. In Section 3.2, when we analyze the infinite horizon economy, we introduce investment.

subject to

$$e_1 + \sum_s p(s) E_2(s) \leq E_1 + \pi_1$$
$$e_2(s,i) \leq E_2(s) + \pi_2(s,i)$$

for all (s,i). The entrepreneur can use initial assets E_1 and the first period return on capital π_1 to consume and buy Arrow securities, which provide insure aggregate shocks. Consumption is given by the realization of the return to capital $\pi(s,i)$ income and the payoff of the assets acquired in the first period, $E_2(s)$.

Technology. Entrepreneurs combine labor and capital to produce output using a CES production function:

$$y(k(s,i), L) = y(s,i) = [\alpha \left(k(s,i)\right)^{\frac{\rho-1}{\rho}} + (1-\alpha) L^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}.$$

where $k(s,i) = g_i g_s K_1$ is the effective stock of capital, and g_i, g_s represent the idiosyncratic and the aggregate shocks.[5] In Appendix B.1 we show that:

$$\pi(s,i) = \alpha(s) g_i Y(s) \tag{3}$$
$$\omega(s) = (1-\alpha(s)) Y(s), \tag{4}$$

where $Y(s) \equiv \mathbb{E}_i(y(s,i))$, $K(s) \equiv \mathbb{E}_i(k(s,i)) = g_s K_1$ and

$$\alpha(s) \equiv \alpha \left(\frac{Y(s)}{K(s)}\right)^{\frac{1-\rho}{\rho}}. \tag{5}$$

Equations (3) and (4) state that profits and wages are a fraction of total output. This fraction, $\alpha(s)$, that we define in (5) is the labor share. Notice from equation (5) that when $\rho = 1$, we are back to the standard Cobb-Douglas case with constant shares, i.e., $\alpha(s) = \alpha$, for all $s \in \mathbb{S}$. In our environment, instead, the capital share will change with aggregate shocks and will be pro-cyclical if $\rho > 1$ and counter-cyclical if $\rho < 1$. Notice also that changes in wages from consumers come not only movements in aggregate output, but also from change in the labor share. The same holds for the entrepreneurs.

[5]We assume that the exogenous shocks affect capital directly so that the entrepreneur's budget constraint is linear in both individual capital and the idiosyncratic shock, which greatly simplifies the entrepreneur's dynamic optimization problem. Later, in Section 3.2, when we consider the infinite horizon model with depreciation rate δ, to maintain the linearity of the entrepreneur's budget constraint, we assume that the exogenous shocks also affect depreciation. Therefore we define the gross return on capital: $R(s) = (1-\delta) g_i + r(s) g_i$.

Contracting. Since entrepreneurs are subject to idiosyncratic risk, they will try to insure it. To that end, we assume that entrepreneurs have access to risk neutral intermediaries who can provide insurance. However, entrepreneurs are subject to moral hazard problems, which puts a limit on how much idiosyncratic risk can be offloaded. To be precise, following the literature we model moral hazard as endowing entrepreneurs with the possibility of diverting resources from the firm to their private accounts at a cost $0 < 1 - \psi < 1$. That is, for each unit of profits that they divert to their private accounts only ψ units transform into consumption goods (or savings). We analyze a setup, similar to DeMarzo and Fishman (2007), where a risk neutral principal provides insurance to the entrepreneurs. The contract stipulates that the entrepreneur must hand over to the financial intermediary a given proportion of her risky profits and in return receives an average of the profits of all firms. Since entrepreneurs can misreport their profits and consume (or save) a proportion ψ of the misreported profits, in Appendix D we show the optimal contract implies that the entrepreneur must retain (or being exposed) to a proportion ψ of the idiosyncratic risky. This is known in the literature as the "skin in the game".[6] As result, we can write the exposure to the idiosyncratic risk in a simple reduced form. Let $\tilde{g}_i$ be the productivity shock to which the firm is exposed. Then, an economy with idiosyncratic risk $\tilde{g}_i$ and possibility of insurance is equivalent to an economy in which there is no possibility of insurance of individual risk and firms are subject to idiosyncratic risk g_i satisfying:

$$g_i = (1 - \psi)\mathbb{E}_i \tilde{g}_i + \psi \tilde{g}_i. \tag{6}$$

Markets. Market clearing requires that:

$$c_1 + e_1 = Y_1 \tag{7}$$

$$c_2(s) + \mathbb{E}_i(e_2(s,i)) = Y_2(s) \tag{8}$$

$$W_2(s) + E_2(s) = 0 \tag{9}$$

$$1 = L^s = L^d. \tag{10}$$

The first constraint, equation (7), is market clearing for goods in period 1. It also implies that the initial asset holdings are such that $W(1) + E(1) = 0$. The second constraint,

[6]DeMarzo and Fishman (2007) assumes that the principal can sign long term contracts (there is commitment) and that both the principal and the agent are risk neutral. In contrast, we consider a risk averse agent who can only commits to short term contracts. For similar setups and results in continuous time see DeMarzo and Sannikov (2006). We also show in the Appendix that as long as insurance contracts cannot be history dependent, this is the best possible insurance independently of whether entrepreneurs have access or not to hidden savings. This contract is akin to an equity contract where the entrepreneur creates a company issue equity for a value amounting to a proportion $1 - \psi$ of the ex-ante value of the company and retains a proportion ψ of the shares.

equation (7), is market clearing for goods in period 2. Note that the idiosyncratic i.i.d. shocks wash out in the aggregate. The final two constraints, equations (9) and (10), specify asset and labor markets clear, respectively. A *Competitive Equilibrium* is an allocation of consumption and labor $\{c_1, e_1, c_2(s), e_2(s,i), L\}_{s\in S, i\in \boldsymbol{I}}$, asset holdings $\{W_2(s), E_2(s)\}_{s\in S}$, asset prices $\{p(s)\}_{s\in S}$ and wages $\{\omega_2(s)\}_{s\in S}$ such that: given prices the consumer maximizes utility by choosing asset holdings and consumption; given prices the entrepreneur maximizes utility by choosing asset holdings, labor, and consumption.

Risk Sharing. Lets start by deriving the optimality conditions for consumers and entrepreneurs. From the first order conditions the programming problem of the consumer and the entrepreneur, we obtain:

$$p(s)u'(c_1) = \Pi(s)u'(c_2(s))$$
$$p(s)u'(e_1) = \Pi(s)\mathbb{E}_i[u'(e_2(s,i))].$$

A key element of the above equations is that, due to the existence of complete set of securities for the aggregate state, the Euler equation holds state by state; for the aggregate state $s \in S$. The two of them together imply:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[u'(e_2(s,i))]}{u'(c_2(s))} \tag{11}$$

Note that equation (11) states that the ratio of average marginal utilities is constant in equilibrium. We now re-express the Arrow-Securities as claims on aggregate output. In particular, define $\phi(s) \equiv \frac{W_2(s)}{Y_2(s)}$. From market clearing it implies that

$$W_2(s) = \phi(s)Y_2(s)$$
$$E_2(s) = -\phi(s)Y_2(s).$$

Let x_1 be the fraction of consumption of workers in the first period at $t = 1$. Then, from market clearing in the goods market in the first period and assuming the preferences of the agents are CRRA with parameter σ, we can rewrite (11) as:

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} = \frac{\mathbb{E}_i[(\overbrace{-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i}^{E_2(s)+\pi_2(s,i)})^{-\sigma}]}{(\underbrace{\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s)}_{W_2(s)+\omega_2(s)})^{-\sigma}}. \tag{12}$$

2.2 Complete Markets and Risk Sharing

How agents share risk in this economy when there are no contracting frictions? For that, we can either assume that the idiosyncratic shock can be fully insured, i.e. $\psi = 0$, or, similarly, that there is no idiosyncratic risk, i.e. $\mathbb{V}ar(\tilde{g}_i) = 0$; either case results in $g_i = 1$ for all i.

Proposition 1. *If entrepreneurs can insure their idiosyncratic risk, the competitive equilibrium is characterized by an allocation*

$$c_1 = x^{CM} Y_1, \;\; e_1 = (1 - x^{CM}) Y_1, \;\; c_2(s) = x^{CM} Y_2(s), \;\; e_2(s) = (1 - x^{CM}) Y_2(s) \tag{13}$$

with

$$x^{CM} = \frac{W_1 + \omega_1 + \sum p(s)\omega_2(s)}{Y_1 + \sum_s p(s) Y(s)},$$

and prices

$$p(s)^{CM} = \beta \Pi(s) \left(\frac{Y_2(s)}{Y_1} \right)^{-\sigma}, \;\; \omega(s)^{CM} = (1 - \alpha(s)) Y_2(s).$$

Asset holdings are given by $W(s) = -E(s)$, $W(s) = \phi^{CM}(s) Y(s)$ where

$$\phi(s)^{CM} = x^{CM} - (1 - \alpha(s)). \tag{14}$$

Proof. See Appendix B.2. □

There are two points that are worth noting from Proposition 1. First, consumption shares are constant over aggregate states. This is a standard result.[7] It is useful to keep this allocation in mind because it generates the efficient reaction of the economy to aggregate shocks, and as such, it is a natural benchmark. In this equilibrium the entrepreneurs fully compensate the workers with contingent payments when the capital income share increases. This compensation is through Arrow-Debreu securities.

Second, note that if α is constant, i.e. $\rho = 1$, the capital income share does not vary over the business cycle and neither do the positions on the Arrow-Debreu securities of the agents, $\phi(s)^{CM} = \phi^{CM}$ for all $s \in \mathbb{S}$. Intuitively, it would be pointless to write contracts contingent on the aggregate state as both types of agents equally suffer the consequences of aggregate shocks and therefore there are fixed gains from trading financial assets. When the capital share varies, the workers and entrepreneurs are affected in different ways by the shocks and therefore trading financial assets contingent on the aggregate

[7] Note that here we are using the change of state variable to express the consumption share as a function of the initial ratio of marginal utilities, m_1, rather than the initial distribution of assets W_1/E_1.

state can make everyone better off.

Implementation with two assets. Suppose there are only two aggregate shocks $s_L < s_H$, and two financial assets, a risk free bond B and a stock-market-indexed risky asset A with payoff $A\alpha(s)Y_2(s)$ for $s = L, H$. The risk free rate is denoted by R_L, and P_A denotes the price of the risky asset. $\{A^c, B^c\}$ is the portfolio allocation of the consumers and $\{A^e, B^e\}$ is the portfolio allocation of the entrepreneurs. In Appendix B.4 we show that the equivalence is given by:

$$R_L^{CM} B^{CM} = -\frac{Y_2(L)Y_2(H)(\alpha(H) - \alpha(L))(1 - x^{CM})}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{15}$$

$$A^{CM} = 1 - \frac{Y_2(H) - Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}(1 - x^{CM}) \tag{16}$$

Notice that if $\alpha(H) \neq \alpha(L)$ the household will take an active position in the risk free asset. If the position is positive or negative depends on the value of ρ, the EIS in production. If $\rho > 1$, the positive productivity shocks are associated with higher α. In this case, the household would take a negative (short) position on the riskless asset and a positive (long) position on the risky asset. Intuitively, she would borrow in the risk free asset to participate in the gains of the entrepreneurs should there be a positive shock to the capital share. This in turn means that the entrepreneurs are holding a positive amount of the risk free asset, and that this amount increases with $1 - x^{CM}$, the share of their own consumption. Alternatively, if $\alpha(H) = \alpha(L)$, i.e. we have a Cobb Douglas production function, the households do not hold risk free assets and the amount of risky assets that are traded depends on the initial distribution of assets, $A = 1 - \frac{1-x^{CM}}{\alpha}$. In particular if $E_1 = W_1 = 0$, then $A = 0$ and there is no asset trading. Instead, if $A \neq 0$ either the entrepreneurs or the households want to transfer resources across time. They do so by using the risky asset, not the risk-less asset.

Take Out and What Follows. We now want to understand how capital biased technological shocks affect portfolio allocations and prices over time. A useful setting is an economy in which there is no growth and each period there are only two possible outcomes for output (so that each period only the pairs $\{\alpha(L), Y(L)\}$, $\{\alpha(H), Y(H)\}$ can happen on the equilibrium path). Suppose we replicate this economy for more than two periods, then the portfolio allocation of the consumer (and, consequently, of the entrepreneur) is constant. Moreover, the risk free rate R_L and the price of the risky asset P_A are constant and therefore we have a constant risk premia. We will see later that if output is stationary but capital biased technological shocks are such that capital shares increase over each replication there will be important consequences for the behavior of prices and quantities with the cycle.

2.3 Incomplete Markets and Risk Sharing

We now characterize the distortions on risk sharing that are a consequence of incomplete markets. These distortions will generate excessive precautionary savings and, as a consequence, lower interest rates and price of risk. To gain some intuition regarding these distoritions we perform a second order Taylor approximation of the right hand side of (12) around the complete markets solution to obtain:

$$\left(\frac{1-x_1}{x_1}\right)^{-\sigma} \simeq \frac{(-\phi(s)+\alpha(s))^{-\sigma}}{(\phi(s)+(1-\alpha(s)))^{-\sigma}}\left(1+\frac{\sigma(1+\sigma)\alpha(s)^2}{(-\phi(s)+\alpha(s))^2}\frac{\mathbb{V}ar(g_i)}{2}\right) \tag{17}$$

for all $s \in \mathbb{S}$.

We can see that the main difference between the incomplete market and the complete market economies is given by the last term of equation (17), which is multiplicative in $\mathbb{V}ar(g_i)$ and is the measure of idiosyncratic uncertainty. More importantly, this term is increasing in $\alpha(s)$. For a given level of idiosyncratic risk, the larger the capital share in the economy, the larger the demand for insurance. At the same time, as $\alpha(s)$ increases, there are less resources available to the workers, and therefore the supply of funds for insurance purposes also decreases. Notice that what creates the different asset positions is the fact that the economy looks like it is exposed to time varying idiosyncratic risk. The larger $\alpha(s)$, the larger the individual risk. However, the mechanism is different; from the perspective of each individual entrepreneur the idiosyncratic risk, $\mathbb{V}ar(g_i)$, remains constant, what happens is that the share of "risky income" over total income increases, and so does the difficulty to insure it. Therefore, we have:

Proposition 2. *For a given initial level of the ratio of marginal utilities, if $\mathbb{V}ar(g_i) > 0$, and $\rho \neq 1$, then:*

a. Precautionary savings: $\phi(s)^{IM} < \phi(s)^{CM}$, for all s.

b. Decreasing risk sharing: ϕ^{IM}, ϕ^{CM} is increasing in α.

c. Increasing precautionary savings $\frac{\partial\phi(\alpha)^{IM}}{\partial\alpha} < \frac{\partial\phi(\alpha)^{CM}}{\partial\alpha} = 1$.

Proof. See Appendix B.3 □

The first part of Proposition 2 states that entrepreneurs hold some precautionary savings (recall that $\phi(s)$ is the position of consumers). The second part of Proposition 2 states that

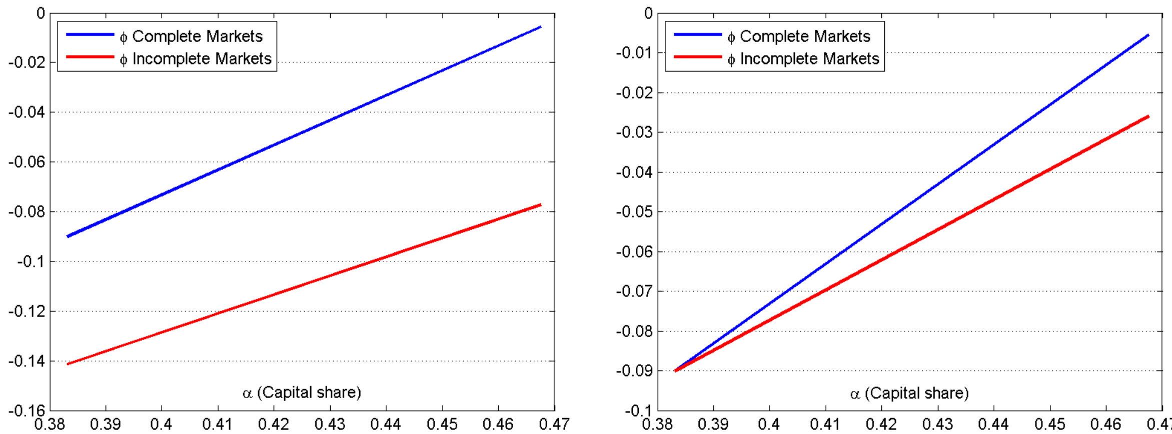

Figure 1: Financial positions: complete vs incomplete markets

as the differences in income shares widen so do the positions on Arrow-Debreu securities demanded in equilibrium. This is true, independently of the existence of uninsured idiosyncratic risk. The last part states that the demand for precautionary savings increases with $\alpha(s)$. This demand is measured as the difference between the asset positions that the entrepreneur holds in the incomplete markets economy and the position that it would hold if markets were complete.

The statements of Proposition 2 are depicted in Figure 1. In the left panel we can see that, in both the complete and incomplete markets economies, workers share the risk of the income shares. Also, in both cases, the trade of financial assets decreases with $\alpha(s)$. Finally, the positive position of the entrepreneurs (negative position of the workers) is larger in the incomplete markets economy. However, part 3) of Proposition 2 is not immediately apparent from the left panel. To clearly show this part, in the right panel we plot exactly the same values as in the left panel, but we have translated the line ϕ^{IM} to coincide with ϕ^{CM} in the first point. After correcting for this "level effect", it is evident how the curves move apart as $\alpha(s)$ increases. The distance between the two lines represents the excess holdings of precautionary savings due to market incompleteness, and how these holdings increase as the labor share decreases. As we will see next, Proposition 2 has important implications for asset prices and trading in financial assets.

Implementation with two assets. As for the case of complete markets, we now want to illustrate how capital biased technological change affects prices and the agents' positions in the risk free and in the risky assets. Again, we study an implementation with two assets and we show that the risk free interest rate decreases as the capital share increases and that our model generates a steep increase in the demand for safe (risk free) assets, reminiscent of the corporate saving glut. Suppose once again that there are only two shocks $s_L < s_H$, and two financial assets, a risk free bond B and a stock-market-indexed risky asset A with payoff $A\alpha(s)Y_2(s)$ for $s = L, H$. We can define $g^{CE}(\alpha, \phi)$ the function such that[8]

$$(-\phi(s) + \alpha(s)g^{CE}(\alpha, \phi))^{-\sigma} = \mathbb{E}_i[(-\phi(s) + \alpha(s)g_i)^{-\sigma}], \tag{18}$$

for all $s \in \mathbb{S}$. This function depends on α and ϕ and is such that $g^{CE}(\alpha, \phi) < 1$ if there is uninsurable idiosyncratic risk, because of the convexity of the marginal utility. Note that if there were complete markets, then $g^{CE}(\alpha, \phi) = 1$ for all levels of $\alpha(\cdot), \phi(\cdot)$.

Before showing the implementation, it is instrumental to characterize the $\phi(s)$ in terms of the certainty equivalent $g^{CE}(\alpha, \phi)$. Performing similar manipulations on the equilibrium condition as in the complete markets equilibrium, plugging (18) into (12), and using the definition of $\phi(s)$, we can show that:

$$\phi(s)^{IM} = x_1[\alpha(s)g^{CE}(\alpha, \phi^{IM}) + (1 - \alpha(s))] - (1 - \alpha(s)), \tag{19}$$

for all $s \in \mathbb{S}$. This equation does not provide a closed form solution for $\phi^{IM}(s)$ but nonetheless gives a clear intuition about the effect of incomplete markets. First, it confirms the result of Proposition 2, so that $\phi^{CM}(s) > \phi^{IM}(s)$, for all s. But, second, this has implications for the implementation. The incomplete markets distortion is multiplied by the consumption ratio x. Thus, the smaller x, the smaller the distortion. In the limit, as $x \to 0$, it must be the case that $\phi^{IM} \to \phi^{CM}$. Also, the distortion decreases the amount of Arrow-Debreu securities that are traded in equilibrium, which shows that the presence of idiosyncratic risk diminishes the quantitative relevance, on average, of changes of α on the trading of financial assets.

To analyze the implementation with two assets we can write the analogous of equations (15) and (16), writing assets positions in terms of the capital share and the certainty equivalent, α and $g^{CE}(\alpha, \phi)$, as:

$$R_L B = R_L^{CM} B^{CM} - x_1 \frac{\alpha(L)Y_2(L)\alpha(H)Y_2(H)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \left[g^{CE}(H) - g^{CE}(L)\right] \tag{20}$$

$$A = A^{CM} + x_1 \left[\left(g^{CE}(L) - 1\right) + \frac{\alpha(H)\alpha(L)\left(g^{CE}(H) - g^{CE}(L)\right)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)}\right] \tag{21}$$

We have purposely written equations (20) and(21), as the efficient allocation plus/minus a distortion, to emphasize the impact of the idiosyncratic risk. First notice that if α is con-

[8] With an abuse of terminology we will sometimes refer to g^{CE} as the "certainty equivalent" even though we are not working with utilities but with marginal utilities

stant, it must be the case that $g^{CE}(H)$=$g^{CE}(L)$. Thus, the distortive term affecting the holdings of the risk free asset vanishes; and since we have already shown that $R_L^{CM}B^{CM}$= 0, it must also be that $R_L B = 0$. Again, as in the complete markets economy, the risk free bond is not used in equilibrium. If it is necessary to transfer resources across periods, this is done using the risky asset. But now, the capitalists need to accumulate some savings to hedge the idiosyncratic risk. They do so buying the risky asset, which leaves less for the workers. Hence, the term $x_1\left(g^{CE}(L)-1\right) < 0$ reducing A.

What about the economy with varying shares? Recall that we are considering an economy with ρ >1, therefore the capital income share must be pro-cyclical, i.e., $\alpha(H) > \alpha(L)$, which in turn implies $g^{CE}(H) < g^{CE}(L)$. Looking at the distortive term, now we have that $R_L B$ is equal to $R_L^{CM}B^{CM} < 0$ plus a positive term. That is, workers borrow less on (capitalist accumulate less of) the risk free asset. Similarly, the distortive effect is also negative on A. The presence of idiosyncratic risk further reduces the workers holdings of risky assets. In short, capitalists hoard on both assets, hindering the possibility of insuring aggregate risk.

It is interesting, that the distortion to the holdings of the risky asset stems from two sources. The first source, captured by the term $\left(g^{CE}(L)-1\right) < 0$, arises just because of the existence of uninsured idiosyncratic risk, and it remains there even when α is constant. The second source, captured by the term $\frac{\alpha(H)\alpha(L)\left[g^{CE}(H)-g^{CE}(L)\right]}{\alpha(H)Y_2(H)-\alpha(L)Y_2(L)} < 0$, arises because of the presence of "time varying" uncertainty. The inefficiency due to uninsured idiosyncratic risk interacts with the stochastic income shares amplifying the distortions.

Finally, notice that the distortions are multiplied by the workers' wealth ratio, x. In particular, as x decreases, so does the extend of the distortion. In the limit, when $x = 0$ (workers have zero net worth) the asset's positions coincide with the efficient ones. This also gives us a hint about the impact of the wealth effects. Since the wealth effects tend, on average, to reduce x over time, we should expect the aggregate asset's holdings converging to those in the complete markets allocation. But because the positions would start as a compressed version of the efficient, it would look like a continuous widening of the aggregate holdings. In other words, one should expect a continuous increase in the firms savings on the risk free asset, accompanied by a continuous increase in the workers equity holdings.

3 Infinite horizon economy

In this section we relax the assumption that the economy lasts for only two periods and we allow for $t \in \mathbb{N}$. Also, we allow for any arbitrary number of aggregate states

$s \in [s^1, s^2, ..., s^N]$. The probability of each state is $Prob(s'|s) = \Pi(s'|s)$. To simplify notation in what follows we characterize the solutions in a recursive fashion. In the two period economy there was no investment and given that after the second period there was no choice to be made, keeping track of the exogenous aggregate shock was enough. However, we also showed that the initial distribution of wealth was a determinant of allocations. In the infinite horizon economy the distribution of wealth will be changing along the business cycle. Thus, we will need to keep track of it, along with the effective stock of capital, to determine the equilibrium. The redefined state space is $s = \{g_s K, x\}$, where x is the ratio of the consumer's wealth to the total wealth in the economy. We formally show in Section 3.3 that these two states variable are enough to characterize the equilibrium. Since both K and x are endogenous variables, the transition function $\Pi(s'|s)$ is an equilibrium object. However, when solving the individual problems, subsection 3.1 and subsection 3.2, the composition of s and how its transition is determined are irrelevant, because they are taken as given by each individual.

3.1 Consumer

In the infinite horizon economy the consumer-worker solves:

$$\begin{aligned} V^c(a,s) &= \max_{\{c(s), a(s'|s)\}} \left\{ u(c(s)) + \beta \mathbb{E}_{s'}[V^c(a(s'|s), s')|s] \right\} \\ st. &\quad c(s) + \sum_{s'} p(s'|s) a(s'|s) \leq a(s) + \omega(s), \end{aligned}$$

where $\omega(s)$ is the wage in state s and $a(s'|s)$ are the Arrow-Debreu securities bought by the consumer in state s, that pay next period contingent on the realization of state s'. The initial financial wealth $a_1 \equiv a(s_0)$ is given. The first order conditions for consumption and financial decisions imply

$$p(s'|s) u'(c(s)) = \beta \Pi(s'|s) u'(c(s')); \quad \forall s'$$

Denote by $\zeta(s)$ the savings rate. We show in Appendix C.1 that the solution is characterized by:

$$c(s) = (1 - \zeta(s))(a + \omega(s) + h(s)) \tag{22}$$

$$a(s'|s) = \phi^c(s'|s)\zeta(s)[a + \omega(s) + h(s)] - \omega(s') - h(s') \tag{23}$$

where $h(s) = \sum_{s'|s} p(s'|s)[\omega(s') + h(s')]$ is the consumer's present value of future incomes, or human wealth. The function $\phi^c(s'|s)$, to be determined, pins down the portfolio allocation. Note that using the budget constraint it must be true that:

$$a'(s) \equiv \sum_{s'|s} p(s'|s)a(s'|s) = \zeta(s)[a + \omega(s) + h(s)] - h(s)$$

The latter implies $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$. In the next sections we will use the consumer's total wealth, $W^c(s) \equiv a + \omega(s) + h(s)$, to characterize the solution. Thus, $\zeta(s)$ is the saving rate out of wealth and $1 - \zeta(s)$ is the implied consumption rate. As we show in Appendix C.1 using the guessed policy functions the consumer's problem solution is fully characterized by:

$$\phi^c(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)}; \quad \forall s, s' \tag{24}$$

Using the condition $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$ and (24) we obtain:

$$(1-\zeta(s))^{-1} = 1 + \sum_{s'|s} \left[(\beta\Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma}(1-\zeta(s'))^{-1}\right]; \ \forall s \tag{25}$$

Taking prices, $p(s'|s)$, and the low of motion of s as given, the last is a recursive equation, linear in $(1-\zeta(s))^{-1}$, which solves for the savings rates. Once $\zeta(s)$ has been found, equation (24) solves for the state contingent assets holdings.

3.2 Entrepreneur

In this section we show that despite being subject to idiosyncratic risk the consumption and saving rates of the entrepreneurs are very simple and akin to those of the consumers.[9] In particular, due to homothetic preferences, savings rate are linear in total wealth, and thus total savings are independent of the distribution of wealth. In other words, there will be aggregation: knowing the average net worth is enough to forecast future aggregate capital. As in the two period model, with the natural extension to an infinity horizon, the entrepreneur solves:

$$\begin{aligned} V^e(E,k;s,i) \quad &= \max_{\{e(s,i),E(s'|s),k'(s,i)\}} \left\{u(e(s,i)) + \beta\mathbb{E}_{s',i'}[V^e(E(s'|s),k';s',i')|s]\right\} \\ st. \quad & e(s,i) + k'(s,i) + \sum_{s'|s} p(s'|s)E(s'|s) \leq E(s) + R(s)kg_i \end{aligned}$$

[9]See Angeletos (2007) for a similar result.

where $R(s)$ is the average gross return on capital, $E(s'|s)$ are Arrow-Debreu securities bought by the entrepreneur in state s, contingent on the realization of state s' the following period.[10] Notice that the return on capital depends only on the aggregate state, because the production function is of the CES type. The initial financial wealth $E_1 \equiv E(s_0)$ is given. Finally, as in the two period economy, g_i is the idiosyncratic shock to which the entrepreneur is subject to. We maintain the assumption that g_i is *i.i.d.* over time. The first order conditions for capital and Arrow-Debreu securities imply:

$$p(s'|s)u'(e(s,i)) = \beta\Pi(s'|s)\mathbb{E}_i[u'(e(s',i))]; \quad \forall s, s' \tag{26}$$

$$u'(e(s,i)) = \beta\mathbb{E}_{s',i}[u'(e(s',i))R(s')g_i|s]; \quad \forall s \tag{27}$$

As in the last subsection, 3.1, we guess and then verify (see Appendix C.2) that the solution is characterized by:

$$e(s,i) = (1-\vartheta(s))W^e(s,i,k) \tag{28}$$

$$k'(s,i) = \nu(s)\vartheta(s)W^e(s,i,k) \tag{29}$$

$$E(s'|s,i) = \phi^e(s'|s)E_1(s,i) \tag{30}$$

where $\vartheta(s)$ is the savings rate, and $\nu(s)$ is the portion of savings invested in capital; also, entrepreneur's total wealth is:

$$W^e(s,j,k) = E(s) + R(s)g_jk$$

In what follows we will refer to $\nu(s)$ as the investment rate. Using the budget constraint we have that total savings, $E_1(s,i)$, must satisfy: $E_1(s,i) \equiv \vartheta(s)(1-\nu(s))W^e(s,i,k)$. Therefore, it must also be true that $\sum_{s'|s} p(s'|s)\phi^e(s'|s) = 1$. The law of motion of individual wealth is:

$$W^e(s',i',k') = \vartheta(s)o(s',i;\phi^e,\nu)W^e(s,i,k) \tag{31}$$

where

$$o(s',i;\phi^e,\nu) \equiv [(1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')g_i]$$

[10]Notice that $R(s)$ is the gross return on capital, which shouldn't be confused with the net $r(s)$, and as such includes any potential depreciation of capital. In equilibrium, it would be true that $R(s) = (1-\delta)g_s + r(s)$, with $r(s) = \frac{\partial y(K,L,S)}{\partial K}$. Therefore, productivity shocks, both g_s and g_i, also affect capital depreciation. This assumption is necessary to obtain the linearity of the budget constraint respect to individual holdings of capital. This assumption is widely used in the literature. See for example Brunnermeier and Sannikov (2014) and Di Tella (2014).

is the ex-post growth rate of wealth. Using both Euler equations for the entrepreneur, equations (26) and (27), we obtain that the portfolio allocation, ϕ^e and $\nu(s)$, are determined by:

$$\mathbb{E}_{s',i|s}\left[((1-\vartheta(s'))o(s',i;\phi^e,\nu))^{-\sigma}\left(R(s')g_i - \frac{1}{\sum_{s'|s} p(s'|s)}\right)\right] = 0 \tag{32}$$

$$\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma} = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))};\ \ \forall s, s' \tag{33}$$

First, note as long as $\vartheta(s)$ is independent of wealth and g_i is i.i.d., the investment rate is also independent of individual wealth and of the current idiosyncratic shock. Second, note that equation (33) pins down $\phi^e(s'|s)$ which is also independent of wealth. Comparing (24) and (33) we see that the consumer's saving rate is affected by $\phi^c(s'|s)$ while for entrepreneurs the equivalent term is $\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma}$, which in turn is affected by both risk aversion and the exposure to idiosyncratic risk. The fact that equation (33) involves an expectation while equation (24) does not, is what generates the different behavior between workers and entrepreneurs. In absence of idiosyncratic risk both agents would react equally to aggregate shocks. Another way of to see the role of the idiosyncratic risk is comparing the saving rates, which for the entrepreneur satisfies:

$$(1-\vartheta(s))^{-1} = 1 + m(s)^{-1}\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma}(1-\vartheta(s'))^{-1}\right];\forall s \tag{34}$$

where $m(s) = \sum_{s'|s} p(s'|s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma}$. Notice that the only difference between (25) and (34) is $m(s)$. If $m(s) = 1, \forall s$, consumers and entrepreneurs would choose the same savings rates. Indeed, that is the case when the moral hazard friction vanishes. However, when the moral hazard prevents the full insurance of idiosyncratic risk, in general $m(s) > 1;\ \forall s$. As result, in equilibrium for any price function $p(s)$, it must be true that $\vartheta(s) > \zeta(s)$: on average the entrepreneurs' wealth grows faster than the consumer's wealth. This creates a downward drift on the consumers to entrepreneurs wealth ratio: x. As we showed in the two period model, this wealth effect has very important quantitative implications generating large changes in the financial assets' positions.[11]

[11] The drift also implies that in the limit the workers end up with zero wealth, while the entrepreneurs hold all the wealth in the economy. This may seem an odd implication of the model. This is a standard result, though, and the literature has alternative strategies to guarantee that there is a well defined stationary distribution of wealth. For example, introducing difference in discount factors, in particular entrepreneurs discount the future more heavily (lower β). See for instance DiTella (2017)), Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2011). Alternatively, we could assume that entrepreneurs become workers with some exogenous probability, keeping their wealth, and are replaced by workers. In this paper the

3.3 Equilibrium

For these allocations to be feasible, they must satisfy the assets' and goods' market clearing conditions, pinning down the equilibrium prices $p(s'|s)$, and $\Pi(s'|s)$ must be consistent with the laws of motion generated by individual decisions. The assets' and goods' market clearing conditions are:

$$a(s'|s) + E(s'|s) = 0; \quad \forall s, s' \tag{35}$$

$$c(s) + e(s) + K'(s) = y(s); \quad \forall s \tag{36}$$

where $e(s) = \int_i e(s,i,k,E)$, $K'(s) = \int_i K'(s,i,k,E)$, $y(s) = \int_i y(s,i,k,E)$ and $E(s'|s) = \int_i E(s'|s,i,k,E)$. Where we have avoided the dependency of the allocations on individual wealth because, as we show in the previous section, the savings and consumption rates are independent of it. However, for the aggregation we take it into account. Using equations for consumption and investment for the consumer and the entrepreneur, (22), (23), (28), (29) and (30), the market clearing conditions (35) and (36), can be written as:

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)\vartheta(s)(1-\nu(s))(1-x) = \frac{w(s') + h(s')}{W^T(s)}; \quad \forall s, s' \tag{37}$$

$$(1-\zeta(s))x + (1-\vartheta(s)(1-\nu(s))(1-x) = \frac{y(s)}{W^T(s)}; \; \forall s \tag{38}$$

where $W^T(s) = W^c(s) + W^e(s)$ and $x = W^c(s)/W^T(s)$. If instead of a CES, we were using a Cobb-Douglas production function, we could show that x would be independent of K. Thus, the state space would not need to include K. By Walras' Law, one of the market clearing conditions is redundant, while the other determines the equilibrium prices $p(s'|s)$.

To find $\Pi(s'|s)$, recall that the aggregate state includes endogenous variables, we need to characterize the endogenous laws of motion. First, recall that $k'(s,j) = \nu(s)\vartheta(s)W^e(s,j,k)$. Aggregating we obtain:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s) \tag{39}$$

Also, it is possible to show that the low of motion of the wealth ratio satisfies:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)}{\iota(s',s) + \nu(s)\vartheta(s)(1-x)R(s')} x(s) \tag{40}$$

distribution of wealth will be moving for two reasons: expected drift and the persistence of the relative shocks to capital $g_s g_i$, and does not affect our qualitative results.

Parameter	Description	Value
σ	Risk aversion	2
β	Discount Factor's	0.95
ρ	Elasticity of Substitution	1.25
α	Capital Share	0.275
δ	Depreciation	0.06
$g_{s,h}, g_{s,l}$	Aggregate Shocks to Capital	1.02, 0.98
$p_{s,h}$	Probability of g_h	0.5
$g_{i,h}, g_{i,m}, g_{i,l}$	Id. Shocks to Capital	0.3, 1, 1.1
$p_{i,h}, p_{i,m}, p_{i,l}$	Id. Shocks to Capital	1/3, 1/3, 1/3
ϕ	Exposure to Id Risk	0.2

Table 1: Baseline Calibration

where $\iota(s', s) = \frac{w(s')+h(s')}{W^T(s)}$. Notice that even though the process for x is not stationary, it is still Markovian. Thus, it is possible to compute its transition probabilities. As result, (39) and (40) together with the exogenous probability distribution over g_s determine the transition probabilities $\Pi(s'|s)$.

An important issue that arises from equation (40) is the possibility of multiple equilibria. The source of potential multiplicity is that s' contains x' itself. In general, $R(s')$ depends only on $g_{s'}K'$, the same is true for $\iota(s', s)$. However, $\phi^c(s'|s)$ will depend on x', i.e., agents would buy AD securities contingent on the expected realization of the distribution of wealth. Thus, the realized distribution, say $\tilde{x}$, must satisfy:

$$\tilde{x} = \frac{\phi^c(\{g_{s'}K', \tilde{x}\}|s)\zeta(s)}{\iota(g_{s'}K', s) + \nu(s)\vartheta(s)(1-x)R(g_{s'}K')}x$$

Without additional knowledge about the shape of $\phi^c(.|s)$, it is not possible to state if the above equation has more than one solution. We address this issue in the numerical implementation. We want to emphasize that this is not a problem that arises only in our environment, but is also present on many studies for economies with financial frictions. This setup just makes it more transparent.

3.4 Numerical calibration.

Regarding the preferences we use standard parameters in the literature. We set the discount factor $\beta = 0.95$ and a degree of risk aversion $\sigma = 2$, which are standard values in the real business cycles literature; see for example Cooley (1995). These values are consistent with risk free interest rate of 5% and the value of risk aversion of standard for business cycle fluctuations.[12]

Regarding the parameters for the production function, we need to find values for ρ, α, δ. For the elasticity of substitution, it has been documented that the labor share appears to be pro-cyclical in the short run, while is counter cyclical in the medium-long run; see for instance León-Ledesma and Satchi (2018).[13] A pro-cyclical labor share would imply an elasticity of substitution of $\rho < 1$, while the pro-cyclical labor share requires $\rho > 1$. Our focus is on the medium-long run and thus we use the estimates of Koh et al. (2016) and Karabarbounis and Neiman (2014), who analyze long term movements. The former estimates an elasticity of substitution between capital and labor $\rho = 1.15$ while the latter finds $\rho = 1.25$. Our baseline calibration is based on the estimation of Karabarbounis and Neiman (2014) and then we consider the alternative value found by Koh et al. (2016). Furthermore, since with a CES technology the average capital income share depends on the capital output ratio, we calibrate α and δ to target these two moments. Thus, we set $\alpha = 0.275$ in order to target, on average, a capital income share of around 0.3, and $\delta = 0.06$ to target a capital-output ratio of around 3.

Regarding the aggregate shock we consider only two values: $g_H = 1.02$ and $g_L = 0.98$ and we assign probability $1/2$ to each realization. The *i.i.d.* structure of the shock simplifies the state space; if this were not to be the case, we would to add another state variable. The fact that we allow for only two possible realizations is to be able to construct the straightforward mapping from the Arrow-Debreu securities economy to the economy with only two assets: a risk free and a risky asset. Adding more realizations would have minimal quantitative effects and would make this mapping less clear. Notice that the variance of the assumed process is $\mathbb{V}ar(g_s) = p(1-p)(g_H - g_L)^2 = \frac{1}{4}0.04^2 = 0.0004$, which is in line with the medium-long term variation of the GDP in the U.S. economy. As described in Section 2.3 what matters for the entrepreneur is the residual risk $\psi^2 \mathbb{V}ar(g_i)$ at which she is exposed. Also, because we are assuming that workers are not subject to idiosyncratic risk, this risk must be interpreted in relative terms. There is ample evidence that firms-entrepreneurs are more exposed to idiosyncratic risk than workers. Following He and Krishnamurthy (2011), we set $\psi = 0.2$ to match the 20% share of profits that hedge funds charge. The id. risk takes three values, $g_{i,h}, g_{i,m}, g_{i,l}$, that are given by $0.3750, 1.25, 1.3750$ with equal probability. This amount to a total id risk of the variance

[12]In this paper, we would like to understand long trends on quantities, and the risk free interest rate; the objective is not to match the equity premium puzzle. A large literature has explored deviations from expected utility (see for example Ju and Miao, 2012), long run risk (see for example Bansal and Yaron, 2004 and Hansen et al., 2008), and disaster risk (see for example Barro, 2009 and Gourio, 2012), as possible explanations of the premium between equities and safe bonds.

[13]For more information about the implications of CES technology for the business cycle see for example Cantore et al. (2014) and Cantore et al. (2015).

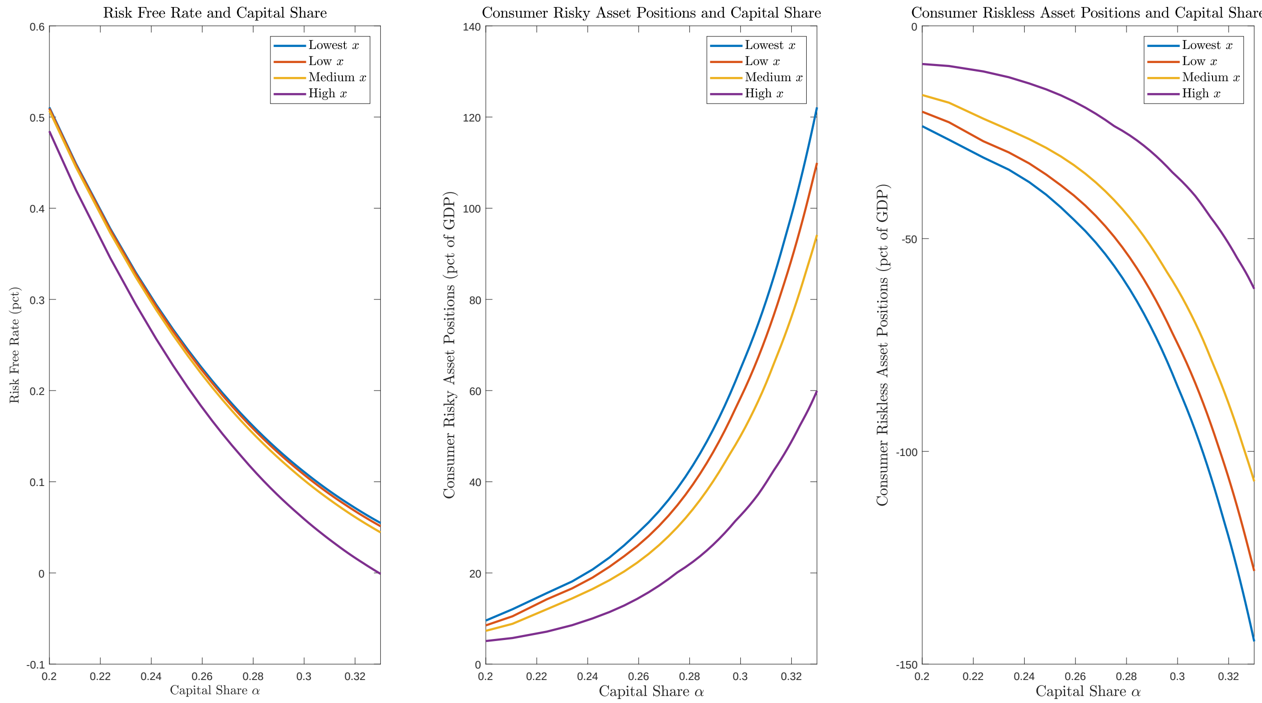

Figure 2: This figure plots the risk free rate, risky and risk-less asset positions for consumers as a function of the labor share. The values of x lowest, low, medium and high are given by 0.0153, 0.2136, 0.4424 ,0.8542.

of the idiosyncratic shock to $\mathbb{V}ar(g_i) = 0.1979$ and thus total id risk is given by to target $\psi^2 \mathbb{V}ar(g_i) = 0.0079$.

3.5 Results

Here we present some of the main results of our paper. As we mentioned in Sections 2 and 3, one of the main implications of the model is the implication for changes in the aggregate portfolio allocation in the economy. In Figure 2 we show the implied worker's holdings of risky and risk-less assets, and the risk free rate.

To construct Figure 2 we simply use the equilibrium aggregate laws of motion generated by the model for the main calibration. Since the model generates $\phi^c(s'|g_s K, x)$, using the implied capital for each value of $g_s K$, we can construct $\phi^c(s'|\alpha, x)$. Then using similar transformations as in section 3, and given that we are considering only two possible aggregate shocks, we can map $\phi^c(s'|\alpha, x)$ into holdings of risky assets $A(\alpha, x)$ and risk-less assets $R(s)B(\alpha, x)$.

First, in the middle panel of Figure 2 we plot the risky asset positions, $A(\alpha, x)$, for different values of x, as a proportion of $Y(s)$. As we can see, larger values of α imply larger holdings of A. As we show in the other panels, the relationship remains for alternative values of x. The smaller x, the larger the implied positions.

Second, in the right panel of Figure 2, we can see the other implication of the model anticipated in the two period model, the pattern observed for $A(\alpha, x)$ is mirrored for $B(\alpha, x)$, with a negative sign. That is, the model predicts exploding changes in the financial positions of households, that can duplicate when moving from values of α around 0.3 to values of α around 0.35. Because of market clearing, the financial positions of the corporate sector are the negative of those corresponding to the households. Thus, if households are borrowing (leveraging) to buy equity, it must be the case that the corporate sector is increasing the issuance of equity and accumulating risk-free assets. This last phenomenon is known as the corporate savings glut. [14]

The next question, after analyzing the policies, is: which are the implied paths for the this economy? To answer this question, we pick an arbitrary path for the capital share in which a succession of positive shocks to g_s generates an increase in the capital share from 0.28 to 0.33 as observed in the data. The results are shown in Figures 3. The simulated paths confirm the results we obtained from the policy functions, but more importantly, show how far the mechanism can take us quantitatively. In particular, a change in the capital share from 0.28 to 0.33 almost triplicates the percentage of risky assets held by the consumers, and implies a drop of the real risk free rate from 12 percent a year to 2 percent a year.

[14] What we add to this discussion is the implication that the accumulation of risk-free assets by part of corporations has to be accompanied by the increasing holdings of risky assets by households. Is this true in the data? As a first attempt to answer this question we constructed, using the Flow of Funds for the U.S. economy, the holdings (direct and indirect) of equity by households. Figure 4 panel B shows that indeed there has been a large increase in the holdings of equity by households from 0.4GDP to 1.4GDP, almost tripling its value. In the same figure we also plot, panel A, all the debt instruments, not related to their main activity, held by corporations. We also see a steady increase in, almost tripling its value from around 0.07GDP to 0.21GDP.

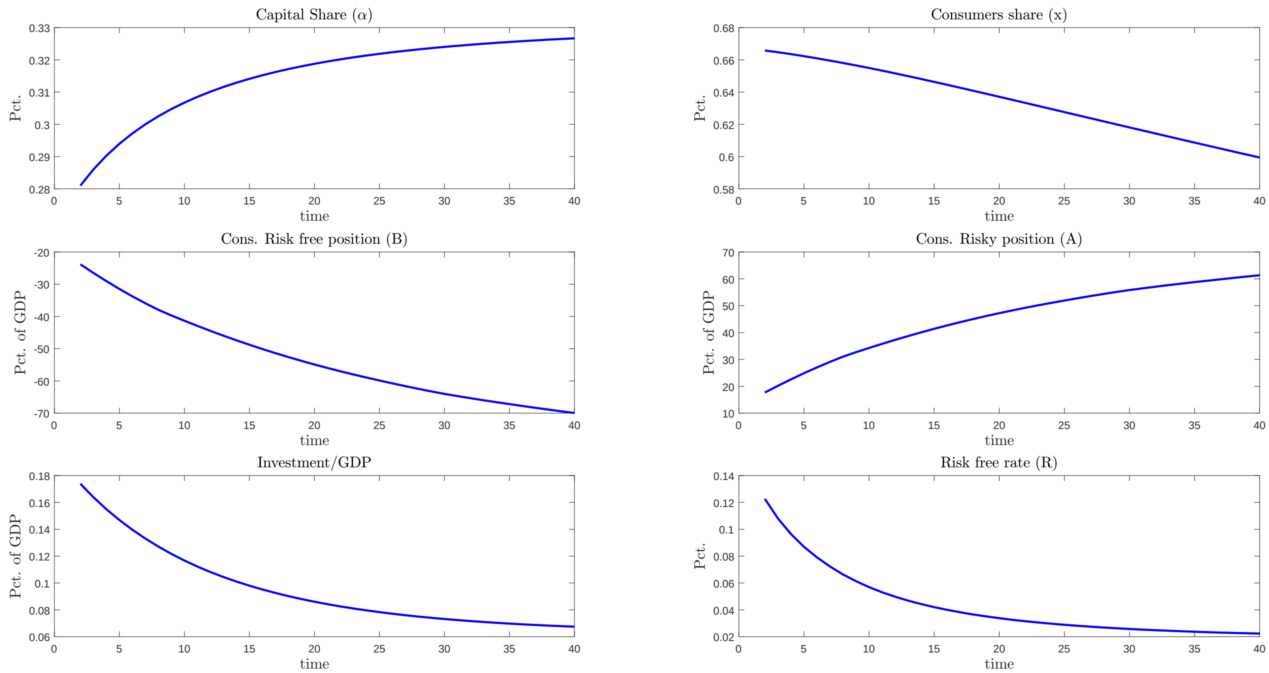

Figure 3: Time path: this figure displays key quantities for our calibrated model after a sequence of capital biased shocks.

4 Conclusions

Kaldor facts led to the prevailing belief that the capital and labor income shares were, besides some small short-run variations, roughly constant. One of the implications of constant income shares is that it is impossible for workers and capitalists to insure each other. With constant income shares, aggregate fluctuations affect both sectors in the same way, and therefore aggregate variations do not affect the distribution of wealth. Recent studies, however, have shown that the labor share moves both in the short and medium-long run.

Motivated by these deviations from the Kaldor facts, in this paper we study how varying income shares affect risk sharing, and which are the predictions over allocations. In particular, we study a standard growth model with financial frictions. We started the discussion with a two-period version of our model. The main result is to show how the combination of limited risk sharing and time-varying shares distorts the allocation of risk, and which assets are used for insurance. Even from the two-period version of the model, it is clear that to share risk, firms want to hoard safe assets, driving interest rates down, and households on the contrary invest in risky assets. Not only that, we show how time-varying shares of different groups in the economy crucially distort their ability to absorb risk from other sectors and divert the asset allocation of other sectors. To explore our channel quantitatively, we then calibrate our model to match long run moments in the US data. We show that the recent US experience of low rates, rising capital shares, accumulation of safe assets by firms and risky assets by households, after 1980's, can be rationalized by persistent capital-biased shocks and limited risk sharing.

The focus of this paper is the medium long-run. However, our model also would lend itself naturally to study how income shares exacerbate or mitigate fluctuations. Also, our model is suitable for studying question about inequality and asset pricing. In particular, ours is a two-factor asset pricing model in which, the capital share and the relative wealth of financial intermediaries are factors pricing the "cross-section" of assets. Both, business cycles and asset prices, are topics for further research.

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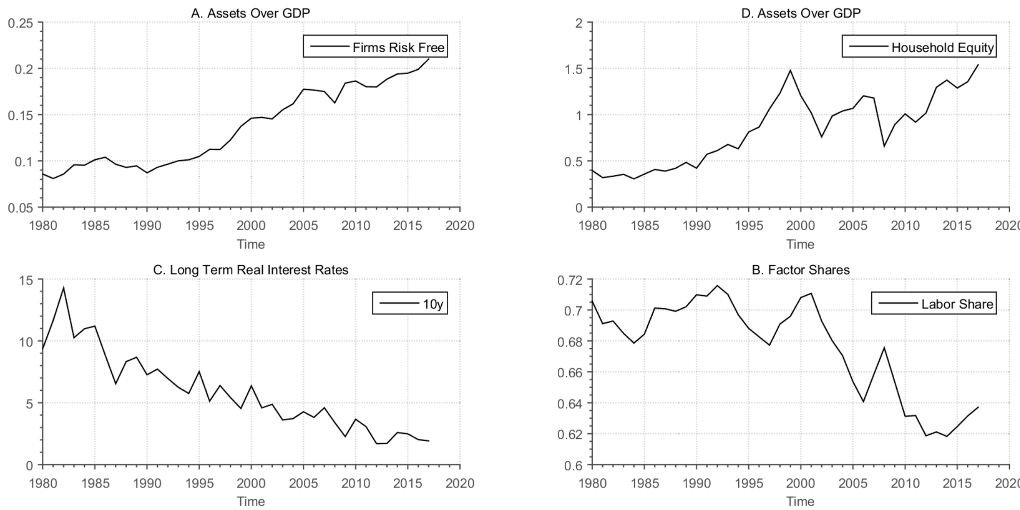


Figure 4: US Data. Data are from 1980-01-01.

A Stylized facts

A.1 Post 1980's

Figure 4 plots four statistics which we take as a starting point for our work. Our aim will be to rationalize the behavior of these four quantities in one theory.[15]

Panel A shows that the share of risk free assets in US firms' portfolios has been increasing in the last 30 years. We define risk free assets as the sum of private foreign deposits, checkable deposit and currency, total time and savings deposits, money market fund shares, security repurchase agreements, commercial paper, treasury securities, agency and GSE backed securities, municipal securities and mutual fund shares. The amount of risk free assets is then normalized by nominal GDP. Our definition of risk free assets reminds the one of broad liquid assets for nonfinancial corporations given by the Board of Governors of the Federal Reserve; ours differs as we consider both financial and nonfinancial corporations and we exclude corporate equity, as we will treat equity differently from other liquid assets. The indicator on corporate risk free debt has received a lot of attention in the last years because of what is now known as the "Corporate savings glut" and, importantly, the behaviour we observe is not peculiar to the US but is evident in many countries around the world. [16]

[15]We choose to show these series since 1980 as we are focusing on secular movements, in Appendix A.2 we have another picture with a longer time span to make the point that the features we are going to discuss are not salient of the last 30 years.

[16]This kind of measure is used for example in the empirical literature on the determinants of corporate cash holdings (see Opler et al. (1999)) and in the literature on intangible capital (see Falato et al. (2013)).

Panel B shows an increase in the share of risky assets in household's portfolios as proportion of GDP; indeed household's equity holdings have almost tripled from 1980 to 2014, with a clear upward trend. To compute the series we normalize the amount of household's directly and indirectly held corporate equities over nominal GDP. This kind of indicator is generally looked at for cross sectional analyses in the household finance literature[17].

Panel C depicts the 10 year US real interest rate, which has been continuously falling since the '80s. This feature has been widely documented; explanations for this falling rates range from demographics, passing from a secular stagnation, to a sudden increase on uncertainty[18]. Long term nominal interest rates and the consumer price index are taken from the FRED database.

Panel D plots the time series of the US labor share, which has been hovering around 0.7 until the 2000, and has experienced a drop in recent years, highlighting that, while one could say that the Kaldor facts hold in the long run, the labor share is moving in the medium run. We chose to compute the labor and capital share similarly to Piketty and Saez (2006)[19]; capital is defined as the sum of consumption of fixed capital and net operating surplus less business current transfer payments, and labor is defined as compensation of employees. We obtain capital and labor shares by normalizing the two quantities over nominal GDP (less taxes). The magnitude and the consequences of the drop in the labor share are a controversial issue which is being analyzed in current studies[20].

A.2 Post WWII and Data Sources

Long term interest rates. Long term interest rates are taken from two sources. First, we use FRED data from 1962 to 2017, serie DGS10, source Board of Governors of the Federal Reserve System (US), release: H.15 Selected Interest Rates. Units are in percent, not

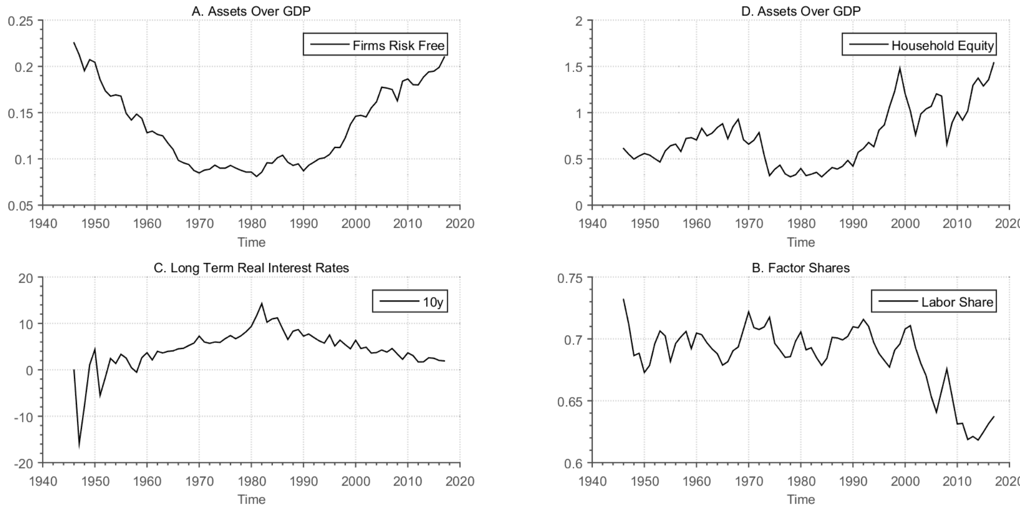
See Chen et al. (2017) for a characterization of patterns of sectoral saving and investment for a large set of countries over the past three decades. Among other possibilities, increased net lending can be associated with accumulation of cash, repayment of debt, or increasing equity buybacks net of issuance, as highlighted by Bates et al. (2009) and Foley et al. (2007).

[17]See Campbell (2006). Most papers use SCF data to look at this statistics (Bergstresser and Poterba (2004), Bertaut and Starr-McCluer (2002), Carroll (2002), Heaton and Lucas (2000), Poterba and Samwick (2001), Tracy and Schneider (2001)) while we use the Flow of Funds data to construct a longer serie that we show Appendix A.2. Tracy et al. (1999) discuss a similar serie.

[18]See for example Karabarbounis and Neiman (2014), and Caballero et al. (2017a), Carvalho et al. (2016b), Summers (2013) and Summers (2014).

[19] There are a number or ways to compute the labor share, see Karabarbounis and Neiman (2014), Blanchard et al. (1997), Blanchard and Giavazzi (2003), Jones et al. (2003), and Bentolila et al. (1999), Harrison (2005) and Rodriguez and Jayadev (2010)). FRED has similar calculations from 1950 on, but we chose to use our own measure to construct a longer serie that we show Appendix A.2.

[20]See Bridgman (2017), Rognlie (2016), Autor et al. (2017).



US Data. Data are from 1946-01-01.

seasonally adjusted, at a monthly frequency. The rates are averages of business days. Inflation is computed from the consumer price index for all urban consumers series CPI-AUCSL taken from FRED as well. The index series is sampled at monthly frequency, with base year 1982-1984, seasonally adjusted. Data before 1962 are taken from Robert Shiller's update of data shown in Chapter 26 of Shiller (1992), and Shiller (2015). For the long term interest rate, the author uses the 10-year Treasury after 1953; before 1953, it is government bond yields from Homer and Sylla (1996). Shiller uses the CPI (Consumer Price Index-All Urban Consumers) published by the U.S. Bureau of Labor Statistics. We compute the monthly inflation as the percentage change of the consumer price index and obtain the real 10 year rate by subtracting the inflation, times 100, from the nominal interest rate.

Labor Share. Data from NIPA Table 1.14; Similarly to Piketty and Saez (2006), capital is defined as the sum of consumption of fixed capital and net operating surplus less business current transfer payments, and labor is defined as compensation of employees. We obtain capital and labor shares by normalizing the two quantities over nominal GDP (less taxes).

Assets. Data are from the board of governors of the federal reserve system. Firms' risk free debt is defined as the sum of private foreign deposits, checkable deposit and currency, total time and savings deposits, money market fund shares, security repurchase agreements, commercial paper, treasury securities, agency and gse backed securities, municipal securities and mutual fund shares. All the series are in table L.102 Nonfinancial Business, Billions of dollars; amounts outstanding end of period, not seasonally adjusted. To compute household equity we use the B.101.e Balance Sheet of Households and Nonprofit Organizations with Equity Detail, Billions of dollars; amounts outstanding end of period, not seasonally adjusted, precisely the entry Directly and indirectly held corporate equities (serie LM153064475). Both quantities are normalized over nominal GDP.

B Proofs two period model

B.1 Capital share in the CES production function

The firms maximizes $\pi(s,i) = \left[\alpha\left(g_i g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha) L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - \omega L$, which implies $L^d(s,i) = \alpha^{\frac{\rho}{\rho-1}} \left[\left(\frac{\omega}{1-\alpha}\right)^{\rho-1} - (1-\alpha)\right]^{\frac{\rho}{1-\rho}} g_i g_s k$. From the labor market clearing condition $1 = L^s = L^d(s) = \mathbb{E}(L^d(s,i))$ we can get the wage

$$\omega(s) = (1-\alpha)\left[\alpha\left(g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)\right]^{\frac{1}{\rho-1}}.$$

Moreover recall that

$$\alpha(s,i) = \frac{\partial y(s,i)}{\partial k}\frac{k}{y(s,i)} = \frac{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)(L)^{\frac{\rho-1}{\rho}}}$$

so $\alpha(s) = \mathbb{E}_i(\alpha(s,i))$ is given by

$$\alpha(s) = \frac{\alpha(g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

and, given that $Y(s) = \mathbb{E}(y(s,i))$, in the same way, the labor share is

$$(1-\alpha(s)) = \frac{\partial Y}{\partial L}\frac{L}{Y} = \frac{(1-\alpha)}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}.$$

Then

$$\omega(s) = (1-\alpha(s))Y(s)$$

Now given that we have the wage, we can find $L^d(s,i)$. We find that $L^d(s,i) = g_i$ and therefore

$$\pi(s,i) = \alpha(s)Y(s)g_i.$$

B.2 Proof of Proposition 1

Proof. We need to find consumption for the entrepreneur and the consumer, and asset prices, such that both consumers are optimizing and markets clear. *Step 0. Consumer* *program.* The consumer solves

$$\max_{\{c_1, c_2(s), A_2(s)\}} \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \sum_s \Pi(s) \frac{c_2(s)^{1-\gamma}}{1-\gamma}$$

$$\begin{aligned} c_1 + \sum p(s) W_2(s) &\leq W_1 + \omega_1 \\ c_2(s) &\leq W_2(s) + \omega_2(s). \end{aligned}$$

The first order condition for assets holdings yields the standard Euler equation:

$$-c_1^{-\gamma} p(s) + \beta \Pi(s) c_2(s)^{-\gamma} = 0,$$

which implies that

$$c_2(s) = \left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}} c_1. \tag{41}$$

The agent chooses to allocate consumption over states of nature depending on the relative price and the likelihood of each event. The demand for insurance in state s is then given by

$$W_2(s) = \left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}} c_1 - \omega_2(s)$$

and from the budget constraint we obtain that:

$$c_1 = \frac{PV(\mathbb{W})}{1 + \sum_s p(s) \left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}}} \tag{42}$$

$$c_2(s) = \frac{\left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}} PV(\mathbb{W})}{1 + \sum_s p(s) \left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}}} \tag{43}$$

$$W_2(s) = \frac{\left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}} PV(\mathbb{W})}{1 + \sum_s p(s) \left(\frac{p(s)}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}}} - \omega_2(s).$$

where

$$PV(\mathbb{W}) = W_1 + \omega_1 + \sum p(s) \omega_2(s).$$

Step 1. Guess consumption, and obtain prices. Guess that at an optimum consumption is given by:

$$c_1 = xY_1 \tag{44}$$

$$c_2(s) = xY(s). \tag{45}$$

Substituting the guess, equations (44) and (45), into the Euler equation for the consumer (41), we obtain equilibrium prices

$$p(s) = \beta\Pi(s)\left(\frac{Y(s)}{Y_1}\right)^{-\gamma}.$$

Step 2: find x. Recall that

$$\begin{aligned}
c_1 &= \frac{PV(\mathbb{W})}{1+\sum_s p(s)\left(\frac{p(s)}{\beta\Pi(s)}\right)^{-\frac{1}{\gamma}}} \\
xY_1 &= \frac{PV(\mathbb{W})}{1+\sum_s \beta\Pi(s)\left(\frac{Y(s)}{Y_1}\right)^{-\gamma}\left(\frac{\beta\Pi(s)\left(\frac{Y(s)}{Y_1}\right)^{-\gamma}}{\beta\Pi(s)}\right)^{-\frac{1}{\gamma}}} \\
x &= \frac{PV(\mathbb{W})}{Y_1+\sum_s \beta\Pi(s)\left(\frac{Y(s)}{Y_1}\right)^{-\gamma}Y(s)} \\
x &= \frac{PV(\mathbb{W})}{Y_1+\sum_s p(s)Y(s)}.
\end{aligned}$$

So, x is the fraction of wealth the consumer. Step 3: Asset holding of the consumer. Note that

$$\begin{aligned}
\phi(s) &= \frac{W_2(s)}{Y_2(s)} \\
&= \frac{c_2(s)-\omega_2(s)}{Y_2(s)} \\
&= \frac{xY_2(s)-\omega_2(s)}{Y_2(s)} \\
&= x-(1-\alpha_2(s)).
\end{aligned}$$

The first line uses the definition of $\phi(s)$, the second line the optimal asset holding for the consumer, equation (43), the third line uses our guess, and the last line the fact that wages are a fraction $(1-\alpha(s))$ of output. Step 4: consumption of the entrepreneur. From market clearing we obtain that

$$e_1 = (1 - x)\, Y_1 \tag{46}$$

$$e_2(s) = (1 - x)\, Y(s). \tag{47}$$

Note that from Walras law, the budget constraint of the entrepreneur will hold. The Euler equation for the entrepreneur holds because

$$(1 - x)\, Y(s) = \left(\frac{\beta \Pi(s) \left(\frac{Y(s)}{Y_1}\right)^{-\gamma}}{\beta \Pi(s)} \right)^{-\frac{1}{\gamma}} (1 - x)\, Y_1$$

holds. □

B.3 Proof of Proposition 2

Proof. First, let's prove the last point, that is, the inefficiency of the response of the economy to aggregate shocks is increasing in the capital share. If markets are complete, it is straightforward to show that, from equation

$$\left(\frac{1 - x_1}{x_1}\right)^{-\sigma} = \frac{(-\phi(s) + \alpha(s))^{-\sigma}}{(\phi(s) + (1 - \alpha(s)))^{-\sigma}}; \quad \forall s$$

if $-\phi(s) + \alpha(s) \neq 0$ (which must be true, otherwise the consumers would die of starvation) we have

$$\frac{d\phi}{d\alpha} = 1.$$

If markets are incomplete, then consider the equation

$$\left(\frac{1 - x_1}{x_1}\right)^{-\sigma} = \frac{(-\phi(s)^{IM} + \alpha(s))^{-\sigma}}{(\phi(s)^{IM} + (1 - \alpha(s)))^{-\sigma}} \left(1 + \frac{\sigma(1+\sigma)\alpha(s)^2}{(-\phi(s)^{IM} + \alpha(s))^2} \frac{\mathbb{V}ar(g_i)}{2}\right); \quad \forall s.$$

Define

$$M \equiv \frac{(-\phi(s)^{IM} + \alpha(s))^{-\sigma}}{(\phi(s)^{IM} + (1 - \alpha(s)))^{-\sigma}} + \frac{\sigma(\sigma+1)\alpha(s)^2(-\phi(s)^{IM} + \alpha(s))^{-\sigma-2}}{(\phi(s)^{IM} + (1 - \alpha(s)))^{-\sigma}} \frac{\mathbb{V}ar(g_j)}{2} - \left(\frac{1 - x_1}{x_1}\right)^{-\sigma} = 0$$

therefore

$$\frac{\partial M}{\partial \phi^{IM}} d\phi^{IM} + \frac{\partial M}{\partial \alpha} d\alpha = 0$$

and

$$\frac{d\phi^{IM}}{d\alpha} = -\frac{\frac{\partial M}{\partial \alpha}}{\frac{\partial M}{\partial \phi^{IM}}}.$$

After some algebra, we can say that

$$\frac{\partial M}{\partial \alpha} = -\frac{\partial M}{\partial \phi^{IM}} + \sigma(\sigma+1)\alpha(s)(-\phi(s)^{IM} + \alpha(s))^{-\sigma-2}\mathbb{V}ar(g_j)$$

then

$$\frac{d\phi^{IM}}{d\alpha} = \frac{\frac{\partial M}{\partial \phi^{IM}} - \sigma(\sigma+1)\alpha(s)(-\phi(s)^{IM} + \alpha(s))^{-\sigma-2}\mathbb{V}ar(g_j)}{\frac{\partial M}{\partial \phi^{IM}}} < 1.$$

Moreover, note that $\frac{d\phi^{IM}}{d\alpha}$ is positive, which proves point 2 of Proposition 2.

We prove point 1 of Proposition 2 using equations (18) and (19) of the main text, which helps us characterizing the $\phi(s)$ in terms of the certainty equivalent $g^{CE}(\alpha, \phi)$. In particular, we report below equation (19)

$$\phi(s)^{IM} = x[\alpha(s)g^{CE}(\alpha, \phi^{IM}) + (1-\alpha(s))] - (1-\alpha(s)); \quad \forall s \tag{48}$$

This equation does not provide a closed form solution for $\phi^{IM}(s)$ but confirms the result of Proposition that $\phi^{CM}(s) > \phi^{IM}(s)$, for all s, since $g^{CE}(\alpha, \phi) < 1$ if there is uninsurable idiosyncratic risk. □

B.4 Derivation Equations (15) and (16)

In equilibrium $A^c + A^e = 0$ and $B^c + B^e = 0$. Consider the problem of the consumer. Note that there is a one-to-one mapping between the second period's payoffs of the AD securities $\phi(s)$ and of the portfolio with two assets:

$$\phi(L)Y_2(L) = R_L B^c + A^c \alpha(L) Y_2(L)$$
$$\phi(H)Y_2(H) = R_L B^c + A^c \alpha(H) Y_2(H).$$

The latter implies positions and prices given by

$$A^c = \frac{\phi(H)Y_2(H) - \phi(L)Y_2(L)}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{49}$$

$$R_L B^c = \frac{Y_2(L)Y_2(H)(\phi(L)\alpha(H) - \alpha(L)\phi(H))}{\alpha(H)Y_2(H) - \alpha(L)Y_2(L)} \tag{50}$$

$$R_L = \frac{1}{\sum_s p(s)} \tag{51}$$

$$P_A = \sum_s p(s)\alpha(s)Y_2(s). \tag{52}$$

Where $p(s)$ is the price of the AD securities in state s, then both the consumer and the entrepreneur are optimizing. Notice that, in this case

$$p(s) = P(s)\left(\frac{Y_1}{Y_2(s)}\right)^{\sigma},$$

for all s, so the price of the AD security in state s only depends on the (exogenous) growth rate of total output Y, and so does the risk free rate R_L. Furthermore, the output growth is stationary, the interest rate is constant through time and in particular does not depend on the capital share.

C Proofs infinite horizon economy

C.1 Proof of consumer's solution

Using the consumer's first order condition and the assumed utility function, it is straightforward to show that:

$$c(s') = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} c(s) = \tilde{\beta}(s',s)c(s); \quad \forall s'$$

Then, using the guesses for consumption and savings we obtain:

$$\begin{aligned}(1-\zeta(s'))(a(s')+\omega(s')+h(s')) &= \tilde{\beta}(s'|s)(1-\zeta(s))(a(s)+\omega(s)+h(s))\\(1-\zeta(s'))\phi^c(s'|s)\zeta(s)W^c(s) &= \tilde{\beta}(s'|s)(1-\zeta(s))W^c(s)\end{aligned}$$

Thus, the individual consumption ratio solves:

$$(1-\zeta(s'))\phi^c(s'|s)\zeta(s) = \tilde{\beta}(s',s)(1-\zeta(s)); \quad \forall s,s' \tag{53}$$

Using (53) it is immediate to arrive to equation (24)

$$\phi^c(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)}; \quad \forall s,s'$$

To solve for the consumption rates we use the present value of (24). Multiplying the last by $p(s'|s)$ and adding over s' we obtain:

$$\begin{aligned}1 = \sum_{s'}\phi^c(s'|s)p(s'|s) &= \sum_{s'}\left[\frac{(\beta\Pi(s'|s))^{1/\sigma}p(s'|s)^{1-1/\sigma}}{(1-\zeta(s'))}\right]\frac{(1-\zeta(s))}{\zeta(s)}\\(1-\zeta(s))^{-1} &= 1+\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma}p(s'|s)^{1-1/\sigma}(1-\zeta(s'))^{-1}\right]\end{aligned}$$

The last is equation (25).

C.2 Proof of entrepreneur's solution

Using the proposed solutions, the implied law of motion of wealth is:

$$
\begin{aligned}
W^e(s', i', k') &= E(s') + R(s') g_{i'} k' \\
W^e(s', i', k') &= \vartheta(s)(1-\nu(s))\phi^e(s'|s) W^e(s,i,k) + \nu(s)\vartheta(s) R(s') g_{i'} W^e(s,i,k) \\
W^e(s', i', k') &= \vartheta(s)[(1-\nu(s))\phi^e(s'|s) + \nu(s) R(s') g_{i'}] W^e(s,i,k)
\end{aligned}
$$

The last equation is (31). Then, using both Euler equations:

$$
\mathbb{E}_{s',i|s}[u'(e(s',i))R(s')g_i] = \frac{\mathbb{E}_{s',i|s}\left[u'(e(s',i))\right]}{\sum_{s'|s} p(s'|s)}
$$

Now use the guessed solution for consumption, (31) and the CRRA preferences to write:

$$
\mathbb{E}_{s',i|s}\left[((1-\vartheta(s'))[(1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')g_i])^{-\sigma}\left(R(s')g_i - \frac{1}{\sum_{s'|s} p(s'|s)}\right)\right] = 0
$$

Introducing the definition of $o(s', i; \phi^e, \nu)$ in the last we obtain (32). To solve for the saving rates, we use the state by state first order condition to obtain:

$$
\begin{aligned}
u'(e(s)) &= \beta \frac{\Pi(s'|s)}{p(s'|s)} \mathbb{E}_i[u'(e(s',i))]; \;\; \forall s, s' \\
u'((1-\vartheta(s))h^e(s,i,k)) &= \beta \frac{\Pi(s'|s)}{p(s'|s)} \mathbb{E}_i[u'((1-\vartheta(s'))h^e(s',i,k'))] \\
u'(1-\vartheta(s)) &= \beta \frac{\Pi(s'|s)}{p(s'|s)} \mathbb{E}_i[u'((1-\vartheta(s'))\vartheta(s) o(s', i; \phi^e, \nu))] \\
(1-\vartheta(s))^{-\sigma} &= \beta \frac{\Pi(s'|s)}{p(s'|s)} [(1-\vartheta(s'))\vartheta(s)]^{-\sigma} \mathbb{E}_i o(s', i; \phi^e)^{-\sigma}
\end{aligned}
$$

Similar manipulations as in the consumer's problem deliver:

$$
(1-\vartheta(s'))\vartheta(s)\left(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}\right)^{-1/\sigma} = \tilde{\beta}(s', s)(1-\vartheta(s)); \;\; \forall s, s'
$$

Given the solution for $\nu(s)$, the last equation, (34) in the text, pins down $\phi^e(s'|s)$, which is also independent of wealth. Comparing (25) and (34) we see that the consumption for consumers is affected by $\phi^c(s'|s)$ while for entrepreneurs the equivalent term is $(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma})^{-1/\sigma}$.

We can perform similar manipulations to the consumer's problem. Multiplying (34) by $p(s', s)$, and adding over s' we obtain:

$$m(s) = \sum_{s'} \left(\mathbb{E}_i o(s', i; \phi^e)^{-\rho}\right)^{-1/\rho} p(s'|s) = \sum_{s'} \left[\frac{(\beta\Pi(s'|s))^{1/\rho} p(s'|s)^{1-1/\rho}}{(1-\vartheta(s'))}\right] \frac{(1-\vartheta(s))}{\vartheta(s)}$$
$$(1-\vartheta(s))^{-1} = 1 + m(s)^{-1} \sum_{s'|s} \left[(\beta\Pi(s'|s))^{1/\rho} p(s'|s)^{1-1/\rho} (1-\vartheta(s'))^{-1}\right]$$

The last equation is (34) in the text.

C.3 Proofs of equilibrium conditions.

Let $W^e(s) = \int_{i,k,E} W^e(s, i, k, E)$, then the aggregate capital follows:

$$K'(s) = \int_{i,k,E} k'(s, i, k, E) = \nu(s)\vartheta(s) \int_{i,k,E} W^e(s, i, k, E) = \nu(s)\vartheta(s) W^e(s)$$

Operating with the asset's market clearing.

$$a(s'|s) + E(s'|s) = 0; \quad \forall s, s'$$

$$\phi(s'|s)\zeta(s)W^c(s) + \phi^e(s'|s)\vartheta(s)(1-\nu(s))W^e(s) = \omega(s') + h(s')$$

Define $x = W^c(s)/W^T(s)$, then

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)\vartheta(s)(1-\nu(s))(1-x) = \frac{\omega(s') + h(s')}{W^T(s)}; \quad \forall s, s'$$

Which is equation (37) Operating with the goods market clearing:

$$\begin{aligned} c(s) + e(s) + k'(s) = &\quad y(s); \quad \forall s \\ (1-\zeta(s))W^c(s) + (1-\vartheta(s))W^e(s) + \vartheta(s)\nu(s)W^e(s) = &\quad y(s); \quad \forall s \\ (1-\zeta(s))x + (1-\vartheta(s)(1-\nu(s))(1-x) = &\quad \tfrac{y(s)}{W^T(s)}; \quad \forall s \end{aligned}$$

Which delivers (38). The next period distribution of wealth, x' is:

$$\begin{aligned} \frac{x(s')}{1-x(s')} &= \frac{W^c(s')}{W^e(s')} = \\ = \frac{\phi^c(s'|s)\zeta(s)W^c(s)}{\mathbb{E}_i o(s', i, s)\vartheta(s)W^e(s)} &= \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s', i, s)\vartheta(s)(1-x)} \end{aligned}$$

Which can be written as:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s', i, s)\vartheta(s)(1-x) + \phi^c(s'|s)\zeta(s)x}$$

Using market clearing (37), this equation can also be written as:

$$x(s') = \frac{\phi^c(s'|s)\zeta(s)}{\iota(s', s) + \nu(s)\vartheta(s)(1-x)r(s')} x(s)$$

where $\iota(s', s) = \frac{w(s')+h(s')}{W^T(s)}$, which is (40) in Section 3.3. For the law of motion of $g_s k$ recall that: $k'(s, i, E) = \nu(s)\vartheta(s)W^e(s, i, k, E)$ and: $E(s', i, s, E) = \phi^e(s', s)\vartheta(s)(1-\nu(s))W^e(s, i, k, E)$. Therefore in every state

$$\frac{E(s', i, s, k, E)}{k'(s, i, k, E)} = \phi^e(s', s)\frac{(1-\nu(s))}{\nu(s)}$$

Which is independent of i. Thus, assuming that $E(s_0)$ is also proportional to k

$$k'(s, i, k, E) = \nu(s)\vartheta(s)W^e(s, i, k, E)$$

As a result we can write:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s)$$

Delivering (39) in Section 3.3.

D Financial Contract

D.1 Optimal Contract

Suppose a 1 period model. There is a risk neutral principal which can provide insurance to the firms. Suppose there are three possible shocks $g_L < g_M < g_H$, with probability γ_i (three shocks can be generalized to any number, with only 2 shocks may not be general).

The firm can enter the insurance contract with the financial intermediary at the beginning of the period, before knowing the realization of g_i. The firm's profits are $\pi k g_i$, the linearity in k and g_i of the CES model is shown in the appendix. Thus, in absence of insurance, the entrepreneur's utility is:

$$\mathbb{E}_i[u(e_i)] = \mathbb{E}_i[u(\pi k g_i)]$$

The principal (financial intermediary) could sign a contract and offer insurance to the entrepreneur. First, suppose that g_i is observable. In this case the contract is very simple. The principal "buy" all the proceeds of the production at a price J. Then, after the shock is realized the entrepreneur hands over to the profits to the principal. Because the principal must break even it must be that $\mathbb{E}_i(\pi k g_i) - J = 0$. Thus, the utility of the entrepreneur is:

$$\mathbb{E}_i[u(e_i^O)] = E_i[u(J)] = u(\mathbb{E}_i[\pi k g_i]) > \mathbb{E}_i[u(\pi k g_i)] = \mathbb{E}_i[u(e_i)]$$

However, the entrepreneur is subject to moral hazard problems: g_i is not observable and must be incentive compatible for him to revel the true realization of g_i. The entrepreneur can report an alternative value of g_i, say $g_{i'}$, and keep the difference for himself. But, transforming these "stolen" profits into consumption is not for free. Each unit of stolen profit transforms into consumption at the rate $0 \leq \phi \leq 1$. Thus, when the entrepreneur steals profits obtains an additional consumption of only $\phi \pi k(g_i - g_{i'})$.

Because of this, the contract must be incentive compatible. Now the principal must give additional payments d_i, contingent on the realization of g_i, to make the contract incentive compatible. Since the entrepreneur will not lie on equilibrium, his consumption is $e_i^c = J + d_i$, while because the principal must break even the contract must satisfy: $\mathbb{E}_i(\pi k g_i - d_i) - J = 0$.

As a result, the optimal contract solves:

$$\max_{\{J, d_i\}} \mathbb{E}_i u(J + d_i)$$

$$\text{st.} \quad \phi \pi k(g_i - g_{i'}) + d_{i'} + J \leq J + d_i; \quad \forall i, i'$$

$$\mathbb{E}_i(\pi k g_i - d_i) - J = 0$$

The first set of constraints are the incentive compatibility, or truth telling, constraints. First notice that only the adjacent constraints matter (single crossing). To see this, consider that the entrepreneur would never lie when observes the low shock. So, only the following can be binding:

$$\begin{aligned}
\phi\pi k(g_H - g_M) + d_M &\leq d_H \\
\phi\pi k(g_M - g_L) + d_L &\leq d_M \\
\phi\pi k(g_H - g_L) + d_L &\leq d_H
\end{aligned}$$

Adding the first two inequalities:

$$\begin{aligned}
\phi\pi k(g_H - g_M) + d_M + \phi\pi k(g_M - g_L) + d_L \leq & \quad d_H + d_M \\
\phi\pi k(g_H - g_L) + d_L \leq & \quad d_H
\end{aligned}$$

Thus, the third constraint is irrelevant, in general is a version of the single crossing property. This can be generalized to any arbitrary number of idiosyncratic shocks. Rewriting the problem we have:

$$\max_{\{J,d_i\}} \sum_i \gamma_i u(J + d_i)$$

$$\text{st.} \quad \phi\pi k(g_H - g_M) + d_M \leq d_H$$

$$\phi\pi k(g_M - g_L) + d_L \leq d_M$$

$$\sum_i \gamma_i(\pi k g_i - d_i) - J = 0$$

Let λ be the multiplier in the break even constraint and μ_i the multiplier in each incentive compatibility. Taking first order conditions:

$$\begin{aligned}
\sum_i \gamma_i u'(J + d_i) &= \lambda \\
\gamma_L u'(J + d_L) &= \gamma_L \lambda + \mu_M \\
\gamma_M u'(J + d_M) &= \gamma_M \lambda + \mu_H - \mu_M \\
\gamma_H u'(J + d_H) &= \gamma_H \lambda - \mu_H
\end{aligned}$$

It is clear that $\mu_L = \mu_H = 0$ cannot be a solution because it violates the IC constraints. Now, suppose $\mu_M = 0$, while $\mu_H > 0$. Then it must be that $d_L = d_M$. If $d_L > d_M$, the IC constraint implies

$$\phi \pi k(g_M - g_L) + d_L - d_M < 0$$

Which is a contradiction. If $d_L < d_M$ a small increase in d_L accompanied by a small reduction on d_M, to keep the break even constraint satisfied, generates a change of welfare of:

$$\gamma_L d_L[u'(e_L) - u'(e_M)] > 0$$

which is true because $u''(.) < 0$ and $e_L < e_M$, thus increasing welfare. A similar argument can be used to show that $\mu_M > 0$ and $\mu_H = 0$ is not possible either. A result, because μ_M and μ_H are both strictly positive, we must have:

$$\begin{aligned} \phi \pi k(g_H - g_M) &= d_H - d_M \\ \phi \pi k(g_M - g_L) &= d_M - d_L \end{aligned}$$

It is easy to see that $d_i = \phi \pi k g_i$, together with $J = (1 - \phi)\mathbb{E}_i(\pi k g_i)$, is a solution for all the equations. And since the problem has a unique solution, it must be the solution.

This contract can be interpreted as an equity contract. Each entrepreneur sells a share $1 - \phi$ of his firm to the intermediary and uses the proceeds to buy an indexed stock market financial instrument. This completely smooths out a proportion $(1 - \phi)$ of the idiosyncratic risk. However, to prevent stealing not all the shares can be sold, the entrepreneur must retain a proportion ϕ of his shares, which is his "skin in the game." This is the best insurance possible with only short term contracts. Note that here we assume that there was no aggregate risk. This result would not be affected by it, since it would affect all the IC constraints proportionally. It would only change the pricing of J.

D.2 Efficiency

We start characterizing the Planner's problem when history cannot be used to provide insurance. Lets set $E_1 = 0$. This at the end is irrelevant since only picks a point in the Pareto frontier. Here what we do is to maximize the entrepreneur's utility subject to satisfy a given level of utility to the workers: $\bar{u}$. Thus, moving $\bar{u}$ from the minimum level to the maximum we can characterize the whole Pareto Frontier. Also, in order to characterize the optimal plan is convenient to write the individual profits in the reduced form: $\pi(s)kg_i = \alpha(s)Y(s)g_i$. The planner solves:

$$\max_{\{J_1, d_{1,i}, J(s), d_{2,i}(s)\}} \left\{ \mathbb{E}_i[u(J_1 + d_{1,i})] + \mathbb{E}_{i,s}[u(J(s) + d_{2,i}(s))] \right\}$$

$$\text{st.} \quad \phi\alpha(s)Y(s)[g_{2,i} - g_{2,i'}] + d_{2,i'}(s) + J(s) \leq J(s) + d_{2,i}(s); \quad \forall i, i' \text{ and } \forall s$$

$$\alpha Y_1[g_{1,i} - g_{1,i'}] + d_{1,i'} + J_1 \leq J_1 + d_{1,i}; \quad \forall i, i'$$

$$u\left(\mathbb{E}_i[\alpha Y_1 g_{1,i} - d_{1,i}] - J_1 + (1-\alpha)Y_1\right) + ...$$
$$... + \sum_s \Pi(s) u\Big(\mathbb{E}_i[\alpha(s)Y(s)g_{2,i} - d_{2,i}(s)] - J(s) + (1-\alpha(s))Y(s)\Big) \geq \bar{u}$$

Notice that in the worker's utility the labor share cancels out when $\mathbb{E}_i g_{t,i} = 1, \ \forall t$. Thus the problem reduces to:

$$\max_{\{J_1, d_{1,i}, J(s), d_{2,i}(s)\}} \left\{ \mathbb{E}_i[u(J_1 + d_{1,i})] + \mathbb{E}_{i,s}[u(J(s) + d_{2,i}(s))] \right\}$$

$$\text{st.} \quad \phi\alpha(s)Y(s)[g_{2,i} - g_{2,i'}] + d_{2,i'}(s) + J(s) \leq J(s) + d_{2,i}(s); \quad \forall i, i' \text{ and } \forall s$$

$$\alpha Y_1[g_{1,i} - g_{1,i'}] + d_{1,i'} + J_1 \leq J_1 + d_{1,i}; \quad \forall i, i'$$

$$u\left(-\mathbb{E}_i[d_{1,i}] - J_1 + Y_1\right) + \sum_s \Pi(s) u\Big(- \mathbb{E}_i[d_{2,i}(s)] - J(s) + Y(s)\Big) \geq \bar{u}$$

The solution for the IC constraints is still the same as before, therefore replacing the optimal d_i's we are left with just the inter temporal smoothing problem:

$$\max_{\{J_1, J(s)\}} \left\{ \mathbb{E}_i[u(J_1 + \phi\alpha Y_1 g_{1,i})] + \sum_s \Pi(s)\mathbb{E}_i[u(J(s) + \phi\alpha(s)Y(s)g_{2,i})] \right\}$$

$$\text{st.} \quad u\left(-\phi\alpha Y_1 - J_1 + Y_1\right) + \sum_s \Pi(s) u\Big(- \phi\alpha(s)Y(s) - J(s) + Y(s)\Big) \geq \bar{u}$$

Let λ be the Lagrange multiplier in the constraint, the foc's are:

$$\mathbb{E}_i[u'(J_1 + \phi\alpha Y_1 g_{1,i})] = \lambda u'\left(-\phi\alpha Y_1 - J_1 + Y_1\right)$$

$$\Pi(s)\mathbb{E}_i[u'(J(s) + \phi\alpha(s)Y(s)g_{2,i})] = \lambda\Pi(s)u'\Big(- \phi\alpha(s)Y(s) - J(s) + Y(s)\Big); \ \forall s$$

Which canceling λ becomes:

$$\frac{\mathbb{E}_i[u'(J_1+\phi\alpha Y_1 g_{1,i})]}{u'\left(-\phi\alpha Y_1 - J_1 + Y_1\right)} = \frac{\mathbb{E}_i[u'(J(s)+\phi\alpha(s)Y(s)g_{2,i})]}{u'\left(-\phi\alpha(s)Y(s) - J(s) + Y(s)\right)} \quad \Rightarrow \quad \frac{\mathbb{E}_i[u'(e_i)]}{u'\left(c_1\right)} = \frac{\mathbb{E}_i[u'(e_{2,i}(s))]}{u'\left(c_2(s)\right)}$$

Which is exactly the same condition as in the competitive equilibrium. Thus, the CE is constraint efficient. The only possibility for the equilibrium to be inefficient is that the introduction of endogenous capital creates a distortion on the optimal investment or that in some way we can introduce history dependency in the contracts.

E Numerical Appendix

a. Using $g_s k$, compute once and for all $Y(s)$, $r(s)$ and $w(s)$

b. Guess $\tilde{\beta}_0(s'|s)$ and $p_0(s'|s)$ and compute $W^T(s)$ and $h(s)$.

c. Also guess a “ratio" $Ra(s',s)$. Start with $Ra(s',s)=1, \forall s, s'$

d. Use $\tilde{\beta}(s'|s)$ and $p_0(s'|s)$ in (25) to solve for $\zeta(s)$, then (24) to get: $\phi(s'|s)$

e. Use $\tilde{\beta}(s'|s)$, $p_0(s'|s)$ and $Ra(s',s)$ to solve for $\vartheta(s)$. To this end use (62).

f. Use $\zeta(s)$, $\vartheta(s)$ and $\phi(s'|s)$ in (57) to solve for $\phi^e(s'|s)$. (see (67))

g. Use $\zeta(s)$ and $\vartheta(s)$ in (**??**) to solve for $\nu(s)$. (see (68))

h. Use $\phi^e(s'|s)$ and $\nu(s)$ in (61) to update the guess for $Ra(s'|s)$.

i. Using (63) and (64) compute a new law of motion of s: $\Pi_1(s'|s)$.

j. Use (65) to compute a new $\tilde{\beta}_1(s',s)$ and $p_1(s',s) = \beta\Pi_1(s'|s)\tilde{\beta}(s',s)^{-\sigma}$

k. f $\Pi_1 = \Pi_0$ and $p_1 = p_0$, stop otherwise update and start again in step 2 with:

$$\Pi_0 = 0.5\Pi_0 + 0.5\Pi_1; \quad p_0 = 0.5p_0 + 0.5p_1; \quad \tilde{\beta}_0 = 0.5\tilde{\beta}_0 + 0.5\tilde{\beta}_1.$$

Online Appendix to "The Macroeconomics of Hedging Income Shares"

Adriana Grasso, Juan Passadore, Facundo Piguillem

F Capital Adjustment Costs

Suppose there are capital adjustment costs given by:

$$\frac{\varpi}{2}\left(\frac{k'}{h^e(k,i)}-v\right)^2 h^e(k,i)$$

For some $v > 0$, the entrepreneur solves

$$V^e(E,k;s,i) = \max_{\{e,E(s'),k'\}} \left\{u(e(s,i)) + \beta \mathbb{E}_{s',i'}[V^e(E(s'|s),k';s',j')|s]\right\}$$

subject to

$$e(s,i) + k' + \frac{\varpi}{2}\left(\frac{k'}{h^e(k,i)}-v\right)^2 h^e(k,i) + \sum_{s'} p(s'|s)E(s'|s) \leq E(s) + r(s)kg_i$$

The variables and the interpretations are the same as before.

F.1 Entrepreneur Solution

First notice that

$$\frac{\partial(\frac{\varpi}{2}\left(\frac{k'}{h^e(k,i)}-v\right)^2 h^e(k,i))}{\partial h^e(k,i)} = -\frac{\varpi}{2}\left(\left(\frac{k'}{h^e(k,i)}\right)^2 - v^2\right)$$

The foc's imply

$$p(s'|s)u'(e(s,i)) = \beta\Pi(s'|s)\mathbb{E}_i\left[u'(e(s',i))\left(1+\frac{\varpi}{2}(\left(\frac{k''}{h^e(s')}\right)^2 - v^2)\right)\right]; \quad \forall s,s'$$

$$[1+\varpi\left(\frac{k'}{h^e}-v\right)]u'(e(s,i)) = \beta\mathbb{E}_{s',i}\left[u'(e(s',i))\left(1+\frac{\varpi}{2}(\left(\frac{k''}{h^e(s')}\right)^2 - v^2)\right)r(s')g_i\right]$$

Notice that $q = [1+\varpi\left(\frac{k'}{h^e(k,i)} - v\right)]$, is similar to the typical investment q. As before, guess that the solution is characterized by:

$$\begin{aligned} e(s,i) &= (1-\vartheta(s))h^e(s,i,k) \\ k'(s,i) &= \nu(s)\vartheta(s)h^e(s,i,k) \\ E(s'|s,i) &= \phi^e(s'|s)E_1(s,i) \end{aligned}$$

Total entrepreneur's wealth is as before:

$$h^e(s,i,k) = E(s,i) + r(s)g_i k.$$

The assumed shape for the adjustment cost is useful because we know k'/h^e is independent of i and equal to $v(s) = \nu(s)\vartheta(s)$. Main difference now is that, using budget constraint:

$$E_1(s,i) \equiv [\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}\left(v(s)-v\right)^2]h^e(s,i,k)$$

And again, we must have $\sum_{s'|s} p(s'|s)\phi^e(s'|s) = 1$. Note that the law of motion of wealth is, as before:

$$\begin{aligned} h^e(s',i',k') &= E(s') + r(s')g_{i'}k' \\ h^e(s',i',k') &= [\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}\left(v(s)-v\right)^2]\phi^e(s'|s)h^e(s,i,k) + \nu(s)\vartheta(s)r(s')g_{i'}h^e(s,i,k) \\ h^e(s',i',k') &= \vartheta(s)[[(1-\nu(s)) - \frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2]\phi^e(s'|s) + \nu(s)r(s')g_{i'}]h^e(s,i,k) \qquad (54) \end{aligned}$$

Using this and putting both Euler equations together, it implies:

$$\frac{\mathbb{E}_{s',i|s}\left[u'(e(s',i))\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)r(s')g_i\right]}{[1+\varpi\left(v(s)-v\right)]} = \frac{\mathbb{E}_{s',i|s}\left[u'(e(s',i))\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)\right]}{\sum_{s'|s} p(s'|s)}$$

Now use the guessed solution for consumption, (**??**) and the CRRA preferences to write:

$$\mathbb{E}_{s',i|s}\left[\left((1-\vartheta(s'))[(1-\nu(s) - \frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2)\phi^e(s'|s) + \nu(s)r(s')g_i]\right)^{-\sigma}\right. \qquad (55)$$

$$\left.\left(\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)r(s')g_i - \frac{[1+\varpi\left(v(s)-v\right)]}{\sum_{s'|s} p(s'|s)}\right)\right] = 0$$

With this equation we can solve for $\nu(s)$, which does not depend on either the individual capital or individual shock as long as g_i is iid. Now we need to solve for savings. Using first Foc:

$$
\begin{aligned}
u'(e(s,i)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'(e(s',i))]\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right);\ \forall s,s' \\
u'(e(s,i)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'((1-\vartheta(s'))h^e(s',i,k'))]\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right) \\
u'(1-\vartheta(s)) &= \beta\frac{\Pi(s'|s)}{p(s'|s)}\mathbb{E}_i[u'((1-\vartheta(s'))\vartheta(s)o(s',i))]\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right) \\
(1-\vartheta(s))^{-\sigma} &= \beta\frac{\Pi(s'|s)}{p(s'|s)}[(1-\vartheta(s'))\vartheta(s)]^{-\sigma}\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)
\end{aligned}
$$

Where

$$
o(s',i;\phi^e) = (1-\nu(s)-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2)\phi^e(s'|s)+\nu(s)r(s')g_i
$$

Using similar manipulations as in the consumer's problem:

$$
\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)\right)^{-1/\sigma} = \frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)};\ \forall s,s' \tag{56}
$$

Given $\nu(s)$ and $\vartheta(s)$, equation (56) solve for $\phi^e(s'|s)$. Multiplying (56) by $p(s'|s)$ and adding up

$$
\sum_{s'}p(s'|s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)\right)^{-1/\sigma} = \sum_{s'}p(s'|s)\frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)};\ \forall s
$$

Operating with the above equation we obtain:

$$
(1-\vartheta(s))^{-1} = 1+m(s)^{-1}\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma}p(s'|s)^{1-1/\sigma}(1-\vartheta(s'))^{-1}\right]
$$

Where:

$$
m(s) = \sum_{s'}p(s'|s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\left(1+\frac{\varpi}{2}(v(s')^2-v^2)\right)\right)^{-1/\sigma}
$$

Notice that if $\varpi=0$ we have the same solution as before. The equation is still linear, with the caveat that $\vartheta(s)$ is included in $m(s)$. With the solution method that we use, this is not a problem. We start guessing $m(s)$ and then we iterate over it.

F.2 Equilibrium

Now **assets market** clearing reads:

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)[\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}(v(s)-v)^2](1-x) = \frac{\omega(s')+h(s')}{W^T(s)};\ \forall s, s' \tag{57}$$

We also have the **goods market clearing** to check:

$$\begin{aligned} c(s) + e(s) + k'(s) + \frac{\varpi}{2}\left(\frac{k'(s)}{h^e(s)} - v\right)^2 h^e(s) &= y(s);\ \forall s \\ (1-\zeta(s))W^c(s) + (1-\vartheta(s))W^e(s) + \vartheta(s)\nu(s)W^e(s) + \frac{\varpi}{2}(v(s)-v)^2 W^e(s) &= y(s);\ \forall s \end{aligned}$$

$$(1-\zeta(s))x + [1-\vartheta(s)(1-\nu(s)) + \frac{\varpi}{2}(v(s)-v)^2](1-x) = \frac{y(s)}{W^T(s)};\ \forall s \tag{58}$$

In theory we should add the individual adjustments costs, but in equilibrium the ratios are all equal, so I avoid the summation to simplify notation.

Recovering $\phi^e(s)$. Once we have $\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}$ how to get ϕ^e? Recall that:

$$\mathbb{E}_i o(s', i; \phi^e)^{-\sigma} = \mathbb{E}_i \left[[(1-\nu(s) - \frac{\varpi}{2\vartheta(s)}(v(s)-v)^2)\phi^e(s'|s) + \nu(s)r(s')g_i]^{-\sigma} \right]$$

Use a second order Taylor approximation around $g_i = 1$ to write:

$$\mathbb{E}_i o(s', i; \phi^e)^{-\sigma} \simeq [o(s', 1; \phi^e)]^{-\sigma} \left[1 + \frac{\sigma(1+\sigma)(\nu(s)r(s'))^2 Var(g_i)}{2o(s', 1; \phi^e)^2} \right]$$

Where $o(s', 1; \phi^e) = o(s', g_i = 1; \phi^e)$. Define the last term as $R(s', s; V_g, \phi^e)$, therefore:

$$\phi^e(s'|s) = \frac{(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma})^{-1/\sigma} [R(s', s)]^{1/\sigma}}{1-\nu(s) - \frac{\varpi}{2\vartheta(s)}(v(s)-v)^2} - \frac{\nu(s)}{1-\nu(s) - \frac{\varpi}{2\vartheta(s)}(v(s)-v)^2} r(s') \tag{59}$$

Let $\tilde{q}(s') = 1 + \frac{\varpi}{2}(v(s')^2 - v^2)$. As before, we can expand the term

$$\mathbb{E}_i o(s', i; \phi^e)^{-\sigma} r(s') g_i \simeq r(s') o(s', 1)^{-\sigma} \left[R(s', s) - \frac{\sigma\nu(s)r(s')}{o(s', 1)} Var(g_i) \right]$$

Using all this in (55) we obtain

$$\sum_{s'|s}\Pi(s',s)\left[r(s')(1-\vartheta(s'))^{-\sigma}o(s',1)^{-\sigma}\tilde{q}(s')\left[R(s',s)-\frac{\sigma\nu(s)r(s')}{o(s',1)}Var(g_i)\right]\right]=$$

$$\frac{q(s)}{\sum_{s'|s}p(s'|s)}\sum_{s'|s}\Pi(s',s)\left[(1-\vartheta(s'))^{-\sigma}o(s',1)^{-\sigma}R(s',s)\right]$$

Using equation (56) we get:

$$\sum_{s'|s}\Pi(s',s)\left[r(s')\left(\tilde{\beta}(s',s)\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{-\sigma}\left[1-\frac{\sigma\nu(s)r(s')}{o(s',1)R(s',s)}Var(g_i)\right]\right]=$$

$$\frac{q(s)}{\sum_{s'|s}p(s'|s)}\sum_{s'|s}\frac{\Pi(s',s)}{\tilde{q}(s')}\left[\left(\tilde{\beta}(s',s)\frac{(1-\vartheta(s))}{\vartheta(s)}\right)^{-\sigma}\right]$$

Doing all the cancellations and replacing $\tilde{\beta}(s',s)^{-\sigma}$:

$$\sum_{s'|s}p(s',s)r(s')\left[1-\frac{\sigma\nu(s)r(s')}{o(s',1)R(s',s)}Var(g_i)\right]=\frac{q(s)}{\sum_{s'|s}p(s',s)}\sum_{s'|s}\frac{p(s',s)}{\tilde{q}(s')} \tag{60}$$

From (56) we have :

$$\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\tilde{q}(s')\right)^{-1/\sigma}=\frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}$$

Using the second order Taylor approximation (see (59)), we can write:

$$[1-\nu(s)-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2]\phi^e(s'|s)+\nu(s)r(s')=\frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}\left[R(s',s)\tilde{q}(s')\right]^{1/\sigma}$$

Multiplying by $p(s'|s)$ and adding up we obtain:

$$1-\nu(s)-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2+\nu(s)\sum_{s'|s}p(s'|s)r(s')=\sum_{s'|s}p(s'|s)\frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}\left[R(s',s)\tilde{q}(s')\right]^{1/\sigma}$$

From equation (60) we obtain:

$$1-\nu(s)-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2+\nu(s)\sum_{s'|s}p(s'|s)r(s')=$$

$$1-\nu(s)-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2+\nu(s)\frac{q(s)}{\sum_{s'|s}p(s',s)}\sum_{s'|s}\frac{p(s',s)}{\tilde{q}(s')}+Prem(s)$$

Where

$$Prem(s)=\sum_{s'|s}p(s',s)\left[\frac{\sigma\nu(s)^2r(s')^2}{o(s',1)R(s',s)}Var(g_i)\right]$$

Thus define the ratio $Ra(s',s)$:

$$Ra(s',s)=\frac{\left[R(s',s)\tilde{q}(s')\right]^{1/\sigma}}{1+Prem(s)+\nu(s)\left[\frac{q(s)}{\sum_{s'}p(s',s)}\sum_{s'}\frac{p(s',s)}{\tilde{q}(s')}-1\right]-\frac{\varpi}{2\vartheta(s)}\left(v(s)-v\right)^2} \tag{61}$$

It is clear than when $\varpi=0$ we are back to the problem before. Thus, the entrepreneur's problem can be solved linearly, given $Ra(s',s)$

$$1=\sum_{s'|s}p(s'|s)\frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}Ra(s',s) \tag{62}$$

Again, in the algorithm we need to iterate over $Ra(s',s)$.

F.3 State space and laws of motion

Again, it is enough to define s as the pair $\{g_s k, x\}$. To construct $\Pi(s'|s)$ we need the endogenous laws of motions. Start with x, notice that:

$$\begin{aligned}\frac{x(s')}{1-x(s')} &= \frac{W^c(s')}{W^e(s')}= \\ =\frac{\phi(s'|s)\zeta(s)W^c(s)}{\mathbb{E}_i o(s',i,s)\vartheta(s)W^e(s)} &= \frac{\phi(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i,s)\vartheta(s)(1-x)}\end{aligned}$$

Which can be written as:

$$x(s')=\frac{\phi(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i,s)\vartheta(s)(1-x)+\phi(s'|s)\zeta(s)x} \tag{63}$$

Using market clearing (57), this equation can also be written as:

$$x(s')=\frac{\phi(s'|s)\zeta(s)}{M(s',s)+\nu(s)\vartheta(s)(1-x)r(s')}x=\frac{\phi(s'|s)\zeta(s)W^T(s)}{W^T(s')}x$$

For the law of motion of $g_s k$ recall that: $k'(s,i) = \nu(s)\vartheta(s)h^e(s,i,k)$ and: $E(s',i,s) = \phi^e(s',s)\vartheta(s)(1-\nu(s))h^e(s,i,k)$. Therefore in every state

$$\frac{E(s',i,s)}{k'(s,i)} = \phi^e(s',s)\frac{(1-\nu(s))}{\nu(s)}$$

Which is independent of i. Thus, assuming that E_1 is also proportional to k

$$k'(s,i) = \nu(s)\vartheta(s)h^e(s,i,k)$$

In short we can write:

$$k'(s) = \nu(s)\vartheta(s)(1-x)W^T(s) \tag{64}$$

Market prices and $\tilde{\beta}(s',s)$. Recall that asset markets clear when

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)[\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}(v(s)-v)^2](1-x) = \frac{\omega(s')+h(s')}{T(s)} = M(s',s)$$

Replacing ϕ and ϕ^e

$$\tilde{\beta}(s',s)\frac{(1-\zeta(s))}{(1-\zeta(s'))}x + \tilde{\beta}(s',s)\frac{(1-\vartheta(s))}{(1-\vartheta(s'))}\left[R(s',s)\tilde{q}(s')\right]^{1/\sigma}(1-x)$$
$$-\nu(s)r(s')\vartheta(s)(1-x) = M(s',s)$$

Therefore:

$$\tilde{\beta}(s',s) = \frac{M(s',s)+\nu(s)r(s')\vartheta(s)(1-x)}{\left(\frac{(1-\zeta(s))}{(1-\zeta(s'))}x + \frac{(1-\vartheta(s))}{(1-\vartheta(s'))}\left[R(s',s)\tilde{q}(s')\right]^{1/\sigma}(1-x)\right)} \tag{65}$$

Then, we can recover the prices using the fact that:

$$p(s'|s) = \beta\Pi(s'|s)\tilde{\beta}(s',s)^{-\sigma} \tag{66}$$

Notice that

$$M(s',s)+\nu(s)r(s')\vartheta(s)(1-x) = M(s',s)+r(s')K(s') = \frac{W^T(s')}{W^T(s)}$$

Get ϕ^e from market clearing. Notice that multiplying (57) by $p(s',s)$ and adding up we obtain:

$$\zeta(s)x + [\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}(v(s)-v)^2](1-x) = \frac{h(s)}{W^T(s)}$$

Thus,

$$[\vartheta(s)(1-\nu(s)) - \frac{\varpi}{2}\left(v(s)-v\right)^2] = \frac{1}{1-x}\left[\frac{h(s)}{W^T(s)} - \zeta(s)x\right]$$

Using the last in (57) we obtain

$$\phi(s'|s)\zeta(s)x + \phi^e(s'|s)\left[\frac{h(s)}{W^T(s)} - \zeta(s)x\right] = \frac{\omega(s') + h(s')}{W^T(s)}$$

Therefore, it is exactly the same as with $\varpi = 0$:

$$\phi^e(s'|s) = \frac{\omega(s') + h(s') - \phi(s'|s)\zeta(s)xW^T(s)}{h(s) - \zeta(s)xW^T(s)} \tag{67}$$

Get $\nu(s)$ from market clearing. Recall, at this point ϑ is supposed to be known. We can use (**??**):

$$(1-\zeta(s))x + [1 - \vartheta(s)(1-\nu(s)) + \frac{\varpi}{2}\left(v(s)-v\right)^2](1-x) = \frac{y(s)}{W^T(s)} = \tilde{y}(s)$$

Since $v(s) = \nu(s)\vartheta(s)$, now it is quadratic in ν (it is linear when $\varpi = 0$)

$$\nu^2\vartheta\frac{\varpi}{2} + \nu(1-\varpi) = \frac{\tilde{y} - 1 + \zeta x}{(1-x)\vartheta} - \frac{\varpi}{2\vartheta}v^2 + 1$$

The solutions are:

$$\nu = \frac{1}{\varpi\vartheta}\left[-(1-\varpi) \pm \sqrt{(1-\varpi)^2 + 2\varpi\vartheta\left(\frac{\tilde{y} - 1 + \zeta x}{(1-x)\vartheta} - \frac{\varpi}{2\vartheta}v^2 + 1\right)}\right]$$

For $\varpi < 1$ only the positive root can be a solution, therefore:

$$\nu(s) = \frac{1-\varpi}{\varpi\vartheta(s)}\left[-1 + \sqrt{1 + \frac{2\varpi}{(1-\varpi)^2}\left(\frac{\tilde{y}(s) - 1 + \zeta(s)x}{(1-x)} - \frac{\varpi}{2}v^2 + \vartheta(s)\right)}\right] \tag{68}$$