The Macroeconomics of Hedging Income Shares

Adriana Grasso — Bank of Italy
Juan Passadore — EIEF
Facundo Piguillem — EIEF and CEPR*

May 2020

Abstract

The recent debate about the falling share of labor income has brought attention to the trends in income shares, but less attention has been devoted to their variability. In this paper, we analyze how their fluctuations can be insured between workers and capitalists, and the corresponding implications for financial markets. We study a neoclassical growth model with aggregate shocks that affect income shares and financial frictions that prevent firms from fully insuring idiosyncratic risk. We examine theoretically how aggregate risk sharing is distorted by the combination of idiosyncratic risk and moving shares. Accumulation of safe assets by firms and risky assets by households emerges naturally as a tool to insure income shares' risk. We calibrate the model to the U.S. economy and show that low interest rates, rising capital shares, and accumulation of safe assets by firms and risky assets by households can be rationalized by persistent shocks to the labor share.

JEL classification: E20, E32, E44, G11.
Keywords: Income shares fluctuation. Risk Sharing. Asset prices. Corporate Savings Glut.

*We would like to thank Vasco Carvalho, Sebastian Di Tella, Claudio Michelacci, Dejanir Silva, and seminar participants at the European Central Bank, EIEF, Paris School of Economics, Universidad Di Tella, Cambridge University, Universidad Carlos III, University of Cologne, CORE 2018 meetings, SED 2017, Bank of Italy, Chicago Fed and the University College of London for helpful comments and suggestions. We thank Maria Tiurina, Roberto Saitto and Marco Castelluccio for excellent research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Italy. All remaining mistakes are ours.

1 Introduction

For several decades the ubiquity and the robustness of the Kaldor facts led to the dominant belief that capital and labor income shares are roughly constant over time. An important implication of this paradigm is the impossibility of insurance between workers and capitalists. Since aggregate shocks affect both agents equally, even if markets existed, aggregate risk would be uninsurable. However, many recent studies find that income shares are moving far more than in Kaldor's original predictions.[1] This opens up new possibilities: if aggregate shocks have different impacts on capitalists and workers, they can be insured. If so, many questions arise: How do these insurance possibilities affect the financial markets? Which kinds of assets could be affected? Last but not least, how quantitatively important are the implications?

How income shares are insured is shaped by their stochastic properties. Many studies show that the labor share is pro-cyclical in the short run and counter-cyclical in the medium-long run.[2] However, the pro-cyclicality is short-lived, tapering off after approximately a year. For this reason, we focus on the more relevant medium-long run (although our theory does not depend on it). We first show theoretically that a counter-cyclical labor share can be insured between capitalists and workers by capitalists accumulating risk-free assets and lending them to workers, who use loans to leverage and buy risky assets. Next, we analyze business cycle dynamics, which yields interesting predictions. Upward changes in the capital share reduce the capitalists' risk absorption, hindering aggregate risk sharing. As a consequence, the demand for precautionary savings increases, decreasing the risk-free interest rate and increasing the risk premium. Finally, we show that these qualitative predictions are also quantitatively sizable.

The channel that we analyze is simple and intuitive, and has been overlooked despite being consistent with several seemingly unconnected findings. There is a growing literature trying to explain what is known as the *Corporate Savings Glut*, shifting the view of corporations from net borrowers to net lenders. Our theory generates this fact and has the additional implication that the *Corporate Savings Glut* must be accompanied by a *Households' Equity Glut*. Indeed, they are two sides of the same coin: optimal portfolio choices to insure income shares. Furthermore, the theory we present is also consistent with widely documented movements in asset prices, including a continuously falling interest rate and an increasing risk premium[3].

[1] See, for instance, Karabarbounis and Neiman (2014) and Rodriguez and Jayadev (2010).

[2] See Ríos-Rull and Santaeulàlia-Llopis (2010), León-Ledesma and Satchi (2018), and Cantore et al. (2019) for estimations and potential explanations.

[3] See Del Negro et al. (2017b) for the U.S. and Del Negro et al. (2017a) for the global trend in the interest

We build on the neoclassical growth model, allowing for income shares that fluctuate persistently over time. The economy is populated by a continuum of entrepreneurs with different endowments of capital and households-workers who inelastically supply labor. Entrepreneurs own the capital, rent the labor and carry out the production. Households work and fund firms through the financial markets. There is a contracting friction; as in DeMarzo and Fishman (2007), the entrepreneurs' returns cannot be verified as they can privately divert resources for consumption. Firms would like to pool the idiosyncratic risk and obtain funding, but they are subject to a "skin in the game" constraint: the lenders force the entrepreneurs to keep a fraction of their investment, in order to deter them from diverting funds to private accounts. Nevertheless, there are enough financial instruments available such that both entrepreneurs and workers can perfectly insure against aggregate risk. However, the contracting friction prevents capitalists from fully insuring the idiosyncratic risk, which affects the agents' willingness to bear aggregate risk.[4]

The key departure from the macro-finance literature is that we move away from standard constant shares technologies (Cobb-Douglas or AK). Since we are focusing on medium-long time horizons (more than one year), we assume that the labor share is counter-cyclical (the capital share is pro-cyclical), so that capitalists benefit more in booms and suffer more in recessions. We want to stress that, for us, is not important whether the labor share has a trend or not; what matters is the existence of unpredictable fluctuations. Similarly, the reason why income shares are changing is immaterial. As long as the proportion of income in the hands of capitalists and workers fluctuates over the business cycle, there is something to insure.

We start by characterizing the optimal risk sharing arrangements. We do so by disregarding the contracting friction, so that entrepreneurs can fully insure the idiosyncratic risk. The only remaining risk is aggregate. To hedge it, workers and capitalists trade state-contingent assets where, for instance, if the capital share increases, entrepreneurs compensate workers with contingent transfers, and vice versa. The transfers are such that the relative wealth (human plus financial) remains constant after any history and after any realization of the shock. Because the equilibrium prices reflect the correct private and social values, an efficient equilibrium with complete insurance is achieved.

A natural follow-up question to this analysis is how the equilibrium can be implemented if state-contingent assets are unavailable. Consider the straightforward case in which at each point in time there are only two possible realizations of the aggregate shock.

---

rate. Both papers attribute most of the fall in the risk-free rate to an increase in the convenience yield.

[4]We assume that workers are not subject to idiosyncratic risk, so the risk must be interpreted in relative terms throughout the paper. There is ample evidence that firms-entrepreneurs are more exposed to idiosyncratic risk than workers, see for example Guiso et al. (2005).

Then, only two assets are needed to implement the optimal insurance contract: a risk-free asset and a risky asset. We show that the equilibrium is implemented by firms taking a long position on the risk-free asset (saving) and households taking a long position on the risky asset (buying equity). Because markets must clear, a positive position by one sector in a given asset implies a negative position by the other. Intuitively, households leverage (borrow from firms) and buy shares in order to participate in the capital share changes. This market allocation is reminiscent of a corporate savings glut, which is complemented by a *households' equity glut*. However, precisely because markets are complete, all wealth effects are absent and, thus, in a stationary economy, financial positions and asset prices are constant and independent of history.

Next, we reintroduce the contracting friction, so that some idiosyncratic risk remains uninsured, but assume that *income shares are constant*. This allows us to isolate the impact of the idiosyncratic risk. The presence of uninsured risk biases capitalists towards positive asset holdings, which in turn pushes the risk-free rate down and the risk premium up. In contrast to the previous benchmark, since capitalists do not internalize how their actions affect the insurance possibilities of workers, there is a pecuniary externality that distorts asset prices away from their social value. Moreover, because income shares are constant, aggregate risk sharing is no longer possible, and thus, aggregate net holdings of assets become degenerate: with constant shares, the risk-free asset is not traded in equilibrium. All the intertemporal transfers are carried out using only the risky asset. The pecuniary externality generates an inefficient output level but does not affect the economy's reaction to aggregate shocks, which is still efficient.

The inefficiency is reflected in a *deterministic* downward trend of the workers' wealth share. We want to stress that with constant shares the wealth effect is invariant to the realization (or history) of aggregate shocks, muting the possibility of "amplification effects", as pointed out in Di Tella (2017). The pecuniary externality generated by the financial friction is orthogonal to both wealth and realizations of the aggregate shock.

In a nutshell, income shares risk alone generates non-trivial portfolio allocations, but due to the lack of wealth effects, the allocation is invariant to the economy's state. Uninsured idiosyncratic risk alone delivers relevant wealth effects, but the allocations are still invariant to aggregate shocks and with degenerate portfolios. A natural question arises: what happens when both risks coexist? The short answer is that *the interaction between the risks maintains the richness of the optimal insurance and adds non-trivial wealth effects, which are highly responsive to aggregate shocks*. There are two reasons for this. First, because of the pecuniary externality, asset prices are distorted, so that perfect insurance is unattainable. However, although imperfect, some insurance is still possible, so financial portfolios are no longer degenerate. Hence, when an aggregate shock occurs, agents with different (and imperfectly insured) portfolios are affected in different ways. This in turn affects the pecuniary externality, which generates a rebalancing of the portfolios.

Intuitively, the presence of uninsured idiosyncratic risk generates precautionary savings that divert resources from the insurance of aggregate risk. This reduces the financial positions that the workers and capitalists would have otherwise chosen. Because firms are more exposed to idiosyncratic risk than workers, as the capital share increases, the aggregate demand for insurance also increases. In addition, since workers are the main suppliers of funds, the decline in the labor share lowers the insurance supply. At first sight, it looks as if aggregate shocks generate time-varying uncertainty. However, from each entrepreneur's perspective the uncertainty remains constant. It is the aggregate weight of the different agents' exposure to uninsured risk that changes. Hence, shocks that increase the capital share also increase the aggregate demand for insurance with three important consequences: a fall in the risk-free rate, an increase in the demand of safe assets by firms and a higher risk premium. As documented by Farhi and Gourio (2018), these three implications are consistent with important recent developments in the U.S. economy.

We again implement the equilibrium using only a risk-free and risky asset. The tendency towards capitalists accumulating risk-free assets and workers accumulating equity remains, despite the additional precautionary savings that significantly disrupts the insurance of income shares. But the wealth effects create an additional channel that amplifies portfolio rebalancing over time. All in all, the economy generates a large positive correlation between firms' long positions on risk-free assets and the capital share. In parallel, households increase their leverage, borrowing from capitalists to increase their risky asset holdings. It is worth noting that *the key difference between the complete and incomplete market economies is the wealth effect.* In the efficient economy, the realization of aggregate shocks does not alter the allocation of total wealth between workers and capitalists. Instead, when capitalists are exposed to idiosyncratic risk, they hold inefficiently high levels of risk-free assets in order to self-insure. Thus, when positive aggregate shocks occur, in addition to the capital share increasing, the wealth distribution tilts in their favor.

Given the "low variance" of income shares, one may be concerned that these predictions, although theoretically interesting, are quantitatively irrelevant. To quantify magnitudes, we calibrate an otherwise standard economy, first by using usual parameter values and then by replicating standard moments and evaluate its performance in terms of financial quantities and prices. We find that a labor share variance of 0.5%, half of what is observed, implies that workers ought to borrow around 1.3GDP and hold equity at around 0.8GDP. This happens with a risk-free rate of 4% or less (depending on the labor share) and with an equity premium which is between 5% and 6%. Comparing these results to the U.S. economy, households held around 1.3GDP in equity in 2018, while total private debt was also around 1.3GDP. Even though we do not target any of the financial markets' moments, and we do not include any other friction and/or motive for trading, we obtain strikingly reasonable quantities. Not only that, the generated moments for the risk-free rate and risk premium are in line with most estimations.

The paper is organized as follows. Section 1.1 reviews the literature. Section 2 highlights the main mechanisms in a tractable two period model. In Section 3, we present a general model and generalize most results. In Section 4 we calibrate and evaluate numerically the general model. The conclusion follows. All proofs are in Appendices.

1.1 Literature Review

This paper is motivated by the recent literature emphasizing changes in the labor share. Since Karabarbounis and Neiman (2014), several studies have pointed out the apparent labor share downward trend. The potential reasons for this trend range from a fall in the price of investment, the growing importance of housing (Rognlie, 2015), rising mark-ups (De Loecker et al., 2020 and Barkai, 2019), or even demographics (Hopenhayn et al., 2018), to the possibility that the labor share is not falling and it is just a measurement issue (Koh et al., 2018).[5] In this paper *we take the changes in the labor share as exogenous*.[6] This reflects the fact that in our analysis it is not important why the labor share is changing as long as it fluctuates. Moreover, we abstract from the potential feedback from the asset markets to the income shares. Finally, we do not focus on the potential existence of a downward trend, but rather on its cyclical properties. This dimension has been mostly overlooked in the literature except, to the best of our knowledge, by Ríos-Rull and Santaeulàlia-Llopis (2010), León-Ledesma and Satchi (2018) and Cantore et al. (2019).

The positive implications of our paper relate to the recent literature that connects low risk-free rates, risk-premia, and changes in the labor share. Caballero et al. (2017) proposes an accounting framework that connects falling short term real rates, a constant marginal product of capital, the labor share decline, and a stable earnings yield from corporations. In contemporaneous works Eggertsson et al. (2018) and Farhi and Gou-

[5]Since Koh et al. (2018) there has been a debate about whether the labor share is falling or not, and if so, to which extent. While the downward trend seems to be robust in the U.S., the labor share appears to be stationary in the rest of the world. See Gutierrez-Gallardo and Piton (2019).

[6]Grossman et al. (2017) argues in favor of a response to declining aggregate productivity when human and physical capital are complements. Oberfield and Raval (2014) find that the elasticity of capital and labor in the U.S. manufacturing sector has been stable around a value that is substantially lower than the one implied by previous estimates.

rio (2018) document and link the simultaneous patterns of a decreasing labor share and risk-free rates with an increasing savings supply and risk premia. Eggertsson et al. (2018) argue that these trends are mostly due to rising markups. In contrast, Farhi and Gourio (2018) use a different methodology and find that even though mark-ups could be playing an important role, it is the risk premia and unmeasured intangibles that are key. In our paper the mechanism generating these facts is completely different. There is no technological or competition factor; all the trends arise due to financial trading to insure income shares. Also Chen et al. (2017) document the corporate savings glut in the U.S. and relate it to the labor share decline. They argue that it is driven by a combination of changes in the real interest rate, the price of investment goods, corporate income taxes and the increase in markups. In our setup the interest rate is endogenous and, hence, it is not a cause but another implication of the theory.

Our paper is also related to the literature on the financial amplification of aggregate shocks, following the seminal work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). We build on the recent contributions of He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014) and Di Tella (2017), where financial frictions and heterogeneity play a key role. We depart from the previous studies by introducing human capital and income shares correlated with the business cycle. These two assumptions allow us to study positive and normative implications of changes in labor and capital shares over the business cycle. Bocola and Lorenzoni (2020) also introduce labor income, but with constant income shares, in a setup that builds on Krishnamurthy (2003). One of their objectives is to explain why financial intermediaries hold so much risk that is amplified once aggregate shocks are realized. In our work, instead, optimal insurance contracts and constant shares do not generate amplification of aggregate shocks; there is amplification only when the income shares are moving. The difference with respect to Bocola and Lorenzoni (2020) is the modeling of the financial friction, which they assume is state dependent collateral constraint on entrepreneurs. In our paper, the financial friction takes the form of a "skin in the game" constraint.

Carvalho et al. (2016) and Auclert et al. (2019) provide a real explanation for low interest rates based on demographics. The channel through which demographics imply a lower interest rate is that increasing the life span implies a higher supply of safe assets for retirement and a lower demand for investment. Our paper focuses on changes in the labor share and in idiosyncratic risk that increase firms' precautionary savings, which in turn depresses the real rate.[7]

[7] The effects of a low real interest rate can be amplified by nominal frictions. The key idea of secular stagnation, proposed by Hansen (1939), is that the real rate needed to achieve full employment is negative,

2 Hedging Income Shares

In this section we study a two-period economy with exogenous capital to clearly identify the implications of changes in income shares over asset prices and quantities due to the insurance channel. In Section 3 we develop an infinite horizon economy with endogenous investment and show that all the findings in this Section have an equivalent result in a more general environment. The general model is also used in the quantitative analysis.

2.1 Simplified Environment

There are two types of agents: households and entrepreneurs. The economy lasts for two periods, $t = 1, 2$. There are two sources of uncertainty: aggregate shocks, indexed by $s \in \mathbb{S}$ and idiosyncratic production shocks, indexed by $i \in \mathbb{I}$, which occur with probability $\Pi(s, i)$,. In this section, for simplicity, we assume that there is no time discounting.

Consumers-workers. Households are endowed with initial assets a_1 and can supply one unit of labor at no utility cost. Labor income in period one is certain and given by ω_1, which denotes the wage rate. Households would like to insure the realization of the aggregate state in the second period. In this period consumers receive $\omega(s)$ as labor income, which is contingent on the realization of the aggregate shock. To insure it the worker has access to a complete set of Arrow-Debreu (AD) securities, denoted by $a_2(s)$, contingent on state s. Each asset can be traded at (endogenous) prices $p(s)$.

Notice that we allow for as many aggregate financial assets – with imperfectly correlated prices – as possible aggregate states. Thus, in principle, with the "correct" equilibrium asset prices, the income shares could be perfectly insured. We believe that this is the proper approach to analyze this problem from a positive point of view. In reality, there are multiple types of financial assets that have their payoffs correlated with aggregate shocks realizations, and do not rely on nor are constrained by individual moral hazard or commitment problems. By properly combining them, any individual could replicate the same insurance target as a complete set of AD securities.[8] Whether complete insurance is achieved or not ultimately depends on the assets' prices.

The consumer maximizes expected utility:

casting on the economy shadows of low growth and high unemployment. See for instance, Eggertsson et al. (2019), Benigno and Fornaro (2018), Schmitt-Grohé and Uribe (2012) and Marx et al. (2019).

[8]It is well known that to complete the markets there must be as many non-state contingent assets, but with imperfectly correlated prices, as possible states. The required number of non-contingent assets decreases as the frequency of trades increases. For instance, as the trade frequency becomes infinitesimal and the underlying risk is characterized by a Brownian motion (in continuous time), only two assets are needed. A Brownian motion can be approximated as the limit of a Binomial process. See for example Merton (1992).

$$\max_{\{c_1, c_2(s), a_2(s)\}} u(c_1) + \mathbb{E}_s(u(c_2(s)))$$

$$s.t. \quad c_1 + \sum_s p(s) a_2(s) \leq a_1 + \omega_1 \tag{1}$$

$$c_2(s) \leq a_2(s) + \omega_2(s) \tag{2}$$

The consumer uses initial assets a_1 and income ω_1 to consume and buy Arrow-Debreu securities in order to insure aggregate shocks. In the second period consumption is given by the realization of income and the payoff of the assets acquired in the first period.

Entrepreneurs-capitalists. Entrepreneurs are endowed with initial financial assets E_1 and exogenous capital income $\pi_2(s, i)$, which is a function of aggregate and idiosyncratic shocks. We start by assuming that capital income is exogenous in order to highlight the insurance mechanism. In the general model of Section 3, we allow capitalists to choose their capital holdings, which generates additional real effects due to insurance possibilities. The entrepreneurs would like to share the idiosyncratic risk with the consumers, but are prevented from doing so due to a financial friction: they can divert the returns of capital to a private account. Entrepreneurs can buy a complete set of AD securities $E(s)$, which are contingent on s but not on i. The problem of the entrepreneur is:

$$\max_{\{e_1, e_2(s,i), E_2(s)\}_{s \in S}} u(e_1) + \mathbb{E}_{s,i}(u(e_2(s, i)))$$

$$s.t. \quad e_1 + \sum_s p(s) E_2(s) \leq E_1 + \pi_1$$

$$e_2(s, i) \leq E_2(s) + \pi_2(s, i)$$

for all (s, i). The capitalist can use initial assets E_1 to consume and buy AD securities. In the second period, consumption is given by the realization of the return to capital, $\pi_2(s, i)$, and the payoff of the assets acquired in the first period, $E_2(s)$.

Profits and Wages. Profits and wages are given by

$$\pi(s, i) = g_i \alpha(s) Y(s) \tag{3}$$

$$\omega(s) = (1 - \alpha(s)) Y(s) \tag{4}$$

where $g_i > 0 \; \forall i$ and $\mathbb{E}(g_i) = 1$. Equations (3) and (4) stress the sources of income variations. First, income for entrepreneurs and consumers will vary as a consequence of aggregate shocks. These shocks change not only the aggregate output, but also the relative claims to it. In the quantitative section we generate time-varying income shares with a CES production function and shocks to the capital quality. Second, capital income is subject to idiosyncratic risk. This feature can be rationalized as the result of an optimal risk-sharing contract between the entrepreneur, with moral hazard, and a principal (the market), as in DeMarzo and Fishman (2007) and Di Tella (2017).[9]

Markets. Market clearing implies:

$$c_1 + e_1 = Y_1 \tag{5}$$

$$c_2(s) + \mathbb{E}_i(e_2(s,i)) = Y_2(s) \qquad \forall s \tag{6}$$

$$a_2(s) + E_2(s) = 0 \qquad \forall s \tag{7}$$

where $Y_1 \equiv \int y_1(i)di$ and $Y_2(s) \equiv \int y_2(s,i)di$, $\forall s$. The first constraint, equation (5), is market clearing for goods in period 1, where $Y_1 = \pi_1 + \omega_1$. It also implies that the initial asset holdings are such that $a_1 + E_1 = 0$. The second constraint, equation (6), is market clearing for goods in period 2. Note that the idiosyncratic *i.i.d.* shocks cancel out in the aggregate. The final constraint, equations (7), specify that asset markets clear. A *Competitive Equilibrium* is an allocation of consumption and labor $\{c_1, e_1, c_2(s), e_2(s,i)\}_{s\in S, i\in \boldsymbol{I}}$, asset holdings $\{a_2(s), E_2(s)\}_{s\in S}$ and asset prices $\{p(s)\}_{s\in S}$ such that: given prices the consumer maximizes utility by choosing asset holdings and consumption; given prices the entrepreneur maximizes utility by choosing financial asset holdings and consumption.

2.2 Aggregate Risk Sharing

We now derive the optimality conditions for workers and entrepreneurs. From the first-order conditions of the individual problems, we obtain:

$$p(s)u'(c_1) = \Pi(s)u'(c_2(s))$$
$$p(s)u'(e_1) = \Pi(s)\mathbb{E}_i[u'(e_2(s,i))]$$

A key element of the above equations is that, due to the existence of a complete set of AD securities for the aggregate state, the Euler equations hold state by state. The two

[9] Because of moral hazard, the optimal contract provides only partial insurance of idiosyncratic risk: entrepreneurs must keep some "skin in the game". See the online Appendix E and Section 3.2 for details.

first-order conditions together imply:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[u'(e_2(s,i))]}{u'(c_2(s))} \qquad \forall s \tag{8}$$

Note that equation (8) states that the ratio of future average marginal utilities is constant across periods. We now re-express the AD as claims on aggregate output. In particular, define $\phi(s) \equiv \frac{a_2(s)}{Y_2(s)}$. Market clearing implies that:

$$a_2(s) = \phi(s)Y_2(s)$$
$$E_2(s) = -\phi(s)Y_2(s)$$

Then, from market clearing in the goods market in the first period, and assuming that the preferences of the agents are CRRA with parameter σ, we can rewrite (8) as:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[(-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i)^{-\sigma}]}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\sigma}} \tag{9}$$

For future reference define the consumer's wealth share in period 1 and 2 as:

$$\begin{aligned} x_1 &= \frac{a_1 + \omega_1 + \sum_s p(s)\omega_2(s)}{Y_1 + \sum_s p(s)Y(s)} \\ x_2(s) &= \frac{a_2(s) + \omega_2(s)}{Y(s)} \end{aligned} \tag{10}$$

The numerator is the consumer's total wealth, and the denominator is the economy's total wealth. In period 1, the consumer's wealth is the initial assets plus the present value of wages. Total wealth in the economy is the current plus the present value of future total output. For the second period, the share of total wealth may depend on the aggregate shock. In the next section, we characterize the equilibrium as a function of the wealth share and discuss the conditions under which it is constant. We say that there is an amplification of aggregate shocks whenever the share of wealth is state-dependent.

2.3 Efficient benchmark

How do agents share risk when there is no idiosyncratic risk? This is equivalent to setting $\mathbb{V}ar(g_i) = 0$. Normalizing $g_i = 1$ for all i, this implies that:

$$\frac{u'(e_1)}{u'(c_1)} = \frac{(-\phi(s)Y_2(s) + \alpha(s)Y_2(s))^{-\sigma}}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\sigma}}$$

which shows that the future wealth ratios are equalized state by state and across time. The complete characterization of the equilibrium is in the following Proposition:

Proposition 1. *If entrepreneurs can insure their idiosyncratic risk, then:*

a. The competitive equilibrium is characterized by prices and asset holdings given by:

$$
\begin{aligned}
p^{CM}(s) &= \Pi(s) g_s^{-\sigma} && (11)\\
\phi^{CM}(s) &= x^{CM} - (1-\alpha(s)) && (12)
\end{aligned}
$$

where $g_s = Y_2(s)/Y_1$.

b. Wealth shares are constant across time and aggregate states, $x_2(s) = x_1 \; \forall s$.

Proof. See Appendix C.2. □

Proposition 1 characterizes the optimal insurance arrangement. There are two points worth noting. First, since $x_2(s) = (1-\alpha(s)) + \phi(s)^{CM} = x^{CM}$, for all s, consumption shares are constant over aggregate states and across periods. The wealth effects of aggregate shocks are thus muted. This is a standard result. Intuitively, entrepreneurs fully compensate the workers with contingent payments when the capital income share increases, and vice versa. This compensation, through AD securities, is such that both types of agents consume a constant proportion of the aggregate resources, which is independent of the current income shares and the history of shocks. Of course, how the resources are distributed depends on the initial distribution of wealth, determined by $a_1$and E_1, but once the share is determined, it remains constant thereafter. Second, note that if α is constant the capital income share does not vary over the business cycle and neither do the agents' positions on the AD securities, i.e. $\phi(s)^{CM} = \phi^{CM}$ for all s. Intuitively, it would be pointless to write contracts that are contingent on the aggregate state as both types of agents are equally affected by the aggregate shocks. In other words, there are no gains from trading financial assets. The agents may want to transfer resources across time, but for that only one asset suffices. When the capital share varies, workers and entrepreneurs are asymmetrically affected by the shocks and therefore trading financial assets contingent on the aggregate state can make everyone better off. In Proposition 3 (Section 3), we show the analogous result for the general model.

Implementation with two assets. Suppose there are only two aggregate shocks $s_L < s_H$, and two financial assets, a risk-free bond B and a stock-market-indexed risky asset A with payoff $A \times \pi_2(s)$ for $s = L, H$. The risk-free rate is denoted by r_L, and P_A denotes the price of the risky asset. $\{A^c, B^c\}$ is the portfolio allocation of the consumers and $\{A^e, B^e\}$ is the portfolio allocation of the entrepreneurs. In Appendix C.4 we show that the equivalence is given by:

$$r_L^* B^* = -\left(\frac{\alpha(H) - \alpha(L)}{\pi_2(H) - \pi_2(L)}\right) Y_2(L) Y_2(H)(1 - x^{CM}) \tag{13}$$

$$A^* = 1 - \left(\frac{Y_2(H) - Y_2(L)}{\pi_2(H) - \pi_2(L)}\right)(1 - x^{CM}) \tag{14}$$

Notice that workers take an active position on the risk-free asset only if $\alpha(H) \neq \alpha(L)$. Whether the position is positive or negative depends on the correlation between the income shares and the output. If positive productivity shocks are associated with higher $\alpha(s)$, the workers would take a negative (short) position on the risk-less asset and a positive (long) position on the risky asset. Intuitively, they would borrow in the risk-free asset to participate in the gains of the entrepreneurs should there be a positive shock to the capital share. By market clearing this in turn means that the entrepreneurs are issuing equity to increase their positive holdings of the risk-free asset. To summarize, the efficient allocation predicts that workers should leverage to buy equity, while corporations ought to accumulate large quantities of risk-free assets. That is, *a corporate savings glut must be accompanied by household equity glut.*

It is worth mentioning that with a Cobb Douglas production function $\alpha(H) = \alpha(L)$, and thus, households do not hold risk-free assets while the traded quantities of risky assets depend instead on the initial distribution of wealth, $A^* = 1 - \frac{1-x^{CM}}{\alpha}$. In particular, if $E_1 = a_1 = 0$, then $A^* = 0$ and there is no asset trading. Instead, if $A^* \neq 0$, then either the entrepreneurs or the households want to transfer resources across time. They do so by using the risky asset, not the risk-less asset. *Varying income shares thus open a wide range of new implications for financial markets, completely absent in theories that rely on the standard Cobb-Douglas and AK technologies.*

2.4 Incomplete Markets

One may wonder whether the previous predictions could vanish when there is idiosyncratic risk. In this section, we show that the main patterns survive. To that end, we characterize the distortions on aggregate risk sharing as a consequence of incomplete insurance of idiosyncratic shocks. These distortions will generate excessive precautionary savings and, as a consequence, they will distort both the efficient transacted quantities and their prices. Let ζ_1 and ϑ_1 be the savings rate out of wealth for the worker and the capitalist, respectively. To gain some intuition regarding these distortions we perform a second-order Taylor approximation of the right-hand side of (9) around the complete markets' solution. Then, using the defined savings rates, for all $s \in \mathbb{S}$ we obtain:

$$\left[\frac{(1-\vartheta_1)}{(1-\zeta_1)}\frac{(1-x_1)}{x_1}\right]^{-\sigma} \simeq \frac{(-\phi(s)+\alpha(s))^{-\sigma}}{(\phi(s)+(1-\alpha(s)))^{-\sigma}}\left(1+\frac{\sigma(1+\sigma)\alpha(s)^2}{(-\phi(s)+\alpha(s))^2}\frac{\mathbb{V}ar(g_i)}{2}\right) \tag{15}$$

The main difference between the incomplete and complete market economies is the last term of equation (15), which is multiplicative in $\mathbb{V}ar(g_i)$ and is the measure of the remaining uninsured idiosyncratic risk. Crucially, this term is increasing in $\alpha(s)$, implying that for a given level of idiosyncratic risk, the larger the capital share, the larger the demand for insurance. At the same time, as $\alpha(s)$ increases, the feasibility constraint implies that there are less resources available to workers, and therefore, the supply of funds for insurance decreases. Notice that what creates the different asset positions is the fact that the economy looks like it is exposed to time-varying idiosyncratic risk. The larger the $\alpha(s)$, the larger the total amount of uninsured idiosyncratic risk. However, the mechanism here is different; from the perspective of each individual entrepreneur, the idiosyncratic risk $\mathbb{V}ar(g_i)$ remains constant, but the share of "risky income" over total income, along with the difficulty of insuring it, increases.

For a more general characterization of the solution define the "certainty equivalent" $g^{ce}(\alpha, \phi; s)$ as the function satisfying:

$$(-\phi(s)+\alpha(s)g^{ce}(\alpha,\phi))^{-\sigma} = \mathbb{E}_i[(-\phi(s)+\alpha(s)g_i)^{-\sigma}] \quad \forall s \in \mathbb{S} \tag{16}$$

This function depends on α and ϕ, and due to the convexity of the marginal utility, is such that $g^{ce}(\alpha, \phi; s) \leq 1 \; \forall s$, with equality only if $\mathbb{V}ar(g_i) = 0$.[10] Equation (16) also points out the relevance of the minimum realization of the idiosyncratic shock, i.e. $\underline{g}_i$. If $\underline{g}_i = 0$, only solutions with $\phi(s) \leq 0$ are admissible, and thus the entrepreneur cannot borrow. Alternatively, if $\underline{g}_i = 1$, the entrepreneur can borrow up to the full expected value of future income. For the remainder of the paper, we assume that $\underline{g}_i > 0$ is sufficiently large such that both borrowing and lending are feasible in equilibrium. Therefore, we have:

[10] With an abuse of terminology we will refer to g^{ce} as the "certainty equivalent" even though we are not working with utilities but with marginal utilities. Technically speaking, any utility function that has a positive coefficient of *prudence* would generate the same outcome.

Proposition 2. *If $\mathbb{V}ar(g_i) > 0$, and $\mathbb{V}ar(\alpha(s)) > 0$, then:*

a. *For any initial wealth ratio x, prices and financial positions satisfy:*

$$p(s) = \Pi(s)\left[1+\alpha(s)(g^{ce}(s)-1)\right]^{-\sigma} g_s^{-\sigma} \quad (17)$$
$$\phi(s) = x-(1-\alpha(s))+\alpha(s)(g^{ce}(s)-1)[x+\Gamma(g^{ce})]+\Gamma(g^{ce}) \quad (18)$$

for some endogenous value $\Gamma(g^{ce}) > 0$. Moreover:

b. *Precautionary savings: $-\sum_s p(s)\phi(s) > -\sum_s p(s)^{CM}\phi(s)^{CM}$*

c. *Increasing precautionary savings: $\frac{\partial\phi(\alpha)}{\partial\alpha} < \frac{\partial\phi(\alpha)^{CM}}{\partial\alpha} = 1$*

d. *Wealth shares are not constant: $x_2(s) \neq x_1$*

Proof. See Appendix C.2.

The results in this proposition are directly comparable to Proposition 1. Comparing equation (11) with (17), it is evident that $p(s) > p^{CM}(s)\ \forall s$, as long as there is some idiosyncratic risk, so that $g^{ce}(s) < 1\ \forall s$. Thus, the distortive factor $[1+\alpha(s)(g^{ce}(s)-1)]^{-\sigma}$ measures the extent of the *pecuniary externality* in the market for aggregate risk. The larger the capital income share, the larger the distortion. This means that insuring aggregate risk becomes more expensive when there is uninsured idiosyncratic risk. This happens because entrepreneurs are hoarding assets to insure the uninsurable risk, which leads to part b) of Proposition 2: *the excess savings due to the precautionary savings motive distorts the insurance markets for aggregate risk.* To interpret part b) recall that $\phi(s)$ are the contingent savings by workers. Thus, due to market clearing, $-\sum_s p(s)\phi(s)$ are the total savings of the entrepreneurs. If the consumers are saving less under incomplete markets, then by market clearing, capitalists must be saving more.[11]

Equation (18) provides additional information on how idiosyncratic risk distorts financial positions. Comparing (18) with (12), we can characterize the distortion, that at the same wealth ratios is $\phi(s)-\phi^{CM}(s) = \alpha(s)(g^{ce}(s)-1)[x+\Gamma(g^{ce})]+\Gamma(g^{ce})$. Intuitively, because $\Gamma(g^{ce}) > 0$ and $(g^{ce}(s)-1) < 0$, when markets are incomplete the schedule of AD securities is shifted upwards and flattens, as stated in part c). As we explain after equation (15), a larger α is analogous to more risk. Hence, entrepreneurs demand more resources (increase their precautionary savings) on those states in which α is larger. This increases the distortion for those states, making it harder to insure them against aggregate risk. As a result, the AD schedule's slope is less steep than in complete markets.

[11]This statement is independent of ϕ's sign. If $\sum_s p(s)\phi(s)$ is negative, workers will borrow more when markets are incomplete.

Last but not least, because aggregate shocks are imperfectly insured, the realization of each shock will have wealth effects, as stated in part d). This can easily be seen by noticing that in the second period, and because it is the last one, the wealth ratio is equal to the income ratio, i.e, $x_2(s) = \phi(s) + 1 - \alpha(s)$. Thus, using equation (18), the wealth effect is equal to the financial distortion:

$$x_2(s) - x_1 = \phi(s) - \phi^{CM}(s) = \alpha(s)(g^{ce}(s) - 1)[x_1 + \Gamma(g^{ce})] + \Gamma(g^{ce})$$

As long as there is some idiosyncratic risk the wealth effect is not zero. And whenever either $\alpha(s)$ or $g^{ce}(s)$ are not constant, the realization of aggregate shocks will have wealth effects. Since this is closely related to the "amplification mechanism" of financial shocks, we discuss it in more detail in Section 2.5. In short, the presence of uninsured idiosyncratic risk generates a *pecuniary externality* that distorts the financial positions in the market for aggregate insurance. In turn, the distorted financial positions generate non-trivial wealth effects that can further affect the outcomes in the economy. Also, the distortion *decreases* the amount of AD securities that are traded in equilibrium. This shows that the presence of idiosyncratic risk *diminishes* the quantitative relevance of changes in α on the trading of financial assets.

Remark about efficiency. We call Proposition 1's allocation efficient because there is no possible Pareto improvement. Analogously, we call Proposition 2's allocation inefficient because a planner who could control all individual consumptions will choose to redistribute the second period capitalist's consumptions to provide insurance. This will improve ex-ante welfare for capitalist, and through changes in prices, will also improve workers' welfare. In this sense, assets price are inefficiently distorted by a pecuniary externality. However, depending on the instruments available to the planner, the incomplete markets allocation could still be *constrained efficient*. In online Appendix E we characterize a contract the make the incomplete market allocation constrained efficient.

Implementation with two assets. As in the complete markets economy, we now illustrate how counter-cyclical labor shares affect prices and agents' positions in risk-free and in risky assets. In the implementation with two assets we show that the risk-free interest rate decreases as the capital share increases and that the model predicts a steep increase in the demand for safe assets, reminiscent of the corporate savings glut. Again, suppose that there are only two shocks $s_L < s_H$, and two financial assets, a risk-free bond B and a stock-market-indexed risky asset A with payoff $A \times \pi_2(s)$ for $s = L, H$.

Equipped with equation (18) we can write the analogous of equations (13) and (14), expressing assets' positions in terms of the capital share and the certainty equivalent as:

$$R_L B = r_L^* B^* + \left(\frac{\alpha(L)\alpha(H)\left[g^{ce}(L) - g^{ce}(H)\right](x_1+\Gamma) + [\alpha(H)-\alpha(L)]\Gamma}{(Y_2(L)Y_2(H))^{-1}(\pi_2(H)-\pi_2(L))} \right) \tag{19}$$

$$A = A^* + \frac{Y_2(H)-Y_2(L)}{\pi_2(H)-\pi_2(L)}\Gamma + \left(\frac{\pi_2(H)[g^{ce}(H)-g^{ce}(L)]}{\pi_2(H)-\pi_2(L)} + g^{ce}(L) - 1 \right)[x_1+\Gamma] \tag{20}$$

We have purposely written equations (19) and (20) as the efficient allocation plus/minus a distortion to emphasize the impact of the idiosyncratic risk. First suppose that α is constant. Then, it must be that $g^{ce}(H)$=$g^{ce}(L)$. Thus, the distortive term affecting the holdings of the risk-free asset vanishes, and since we have already shown that $r_L^* B^* = 0$, it must also be that $r_L B = 0$. As in the complete markets economy, the risk-free bond is not used in equilibrium. If it is necessary to transfer resources across periods, it is done using the risky asset. But now, the capitalists need to accumulate some savings to hedge the idiosyncratic risk. They do so by buying the risky asset, which leaves less for the workers. Hence, the last negative term in (20), due to $g^{ce}(L) < 1$, reducing A.

What happens when the labor share is counter-cyclical? Then the capital share must be pro-cyclical, i.e., $\alpha(H) > \alpha(L)$, implying $g^{ce}(H) < g^{ce}(L)$. Looking at the distortive term, $r_L B$ is now equal to the negative $r_L^* B^*$ plus a positive term. *Workers borrow less* on (capitalists accumulate less of) the risk-free asset. The distortive effect is also negative on A. The idiosyncratic risk further reduces workers' holdings of risky assets. In short, capitalists hoard both assets, hindering the possibility of insuring aggregate risk.

It is interesting that the distortion to the holdings of the risky asset stems from two sources. The first source, captured by the term $g^{ce}(L) < 1$, arises just because of the existence of uninsured idiosyncratic risk, and it remains even when α is constant. The second source, captured by the term $\frac{\pi_2(H)[g^{ce}(H)-g^{ce}(L)]}{\pi_2(H)-\pi_2(L)} < 0$, arises because of the presence of "time-varying" uncertainty. The inefficiency due to uninsured *idiosyncratic risk interacts with the stochastic income shares, amplifying the distortions*.

Empirical predictions. The implication that the accumulation of risk-free assets by corporations must be accompanied by households' increasing holdings of risky assets is an interesting and testable prediction of our theory that we fully address in the quantitative exercise. As a preview of our findings, we constructed, using the Flow of Funds for the U.S. economy, the holdings of direct and indirect equity by households (see Appendix B.1 for details). In Section 4.2, Figure 4 panel B, we show that there has indeed been a large increase in households' equity holdings, from 0.4GDP to 1.4GDP, an almost tripling of its value. In the same figure (panel A) we also plot all corporations' debt instruments that are unrelated to their main activity. We also see a steady increase of those, from around

0.07GDP to 0.21GDP. We select 1980 as the initial date in order to compare our results to other findings in related literatures that focus on a similar time frame. Nevertheless, in Appendix B.2 we show that the same pattern holds in a longer time span, which stresses that our mechanism is not just a salient feature of the last 30 years.

2.5 Pecuniary externality and the amplification effect

The previous discussion hints at the fact that the pecuniary externality is independent of the aggregate shock when the labor share is constant, so that there is no amplification effect in the sense of Bernanke and Gertler (1989). It is straightforward to show from equation (9) that when α is constant, so is ϕ, in the sense that $\phi(s) = \phi$ for all s. As a result, the certainty equivalent solving equation (16) is also a constant $g^{ce}(\phi)$, independent of the aggregate state. Thus, whenever $x_1 = x_1^{CM}$ we can write (17) and (18) as:

$$
\begin{aligned}
p(s) &= p^{CM}(s)\,[1+\alpha(g^{ce}-1)]^{-\sigma} \\
\phi &= \phi^{CM} + \alpha(g^{ce}-1)[x+\Gamma(g^{ce})] + \Gamma(g^{ce})
\end{aligned}
$$

Loosely speaking, the last two equations show that the incomplete markets economy is a "scaled" version of the efficient one, and that the scale factor is invariant to the aggregate shock's realization. The pecuniary externality affects prices through the constant factor $[1+\alpha(g^{ce}-1)]^{-\sigma}$, *increasing all assets' prices by the same proportion*, and hindering the possibility of insuring the aggregate shock. Moreover, *the wealth effects are muted*, which can easily be seen noting that $x_2 - x_1 = \alpha(g^{ce}-1)[x+\Gamma(g^{ce})] + \Gamma(g^{ce})$, as it is independent of s. Hence, the relative variances of prices and quantities are the same as in the complete markets economy: *when the income shares are constant, there are no amplification effects.*

In Appendix C.2 we show that the only implication of the pecuniary externality is to add a deterministic downward drift to the consumption (wealth) of the worker, determined by $[1+\alpha(g^{ce}-1)]^{-\sigma}$. In the general infinite horizon model this implies a continuously falling relative share of workers' wealth. As it is standard in the literature, we correct the downward drift by introducing a lower capitalist's discount factor β^e, rendering the economy stationary. In this simplified economy, if we assume that the entrepreneur discounts the future at rate $\beta^e = [1+\alpha(g^{ce}-1)]^{\sigma} < 1$, we can show that with constant shares $x_2 = x_1$, $p(s;\beta^e) = p^{CM}(s;1)$ and $\phi(\beta^e) = \phi^{CM}(1)$. In this sense, the resulting *outcomes are observationally equivalent to a complete markets efficient economy*. This is not just a property of the two-period model. In Proposition 4 of Section 3.4, we present a formal proof for the infinite horizon economy with endogenous investment.

We want to stress the relevance of varying income shares not only as a driver of financial markets' quantities and prices, but also as an amplifier of aggregate shocks. For instance, Di Tella (2017), in an economy without human capital, generates amplification effects by making the idiosyncratic risk state dependent. The relevance of labor income is pointed out by Bocola and Lorenzoni (2020). They study an economy with constant shares in which firms must borrow to pay investment and workers in advance, and consumers are net suppliers of funds. Since firms are subject to a collateral constraint, when the constraint is binding, firms also decrease wages, which decreases the consumers willingness to supply insurance, tightening even more the collateral constraint. Thus, the induced correlation between labor income and the financial friction leads to amplification.

3 General model

In this section we present the general model, allowing for any arbitrary $t \in \mathbb{N}$, any arbitrary number of aggregate states $s \in [s^1, s^2, ..., s^N]$ and an endogenous investment decision. To simplify notation, in what follows we characterize the solutions in a recursive fashion. In the two-period economy there was no investment, and given that after the second period there was no choice to be made, keeping track of the exogenous aggregate shock was enough. However, we also showed that the initial distribution of wealth was a determinant of the allocations. In the infinite horizon economy, the distribution of wealth will be changing along the business cycle. Thus, we will need to keep track of it, together with the effective stock of capital, to determine the equilibrium. The redefined state space is $s = \{g_s K, x\}$, where x is the ratio of the consumer's wealth to the total wealth in the economy. We formally show in Section 3.3 that these two state variables are enough to characterize the equilibrium. Since both K and x are endogenous variables, the transition function $\Pi(s'|s)$ is an equilibrium object. However, when solving the individual problems, in Subsection 3.1 and Subsection 3.2, the composition of s and how its transition is determined are irrelevant, because each individual takes them as exogenous.

3.1 Consumer-Worker

As in the two-period economy, the worker is endowed with one unit of labor that is inelastically supplied.[12] When time is infinite the consumer-worker solves:

$$\begin{aligned} V^c(a,s) &= \max_{\{c(s),a(s'|s)\}} \left\{ u(c(s)) + \beta \mathbb{E}_{s'}[V^c(a(s'|s),s')|s] \right\} \\ st. &\quad c(s) + \sum_{s'} p(s'|s)a(s'|s) \leq a(s) + \omega(s); \qquad \forall s, s' \end{aligned}$$

where $\omega(s)$ is the wage and $a(s'|s)$ are the AD securities bought by the consumer in state s, that pay off in the next period contingent on the realization of s'. The initial financial wealth $a_1 \equiv a(s_0)$ is given. The first-order conditions for consumption and financial decisions imply:

$$p(s'|s)u'(c(s)) = \beta\Pi(s'|s)u'(c(s')) \quad \forall s, s'$$

Denote by $\zeta(s)$ the savings rate out of (total consumer's) wealth. We show in Appendix F that the solution is characterized by:

$$c(s) = (1-\zeta(s))(a+\omega(s)+h(s)) \tag{21}$$

$$a(s'|s) = \phi^c(s'|s)\zeta(s)[a+\omega(s)+h(s)] - \omega(s') - h(s') \tag{22}$$

where $h(s) = \sum_{s'|s} p(s'|s)[\omega(s') + h(s')]$ is the consumer's present value of future income, or human wealth. The function $\phi^c(s'|s)$, to be determined, pins down the portfolio allocation. Note that by using the budget constraint it must be true that:

$$a'(s) \equiv \sum_{s'|s} p(s'|s)a(s'|s) = \zeta(s)[a+\omega(s)+h(s)] - h(s)$$

The latter implies $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$. In the next sections we will use the consumer's total wealth, $W^c(s) \equiv a + \omega(s) + h(s)$, to characterize the solution. Thus, $\zeta(s)$ is the savings rate out of wealth and $1-\zeta(s)$ is the implied consumption rate. In Appendix F we show that the consumer's asset positions are characterized by:

$$\phi^c(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)} \qquad \forall s, s' \tag{23}$$

[12] Allowing for an endogenous labor supply would not affect the bulk of our analysis. It would certainly change our calibration but, given that we are targeting the level and fluctuations of the income shares, the conclusions would remain valid as long as they are driven by technology.

Using the condition $\sum_{s'|s} p(s'|s)\phi^c(s'|s) = 1$ and (23) we obtain:

$$(1-\zeta(s))^{-1} = 1 + \sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma} p(s'|s)^{1-1/\sigma}(1-\zeta(s'))^{-1}\right] \qquad \forall s \tag{24}$$

Taking prices, $p(s'|s)$, and the law of motion of s as given, the latter is a recursive equation, linear in $(1-\zeta(s))^{-1}$, which solves for the savings rate. Once $\zeta(s)$ has been found, equation (23) solves for the state contingent assets holdings. For future reference, we define:

$$\tilde{\beta}(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \qquad \forall s, s'$$

3.2 Entrepreneur-Capitalist

Technology. Entrepreneurs combine labor and capital to produce using a constant returns to scale technology:

$$y(s,i,k,l) = F(g_i g_s k, l) + (1-\delta) g_i g_s k$$

where k is stock of capital, l is labor, δ is the depreciation rate and g_i, g_s represent the idiosyncratic and the aggregate shocks, respectively. Denote by $k(s,i) = g_i g_s k$ the effective capital stock. The firm hires labor in competitive markets. In Appendix C.1 we show that the income from capital, $F(g_i g_s k, l) + (1-\delta) g_i g_s k - \omega(s) l(s)$ can be written as:

$$\pi(s,i) = g_i R(s) k(s) \tag{25}$$

where $R(s) = (1-\delta)g_s + r(s)$, with $r(s) = \frac{\partial y(s, \mathbb{E} g_i, k, l)}{\partial k}$. Since we assume that $E(g_i) = 1$, the aggregate capital income share is affected only by the aggregate shock. Notice the linearity of profits in the capital stock and the idiosyncratic shock. This is instrumental in the characterization of the equilibrium as it allows for linear decision functions.

Contracting. Since entrepreneurs are subject to idiosyncratic risk, they will try to insure it. To that end, we assume that entrepreneurs have access to risk neutral intermediaries who can provide insurance. However, due to moral hazard, there is a limit on how much idiosyncratic risk can be offloaded. To be precise, we model moral hazard as endowing entrepreneurs with the possibility of diverting resources from the firm to their private accounts at a cost $0 < 1-\psi < 1$. For each unit of profit that they divert, only ψ units are transformed into consumption goods (or savings). Following DeMarzo and Fishman (2007), the insurance provider is risk neutral. The contract stipulates that the entrepreneur must hand over to the financial intermediary a given proportion of her risky profits, receiving an average of the profits of all firms in return. Since entrepreneurs can misreport their profits and consume a proportion ψ of the misreported profits, in Appendix E we show that the optimal contract implies that the entrepreneur must retain (or be exposed to) a proportion ψ of the idiosyncratic risk. This is known in the literature as a "skin in the game" constraint.[13] As a result, we can write the exposure to the idiosyncratic risk in a simple reduced form. Let $\tilde{g}_i \geq 0$ be the productivity shock to which the firm is exposed. Then, an economy with idiosyncratic risk $\tilde{g}_i$ and restricted insurance is equivalent to an alternative economy in which individual risk is not insurable and firms are subject to idiosyncratic risk g_i satisfying:

$$g_i = (1-\psi)\mathbb{E}_i\tilde{g}_i + \psi\tilde{g}_i \geq 0 \tag{26}$$

Entrepreneurs Program. As in the two-period model, with the natural extension to an infinite horizon, the entrepreneur solves:

$$V^e(E,k;s,i) = \max_{\{e(s,i),E(s'|s),k'(s,i)\}} \left\{u(e(s,i)) + \beta\mathbb{E}_{s',i'}[V^e(E(s'|s),k';s',i')|s]\right\}$$

$$s.t. \quad e(s,i) + k'(s,i) + \sum_{s'} p(s'|s)E(s'|s) \leq E(s) + g_i R(s)k; \qquad \forall i,s,s'$$

where $R(s)$ is the average gross return on capital and $E(s'|s)$ are AD securities bought by the entrepreneur in state s, with payoffs contingent on the realization of state s' in the following period.[14] The initial financial wealth $E_1 \equiv E(s_0)$ is given. Finally, as in the two-period economy, g_i is the idiosyncratic shock that the entrepreneur is exposed to. We maintain the assumption that g_i is *i.i.d.* over time. In this section we show that, despite being subject to idiosyncratic risk, the consumption and savings rates of the entrepreneurs

[13] DeMarzo and Fishman (2007) assume that the principal can sign long-term contracts (there is commitment) and that both the principal and the agent are risk neutral. In contrast, we consider a risk averse agent who can only commit to short term contracts. For similar setups and results in continuous time see DeMarzo and Sannikov (2006). We also show that as long as insurance contracts are not history dependent, this is the best possible insurance independently of whether or not the entrepreneurs have access to hidden savings. This contract is akin to an equity contract in which the entrepreneur creates a company, issues equity for a proportion $1-\psi$ of its ex-ante value and retains a proportion ψ of the value of the company. See Di Tella (2019) for an example of how a social planner could improve the allocations using taxes.

[14] Recall that $R(s)$ is the gross return on capital, which shouldn't be confused with the net return $r(s)$. As such, it includes any potential depreciation. In equilibrium, it will be true that $R(s) = (1-\delta)g_s + r(s)$, with $r(s) = \frac{\partial y(K,L,S)}{\partial K}$. Therefore, productivity shocks also affect capital depreciation. This assumption is widely used in the literature. See for example Brunnermeier and Sannikov (2014) and Di Tella (2017).

are simple and akin to those of the consumers. In particular, due to homothetic preferences, savings rates are linear in total wealth, and thus total savings are independent of the distribution of wealth. In other words, there will be aggregation: knowing the average net worth is enough to forecast future aggregate capital. Notice that here we are using the result in Appendix C.1 to express individual returns as a linear function of the individual holdings of capital. This step is crucial to obtain aggregation and characterize the equilibrium with only two state variables.

Solving the Program. The first-order conditions for capital and securities imply:

$$p(s'|s)u'(e(s,i)) = \beta\Pi(s'|s)\mathbb{E}_i\left[u'(e(s',i))\right] \tag{27}$$

$$q(k',W^e)u'(e(s,i)) = \beta\mathbb{E}_{s',i}\left[u'(e(s',i))R(s')g_i\right] \tag{28}$$

As before we guess and then verify (see Appendix F) that the solution is characterized by:

$$e(s,i) = (1-\vartheta(s))W^e(s,i,k) \tag{29}$$

$$k'(s,i) = \nu(s)\vartheta(s)W^e(s,i,k) \tag{30}$$

$$E(s'|s,i) = \phi^e(s'|s)E_1(s,i) \tag{31}$$

where $\vartheta(s)$ is the entrepreneur's savings rate, and $\nu(s)$ is the portion of savings invested in capital. The entrepreneur's total wealth is:

$$W^e(s,i,k) = E(s,i) + R(s)g_i k$$

In what follows we will refer to $\nu(s)$ as the investment rate. Using the budget constraint we have that total savings, denoted by $E_1(s,i)$, is defined as: $E_1(s,i) \equiv \vartheta(s)(1-\nu(s))W^e(s,i,k)$. Therefore, it must also be true that $\sum_{s'|s} p(s'|s)\phi^e(s'|s) = 1$. The law of motion of individual wealth is:

$$W^e(s',i',k') = \vartheta(s)o(s',i;\phi^e,\nu)W^e(s,i,k) \tag{32}$$

and the ex-post growth rate of wealth

$$o(s',i;\phi^e,\nu) \equiv (1-\nu(s))\phi^e(s'|s) + \nu(s)R(s')g_{i'}$$

Using both Euler equations for the entrepreneur, equations (27) and (28), we obtain that the portfolio allocation, $\phi^e(s'|s)$ and $\nu(s)$, is determined by:

$$\mathbb{E}_{s',i|s}\left[\left[(1-\vartheta(s'))o(s',i;\phi^e,\nu)\right]^{-\sigma}\left(R(s')g_i-\frac{1}{\sum_{s'|s}p(s'|s)}\right)\right]=0 \tag{33}$$

$$(1-\vartheta(s))^{-\sigma}=\beta\frac{\Pi(s'|s)}{p(s'|s)}[(1-\vartheta(s'))\vartheta(s)]^{-\sigma}\mathbb{E}_i o(s',i;\phi^e)^{-\sigma} \tag{34}$$

Some features are worth noting about these two equations. First, as long as $\vartheta(s)$ is independent of wealth and g_i is *i.i.d.*, the investment rate is also independent of individual wealth and the current idiosyncratic shock. Second, equation (34) pins down $\phi^e(s'|s)$, which is also independent of wealth. Comparing (23) and (34) we see that the consumer's savings rate is affected by $\tilde{\beta}^\sigma(s'|s)$ while for entrepreneurs the equivalent term is $\tilde{\beta}^\sigma(s'|s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma}$, which depends on both risk aversion and the exposure to uninsured idiosyncratic risk. The fact that equation (34) involves an expectation while equation (23) does not is what generates the different behavior across workers and capitalists. In the absence of uninsured idiosyncratic risk both agents would react equally to aggregate shocks. Another way to see the role of idiosyncratic risk is by comparing the savings rates, which for the entrepreneur satisfy:

$$(1-\vartheta(s))^{-1}=1+m(s)^{-1}\sum_{s'|s}\left[(\beta\Pi(s'|s))^{1/\sigma}p(s'|s)^{1-1/\sigma}(1-\vartheta(s'))^{-1}\right]\quad\forall s \tag{35}$$

where $m(s)\equiv\sum_{s'}p(s'|s)\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-1/\sigma}$. The only difference between (24) and (35) is $m(s)$. If $m(s)=1\forall s$, the moral hazard friction vanishes and consumers and entrepreneurs choose the same savings rates. However, in general, $m(s)>1\forall s$ and the moral hazard prevents the full insurance of idiosyncratic risk. As a result, in equilibrium, for any price function $p(s)$, it must be true that $\vartheta(s)>\zeta(s)$: on average, the capitalist's wealth grows faster than the consumer's wealth. This creates a downward drift on the consumer's wealth ratio, x. As we showed in the two-period economy, this wealth effect has important quantitative implications, generating large changes in the position of financial assets.[15]

[15] The drift also implies that in the limit the workers end up with zero wealth, with entrepreneurs holding almost all wealth. This may seem like an odd model prediction, but it is the natural outcome of combining agents with heterogeneous exposure to risk. To be able to construct equilibria with non-degenerate stationary distributions of wealth, the literature has resorted to alternative strategies. One solution is to introduce different β's, with capitalists discounting the future more (lower β), as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012). Alternatively, one can assume that with some exogenous probability entrepreneurs become workers, while maintaining their wealth, and are replaced by workers who become entrepreneurs. In the next section, for our quantitative exercise, we follow the first approach.

3.3 Equilibrium

For the allocations to be feasible, they must satisfy the assets' and goods' market clearing conditions, pinning down the equilibrium prices $p(s'|s)$. Furthermore, $\Pi(s'|s)$ must be consistent with the laws of motion generated by individual decisions. The assets' and goods' market clearing conditions are:

$$a(s'|s) + E(s'|s) = 0 \qquad \forall s, s' \tag{36}$$

$$c(s) + e(s) + K'(s)) = y(s) \quad \forall s \tag{37}$$

where $e(s) = \int_i e(s,i,k,E)$, $K'(s) = \int_i k'(s,i,k,E)$, $y(s) = \int_i y(s,i,k,E)$ and $E(s'|s) = \int_i E(s'|s,i,k,E)$. We have avoided the dependency of the allocations on individual wealth because, as shown in the previous section, the savings and consumption rates are independent of it. However, for the aggregation we take it into account. Using the consumer's and the entrepreneur's equations for consumption and investment, (21), (22), (29), (30) and (31), the market clearing conditions, (36) and (37), can be written as:

$$\phi^c(s'|s)\zeta(s)x + \phi^e(s'|s)\vartheta(s)(1-\nu(s))(1-x) = \frac{w(s') + h(s')}{W^T(s)} \quad \forall s, s' \tag{38}$$

$$(1-\zeta(s))x + [1-\vartheta(s)(1-\nu(s))](1-x) = \frac{y(s)}{W^T(s)} \qquad \forall s \tag{39}$$

where $W^T(s) = W^c(s) + W^e(s)$ and $x = W^c(s)/W^T(s)$. By Walras' Law one condition is redundant, while the other determines the equilibrium prices $p(s'|s)$. In Appendix F, equation (86), we show that the Arrow-Debreu prices satisfy:

$$p(s'|s) = \beta\Pi(s'|s)\left(\frac{(1-\zeta(s))}{(1-\zeta(s'))}x + \frac{(1-\vartheta(s))}{(1-\vartheta(s'))}\mathbb{R}(s',s)^{1/\sigma}(1-x)\right)^{\sigma}\left(\frac{W^T(s)}{W^T(s')}\right)^{\sigma} \tag{40}$$

where $\mathbb{R}(s',s) \geq 1$ is an adjustment for the presence of idiosyncratic risk. Thus, when $\mathbb{V}ar(g_i) = 0$, $\mathbb{R}(s',s) = 1$.[16] All of the elements in this equation are endogenous, which complicates the interpretation. However, in the following sections we show how this equation changes under different assumptions, clarifying the economic mechanisms at play.

To find $\Pi(s'|s)$, recall that the aggregate state includes endogenous variables, thus, we need to know their laws of motion. Since the individual law of motion of capital is

[16] See equation (77) in Appendix F, where we provide a definition, making explicit the dependency of it on both $\mathbb{V}ar(g_i)$ and $\phi^e(s',s)$.

$k'(s,j) = \nu(s)\vartheta(s)W^e(s,j,k)$, aggregating yields:

$$K'(s) = \nu(s)\vartheta(s)(1-x)W^T(s) \tag{41}$$

Also, in Appendix F we show that the law of motion of the wealth ratio satisfies:

$$x(s'|s) = \phi^c(s'|s)\zeta(s)\frac{W^T(s)}{W^T(s')}x \tag{42}$$

Notice that even though the process for x is not stationary, it is still Markovian. Thus, it is possible to compute its transition probabilities. As result, (41) and (42) together with the exogenous probability distribution over g_s determine the transition probabilities $\Pi(s'|s)$.

Introducing capital adjustments costs. To avoid cluttered notation we have presented the model without capital adjustments costs. In Appendix F we show how all previous conditions must be modified when is costly to adjust the capital stock. Let $\chi(k', W^e)$ be the units of output that a capitalist with wealth W^e must spend to invest in k' units of capital. To maintain the parsimonious characterization of the equilibrium we use the following functional form:

$$\chi(k', W^e) \equiv \frac{\varpi}{2}\left(\frac{k'}{W^e} - v\right)^2 W^e \tag{43}$$

The standard adjustment cost function is of the form $\left(\frac{k'}{k} - \delta\right)^2$. However, here we have chosen to write the adjustment cost relative to individual net-worth rather than individual holdings of capital. We do so because all decision functions are proportional to net-worth, so this choice preserves the linearity, and consequently, the reduced dimensionality of the state space. Regarding the constant, instead of setting it equal to δ, so that the adjustment cost is zero around the steady state level of capital, we set it such that the adjustment cost is zero around the optimal proportion of investment in physical capital with respect to net-worth. Nevertheless, this function has the appealing feature that the cost of investment grows sharply as the wealth of the capitalist decreases. One can think about it as an additional financial friction obstructing the optimal level of investment.

3.4 Benchmark economies

In this section we provide some important theoretical results that are useful for understanding the quantitative implications. We prove the equivalent to Proposition 1 for the infinite horizon economy with endogenous investment. We also prove formally, in Proposition 4, the intuition provided in Section 2.5, where we argue that the economy with constant shares features no amplification effects with degenerate asset positions.

Efficient risk sharing. Now that we have all the ingredients for the full model we can show the mapping from the two-period model to the general setup. We start by showing the analogous to Proposition 1:

Proposition 3. *Suppose $\mathbb{V}ar(g_i) = 0$, or alternatively that all idiosyncratic risk is fully insured, then:*

$$x(s'|s) = x; \qquad \forall s, s'$$

Proof. See Appendix C.6. □

Proposition 3 contains the same information as Proposition 1 but in a more general environment. After any aggregate shock s, and after any history of shocks, the portfolio holdings are such that all the implied payment transfers leave both types of agents with the same *relative* wealth. In Proposition 1 we were able to characterize the solution in a sharper way because the second period wealth was equal to income. Thus, the compensation consisted of only the difference between the income share and the wealth ratio x. In the general setup the compensation embeds not only the current difference in income but also the present value of all the expected future changes.

The analogy is also evident in the implied asset prices. Using equation (40), because with complete markets $\zeta(s) = \vartheta(s)$ and $\mathbb{R}(s', s, \phi^e) = 1$, the efficient Arrow-Debreu prices are given by:

$$p(s'|s) = \beta\Pi(s'|s)\left(\frac{1-\vartheta(s)}{1-\vartheta(s')}\right)^{\sigma}\left(\frac{W^T(s)}{W^T(s')}\right)^{\sigma}$$

The differences with respect to Proposition 1 are: 1) the price depends on the ratio of wealth rather than the ratio of second period output, and 2) the ratio of the marginal propensity to consume also appears. This is because in Proposition 1 there was no investment decision, while here the resources diverted to investment change over the business cycle. If the consumption ratios were constant, the prices would just reflect the random growth in total wealth. This happens because the changes in $W^(s)$ reflect the common component of the shock, and therefore cannot be insured. All its changes are directly translated to prices.

Constant income shares. Next we derive some implications for the constant income share economy. From equation (38) it is evident that the portfolio choice depends on the realization of the growth rate of wealth. To understand its relationship with the income shares, we express, as we did in Section 2.2, labor income as $\omega(s) = (1 - \alpha(s))Y(s)$. Here $\alpha(s)$ could have any structure, potentially depending on aggregate capital, i.e., there is no additional assumption, it is just a re-labeling. Using this, we can express the worker's human wealth per unit of output as:

$$\tilde{h}(s) = \sum_{s'|s} p(s'|s) \left([1 - \alpha(s') + \tilde{h}(s')]\tilde{g}(s'|s) \right)$$

where $\tilde{h}(s) = \frac{h(s)}{Y(s)}$ and $\tilde{g}(s'|s) = \frac{Y(s')}{Y(s)}$. Using the above expression we can write the right-hand side of (38) as:

$$\frac{w(s') + h(s')}{W^T(s)} = \tilde{g}(s'|s) \frac{[1 - \alpha(s') + \tilde{h}(s')]}{R(s)(K/Y) + (1 - \alpha(s)) + \tilde{h}(s)} \tag{44}$$

With these intermediate calculations we can state the following proposition:

Proposition 4. *Suppose $Prob(\tilde{g}(s'|s)|s = s_j) = Prob(\tilde{g}(s'|s)|s = s_i)$, for all j, i, s' and $\mathbb{V}ar(\alpha(s)) = 0$ (i.e. the income shares are constant and the output exhibits independent increments). Then there exists $\beta^e < \beta$ such that:*

a. $x(s'|s) = x; \quad \forall s, s'.$

b. $p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}; \quad \forall s, s'.$

Proof. See Appendix C.7. □

Most of the information in this proposition is similar to the discussion about constant income shares in Section 2.5. Because of the precautionary savings motive, as capitalists are more exposed to uninsured idiosyncratic risk, they tend to accumulate more assets than workers. In the limit, this force pushes the proportion of wealth in the hands of the workers towards zero. To compensate for this downward drift one can assign a smaller discount factor β^e to the entrepreneur. In Appendix C.7 we show that this alternative discount factor satisfies:

$$\beta^e = \beta \frac{(1 + D)^{-\sigma}}{\mathbb{E}_i(1 + Dg_i)^{-\sigma}}$$

where $D > -1$ is the ratio of risky physical investment to financial assets in the capitalist's portfolio, capturing the capitalist's exposure to idiosyncratic risk. Only when $\mathbb{V}ar(g_i) = 0$ it is true that $\beta^e = \beta$, while for any strictly positive variance $\beta^e < \beta$. With this adjustment, Proposition 4 generates the same prices and allocations as an analogous economy with complete markets ($\mathbb{V}ar(g_i) = 0$) and $\beta = \beta^e$. In this sense, the economy with constant income shares generates business cycle fluctuations that are efficient. As we discussed in Section 2.5, *fluctuations in the income shares are key for generating non-degenerate financial portfolios which interact with the pecuniary externality and thus amplify the effect of aggregate shocks through changes in the quantities*. The assumption that the growth shocks are independent across time guarantees that there are no *indirect effects through prices*. If the output exhibited persistent growth rates, this would create predictable changes in asset prices that would in turn affect the quantities. Nevertheless, the largest share of the amplification effect arises from the direct effect on quantities due to insurance motives.

Similar results can be found in the literature; Proposition 4 is a generalization of Di Tella (2017), which presents a similar result in a continuous time environment with an AK technology (and hence no labor supply) and aggregate productivity shocks that follow a Geometric Brownian Motion. We extend the result to a discrete time environment, allowing for any constant returns to scale technology and an additional factor of production (labor), as long as the income shares are constant. Also Bocola and Lorenzoni (2020) provide a similar result in a discrete time environment, but they maintain the AK assumption and assume that the aggregate productivity shock is *i.i.d.* (in levels) over time.

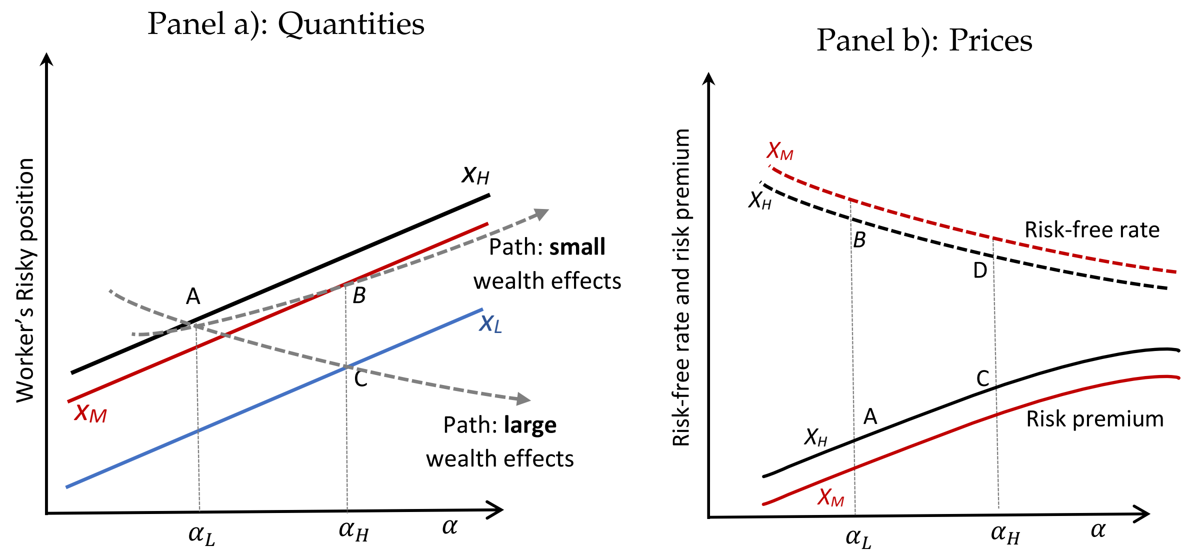
3.5 The economy with moving shares

Obtaining analytical results with varying income shares and idiosyncratic risk, as in Proposition 2, is less straightforward in the general model. However, using the insights from this proposition and the equilibrium equations for the general model we can provide some intuition for the expected outcomes. We start by analyzing the effects of an increase in the capital share on the desired financial positions. As we explained in Section 2.4, the assets' positions widen with α, so that the larger α, the larger the worker's positive position in the risky asset, and the more leverage the workers has in the risk-free asset.

For instance, suppose that the economy is initially in a state with capital share α_L and distribution of wealth x_H. In Panel a) of Figure 1, we depict the hypothetical worker's risky position at point A. Now suppose there is a shock that increases the capital share to $\alpha_H > \alpha_L$. Absent wealth effects, the worker's position would move along the black line labeled x_H. In this case, a positive α shock unambiguously increases the worker's position on the risky asset. The same argument applies for the worker's debt. This would be the outcome with complete markets, since x would remain constant after any shock.

However, when markets are incomplete, a positive α shock would increase the capitalist's wealth, and therefore x would fall. Since a larger $1 - x$ implies that more resources must be devoted to insure the idiosyncratic risk, capitalists are less willing to trade on the

Figure 1: Wealth effects and financial markets



insurance of the income shares' risk. This effect dampens the increase in the trading of the risky asset; it does so in such a way that after the shock the worker's risky position may end up being smaller. If the wealth effect is "small", say x moves to a new level $x_M < x_H$, such that the demand now lies on the red line labeled x_M, then the new position would be located at a point such as B in Panel a) of Figure 1. Despite the dampening effect, the worker's risky positions would be positively correlated with the capital share. But if the wealth effect is large enough, e.g. the wealth distribution moves to $x_L < x_M < x_H$, then the economy would end up at point C, where the worker's risky position and α are negatively correlated. Thus, the extent of uninsured idiosyncratic risk and its implications for the wealth effects are crucial components of the quantitative implications.

In Panel b) of Figure 1 we show the expected patterns for asset prices. To this end it is important to bear in mind equation (40). In the previous section neither idiosyncratic risk nor the distribution of wealth played any role, but now these two components have important implications. Our setup does not explicitly include a risk-free rate, but it is straightforward to show that if there were a risk-free asset, its return would be given by $1/\sum_{s'} p(s'|s)$. Thus, an increase in the "average" price is equivalent to a fall in the risk-free rate. As we explain in the two-period model, and increase in α acts as an endogenous increase in uncertainty, which is reflected in a larger factor $\mathbb{R}(s', s)$ in equation (40). This direct impact is the main component generating a decreasing risk-free rate as shown in Panel b). Ceteris paribus, the new rate would move from point B to point D in the figure. However, there are two additional effects. First, x drops, say to $x_M < x_H$. Then the weight on $\mathbb{R}(s', s)$ increases, i.e., there is more idiosyncratic risk in the economy, which also raises the average prices. However, because of the increased risk, the capitalists' consumption slows down, which puts a downwards pressure on prices. Taken together, these simultaneous effects could dampen the fall in the risk-free rate, as shown in Panel b) with the line labeled x_M, or reinforce it. In general, we expect an important decline in the interest rate when the capital share increases.

Moreover, in Appendix F (see equation (80)) we show that the risk premium satisfies:

$$Prem(s) = \sum_{s'|s} p(s', s) \left[\frac{\sigma \nu(s)^2 r(s')^2}{o(s', 1)\mathbb{R}(s', s)} \mathbb{V}ar(g_i) \right]$$

In this case the larger α has a direct and sizeable impact on increasing $r(s)$. Without wealth effects the risk premium should increase as depicted in the shift from A to C in Panel b). But there are two additional effects related to wealth. First, because the wealth share of the agents not exposed to idiosyncratic risk drops, it becomes increasingly harder to insure it, and therefore the risk premium could rise further. Second, the higher exposure to the idiosyncratic risk generates a portfolio rebalancing, in which the capitalist invests less in capital so that $\nu(s)$ falls. Thus, the risk premium may fall as indicated in Figure 1. Which effect dominates is a quantitative question that we analyze in the next section.

4 Quantitative Implications

We now turn to a calibrated economy to show quantitative results. In a nutshell, we showed that income shares' risk alone generates non-trivial portfolio allocations, but due to the lack of wealth effects, the allocation is invariant to the state of the economy. Uninsured idiosyncratic risk alone delivers relevant wealth effects, but the allocations are still invariant to aggregate shocks and imply degenerate portfolios. In Section 3.5 we discussed how the interaction between them maintains the richness of the portfolio allocations and adds non-trivial wealth effects which are highly responsive to aggregate shocks.

4.1 Calibration

In this section we do not attempt to match either the assets' positions or prices of those assets. There are multiple factors affecting the financial markets from which we are abstracting. For instance, households may want to accumulate risk-free assets for liquidity reasons, but in this paper we have not included a demand for liquidity. For this reason, we calibrate the model to replicate "untargeted" standard moments and we evaluate its predictions for the financial markets.

We set the workers' discount factor to $\beta = 0.96$ and the risk aversion parameter to $\sigma = 2$. These are both standard parameters in the literature. Since the worker discounts the future less, it would be the agent determining the average risk-free rate. Thus, our choice implies a risk-free rate of around 4% in the stationary equilibrium. Moreover, in our setup σ also pins down the intertemporal elasticity of substitution (IES). As shown by Crump et al. (2015) most empirical studies point towards an IES of 0.5.[17]

Regarding β^e, we calibrate it to replicate the implied average x in the data. We construct x using the flow of funds tables. Since $x = \frac{W^c}{W^T} = \frac{a/y+(1-\alpha)+h/y}{1+(1-\delta)k/y+h/y}$, we use households' financial assets over GDP as a measure of a/y. We approximate h/y as $h/y = (1 + E(r))E(1 - \alpha)/E(r)$, which is the exact value for the human wealth to GDP ratio in a deterministic economy. For the risk-free rate we use the Fred AAA 10 year corporate bond. Then using the capital to GDP ratio from Fred in every period we obtain that the average x is around 0.82. This approach generates $\beta^e = 0.885$.

To discipline the relationship between wages, the capital stock and capital returns we use a CES production function with parameters $\{\rho, \alpha_k\}$, which can generate meaningful movements in the labor share:

$$F(K, L) = \left[\alpha_k K^{\frac{\rho-1}{\rho}} + (1 - \alpha_k)L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

There are many reasons why the labor share could be procyclical in the medium-long run, which are not yet fully understood. It could be due to purely technological reasons, as in Karabarbounis and Neiman (2014) and Koh et al. (2018), or changes in bargaining power during the business cycle, as in Ríos-Rull and Santaeulàlia-Llopis (2010), or just wage rigidities as in Cantore et al. (2019).[18] Since our model does not include additional frictions, we rely on ρ to obtain additional variation in the labor share.[19] Moreover, with this technology the capital income share also depends on the capital-output ratio. Thus, we set $\rho = 1.5$ and we choose α_k and δ to jointly target the average capital-output ratio and the average capital share. This generates $\alpha_k = 0.28$ and $\delta = 0.048$. The resulting average

[17]This would affect the resulting equity premium. A large literature has explored alternative solution such as long-run risk, Bansal and Yaron (2004) and Hansen et al. (2008), and disaster risk, Barro (2009) and Gourio (2012), as possible explanations for the premium between equities and safe bonds.

[18]Karabarbounis and Neiman (2014) and Koh et al. (2018) find values of ρ between 1.15 and 1.4; Cantore et al. (2019) and Ríos-Rull and Santaeulàlia-Llopis (2010) find that the labor share is pro-cyclical in the medium-long run. León-Ledesma and Satchi (2018) show how to model an economy in which the ES is different in the short run than in the medium-long run based only on technology choices.

[19]The variance of the capital share in the data ranges from 1.4% to 2%, depending on how the labor share is computed. To obtain this value we need to force ρ to reach unrealistically high values. For instance, Karabarbounis and Neiman (2014) finds that ρ could range from 1.15 to 1.4.

capital share is 0.37, in line with most estimates that include sufficiently long time series.[20] We obtain a capital-output ratio of around 2.3, which is relatively low. Nevertheless, the computed K/Y and depreciation rate are consistent with the average values in the Penn World Tables (PWT8.0), as in Inklaar and Timmer (2013), who estimate a depreciation rate between 0.045 and 0.05 and a capital-output ratio of roughly 2.5.

To construct the mapping to the intuitive riskless and risky asset positions we assume that the aggregate shock can take on two values: $g_H = 1.02$ and $g_L = 0.98$, each occurring with probability 1/2. Notice that the variance of the assumed process is $\mathbb{V}ar(g_s) = p(1-p)(g_H - g_L)^2 = \frac{1}{4}0.04^2 = 0.0004$, which is in line with the medium-long term variation of GDP in the U.S. economy. The *i.i.d.* structure of the shock simplifies the state space without losing realism, since the generated output is still close to a random walk. With only two possible realizations of the aggregate shock we can construct the straightforward mapping from the economy with Arrow-Debreu securities to that with only two assets: a risk-free and a risky asset. Adding more realizations would have minimal quantitative effects and would make this mapping less clear.

As described in Section 2.4 what matters for the entrepreneur is the residual risk $\psi^2\mathbb{V}ar(g_i)$ to which she is exposed. Since we are assuming that workers are not subject to idiosyncratic risk, this risk must be interpreted in relative terms. There is ample evidence that firms are more exposed to idiosyncratic risk than workers. Following He and Krishnamurthy (2012), we set $\psi = 0.2$ to match the 20% share of profits that hedge funds charge. The total variance of the idiosyncratic shock is $\mathbb{V}ar(g_i) = 0.04$ and thus the total idiosyncratic risk borne by entrepreneurs is given by $\psi^2\mathbb{V}ar(g_i) = 0.0016$. We summarize our calibrated parameters in Table 1.

4.2 Results: the corporate and household gluts

Table 2 displays several moments of the calibrated economy. Panel A shows the targeted values in the data and the corresponding model predictions. On average, the capital share is around 0.37, the capital-output ratio is 2.3 and the workers' share of wealth is 0.83, very close to the targeted moments. Panel B displays the values for the risk-free rate, the investment rate and the workers' portfolio allocations. Although these are all non-targeted statistics, the calibrated economy delivers sensible predictions for each quantity. The risk-free rate is on average 4%, investment is 11% of GDP and, on average, the workers hold

[20] The current debate about the labor share focuses on the last 20 years or so and estimates lower values. In our environment the capital share is very close to a random walk but is still stationary. As we show in the next section, long periods of time (say 20 to 30 years) with either constantly increasing or decreasing capital shares arise naturally.

Table 1: Baseline Calibration

Parameter	Description	Value
σ	Risk aversion	2
β	Discount Factor Consumers	0.96
β^e	Discount Factor Entrepreneurs	0.885
ρ	Elasticity of Substitution	1.50
α_k	Capital Share Parameter	0.28
δ	Depreciation	0.048
$g_{s,h}, g_{s,l}$	Aggregate Shocks to Capital	1.02, 0.98
p_s	Probability of g_s	0.5
$Var(g_i)$	Variance of Id. Shocks to Capital	0.04
ψ	Exposure to Id. Risk	0.20

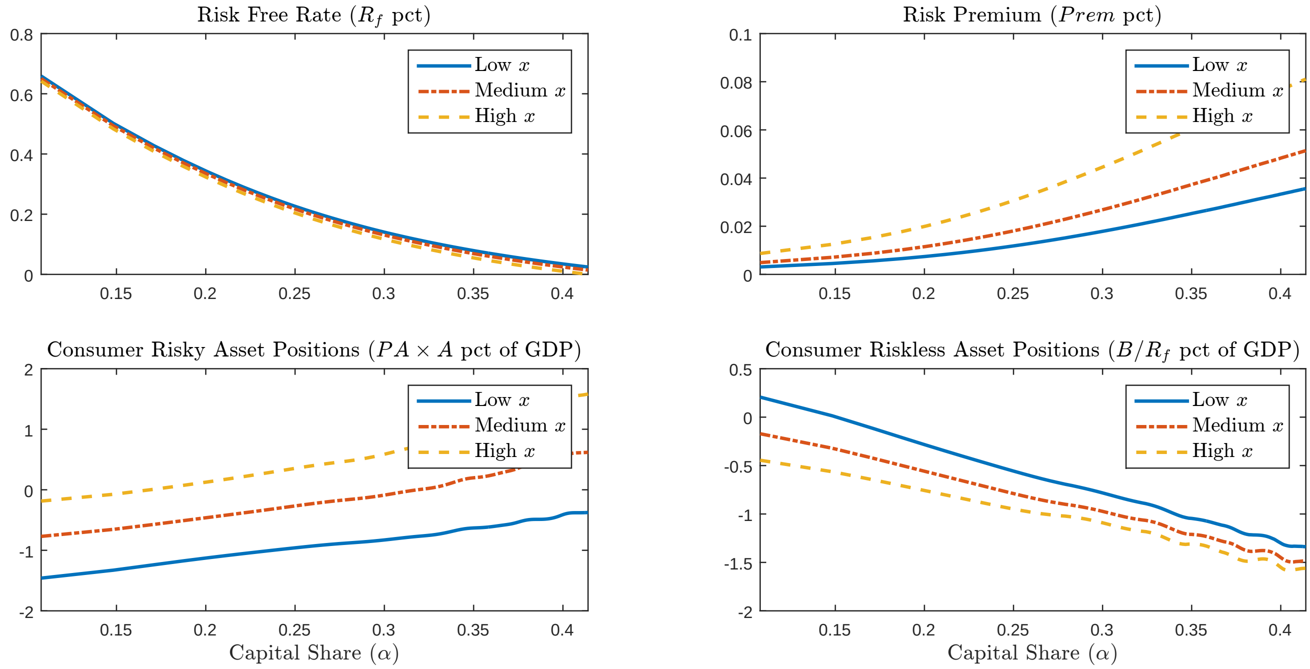
Table 2: Simulated moments

Quantity	Description	Data	Model
Panel A - Targeted Moments - Means			
$K(s)/Y(s)$	Capital-Output Ratio	2.50	2.31
$\alpha(s)$	Capital Share	0.37	0.37
$x(s)$	Workers' Wealth Share	0.82	0.83
Panel B - Non-targeted Moments - Means			
$B(s)/Y(s)$	Worker Risk-less Asset Position		-1.36
$PA(s)A(s)/Y(s)$	Worker Risky Asset Position		0.79
$r(s)$	Risk-Free Rate		0.043
$I(s)/Y(s)$	Investment		0.113
$Prem(s)$	Risk Premium		0.05

a positive amount of risky assets (equities) and finance this position by borrowing on the risk-less asset. The assets' positions have been constructed using the equilibrium laws of motion in a similar fashion to Section 2.4.

The main takeaway from Table 2 is that the mechanism of this paper is able to generate large and reasonable financial positions with apparently low variations in the income shares. Situating the values in the context of the U.S. economy, households held around 1.3GDP in equity in 2018, while the total private debt was also around 1.3GDP. Moreover, the model delivers sensible results for financial positions while also implying an underlying risk-free rate and risk premium that are not far from the actual observations. Last but not least, the asset positions and prices exhibit the right sign for the correlations with both GDP and the capital share.

Figure 2: Equilibrium policy functions

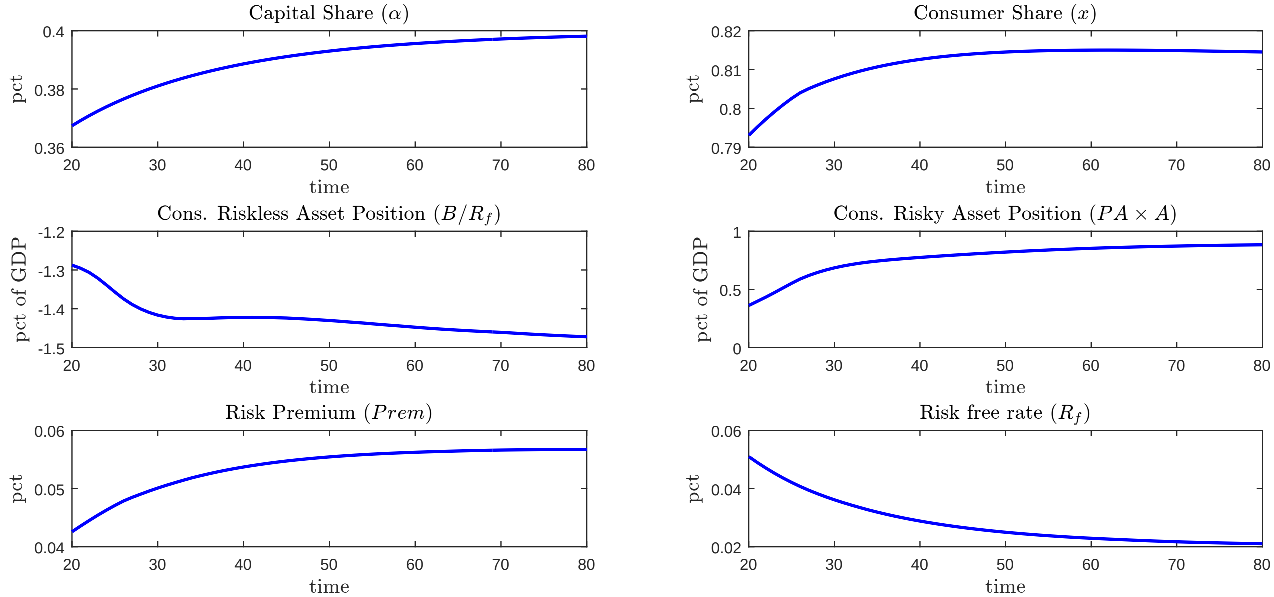


Note: This figure plots, from left to right and from top to bottom, the risk free rate, the risk premium and the risky and risk-less asset positions for consumers as a function of the capital share. The values of x low, medium and high are given by 0.71, 0.79, 0.85.

As mentioned in Sections 2 and 3.5, the combination of a pro-cyclical capital share and limited risk sharing has important implications for the changes in the aggregate portfolio allocations. In Figure 2 we plot the calibrated version of Figure 1. First, in the top left panel we show that the wealth effect on the risk-free rate is mild. As a result, the risk-free rate falls sharply as the capital share increases. The combination of high exposure to idiosyncratic risk and large quantities of accumulated capital pushes the return on capital downwards. As expected, and anticipated in Section 3.5, the risk premium increases as the capital share rises as pictured in the top right panel of Figure 2. Unlike the risk-free rate, here the wealth effects are more relevant. For a given capital share, the risk premium is higher the higher is the consumer's wealth share.

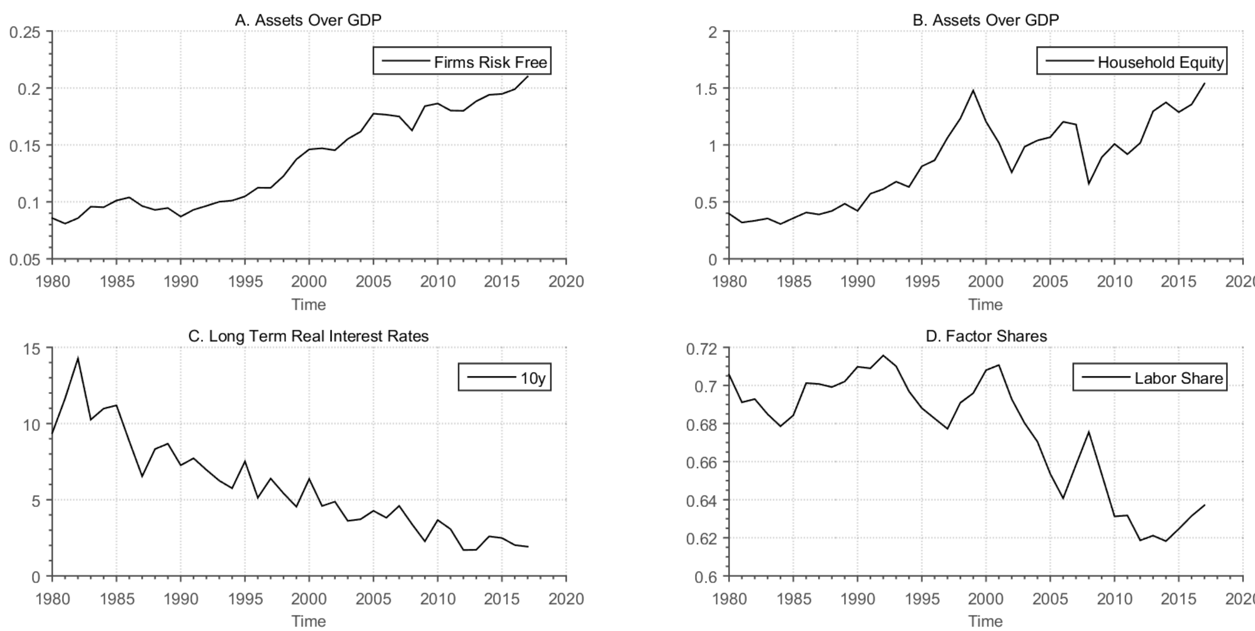
In the bottom left panel of Figure 2 we plot the risky asset positions, $PA(s) \times A(s)/Y(s)$, for different values of x. As we discussed in Section 3.5 the position on the risky asset widens as $\alpha(s)$ increases and the wealth effects become more prominent. For instance, when x falls from 0.85 to 0.79, the risky position decreases by around 0.5 GDP (the difference between the yellow dashed line and the red dotted line). In fact, if x is sufficiently low, the worker will also be borrowing in the risky asset. The flip side of the accumulation of risky assets is the borrowing in the risk-free asset. In the bottom right panel of Figure 2 we depict the implied patterns for the workers' holdings of risk-free assets. The pattern

Figure 3: Model generated fall in the labor share



Note: This figure displays key quantities for our calibrated model after a path of observed changes in α as in the last 20 years.

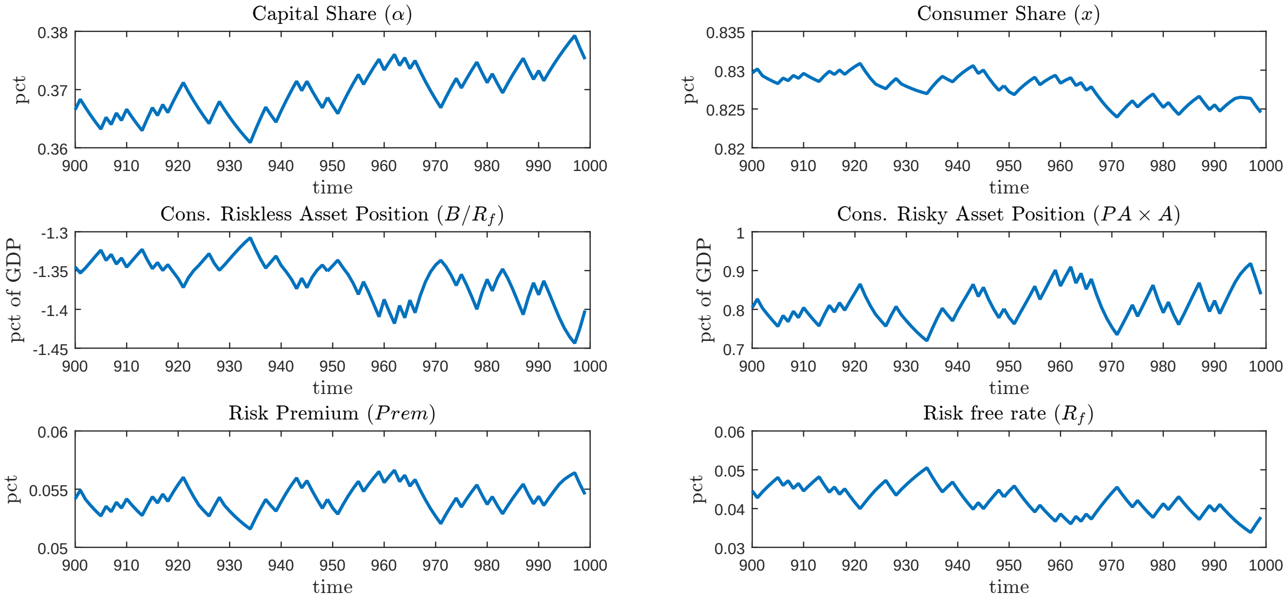
Figure 4: Corporate savings and household equity gluts



U.S. Data. Data are from 1980 on.

for $A(s)$ is mirrored by $B(s)$ with the opposite sign. As the capital share increases, the financial positions sharply widen, increasing the leverage that is used to accumulate risky assets. Because of market clearing, the capitalists' financial positions are the negative of

Figure 5: Time path



Note: This figure displays key quantities for our calibrated model after a sequence of shocks.

the workers' positions. Thus, if households are borrowing to buy equity, it must be that the corporate sector is increasing the issuance of equity and accumulating risk-free assets.

The increased holdings of risk-free assets by corporations is known as the *corporate savings glut*. What we add to this discussion is the implication that it must be accompanied by increasing holdings of risky assets by households: a *household's equity glut*. Given the discussion in Section 3.5 and the large wealth effects observed in Figure 2, one may wonder if the time paths implied by the model generate a positive or negative correlation between the capital share and the capitalists' holdings of risk-free assets. To shed light on this issue we reproduce an increasing path for α of the same magnitude as observed in the data since 1980. We plot the implied paths for the worker's wealth share, risk-free assets, risky assets, risk-free rate and the risk premium. The results can be seen in Figure 3. With a standard calibration, the wealth effects are not enough to overturn the patterns predicted by the complete markets economy. As the labor share falls, capitalists accumulate more risk-free assets, and lend these funds to workers who, leveraging, invest in equity to insure against changes in the income shares. At the same time, as the capital share increases, it becomes harder for capitalists to insure the idiosyncratic risk, which put a downwards pressure on the risk-free rate. Because capital is a risky asset, to decrease their exposure to risk capitalists also invest less.

Are these patterns consistent with the data? In Appendix B.1 we describe in detail how, using widely available data, we construct model-consistent measures for the U.S. economy. In Figure 4 we show the results for the period starting in 1980. We choose this

Table 3: Implied correlations

Capital Share	Risk Free Position	Risky Position	Risk Free Rate	Risk Premium
1.00	-0.97	0.85	-0.99	0.81
-0.97	1.00	-0.91	0.98	-0.86
0.85	-0.91	1.00	-0.90	0.99
-0.99	0.98	-0.90	1.00	-0.88
0.81	-0.86	0.99	-0.88	1.00

starting point in this section for ease of comparison with other literatures that focus on a similar time frame. Nevertheless, in Figure 6 of Appendix B.1 we show the equivalent calculations starting in 1946. In short, Figure 4 shows a striking similarity, in terms of patterns, to those predicted by our theory. The corporate savings glut is accompanied by a households' equity glut and falling interest rates.

It is also evident from the figures that the levels of our calibrated quantities are relatively distant from the actual observations. We have too many risk-free assets and too few risky assets. As we mentioned before, the purpose of the exercise is not to exactly match all the moments but to assess the quantitative relevance of this mechanism. To properly match all the moments, we would need to add additional motives to hold and trade financial assets, which is beyond the scope of this paper. Nevertheless, we show that *seemingly small variations in the income shares have a large impact on the financial markets*.

The path chosen for α in Figure 3, although observed in data, may appear arbitrary within the structure of the model. Could random paths with *i.i.d* aggregate shocks generate a similar pattern? As mentioned in Section 3.3, the g_s shock is such that the *growth rates are i.i.d.*, so that the *levels* are close to random walks. Figure 5 depicts a typical path from a simulation of the model. Although the shocks are *i.i.d.*, the model generates long time spans of an increasing capital share (top left panel) and a decreasing consumers' share of wealth (top right panel). In addition, the consumers' risky and riskless asset positions (middle panels) are almost perfectly negatively correlated. This can be seen in detail in Table 3, which displays the model-implied correlations[21]. The predictions derived analytically in Section 2.4, reaffirmed in Section 3.3 and clearly depicted in Figure 3 are part of the typical random paths of this economy.

[21] Additional correlations for other variables can be found in Table 4 in Appendix A

5 Conclusions

The Kaldor facts led to the prevailing belief that the capital and labor income shares were, aside from some small short-run variations, roughly constant. An important implication of this belief is the impossibility for workers and capitalists to insure each other. With constant income shares, aggregate fluctuations affect both sectors equally and only common uninsurable shocks are left. Recent studies, however, have shown that the labor share moves in both the short and medium-long run.

In this paper we study how varying income shares affect risk sharing, and examine the resulting predictions over allocations and prices. We argue that variation in the income shares creates an important motive to share risk between capitalists and workers. Since both are differentially affected by aggregate shocks, they have incentives to trade in the financial markets to insure changes in their relative income. When the labor share is counter-cyclical, the optimal insurance contract can be implemented by workers borrowing in risk-free assets and buying equity to participate in the capitalists' gains. The presence of uninsured idiosyncratic risk for capitalists decreases their willingness to trade, hampering the implementation of the optimal contract. Nevertheless, we show in a calibrated model that this channel is quantitatively large, to the extent that it can by itself account for some observed long-term patterns in the financial markets: the corporate savings glut, the households' equity glut and the falling interest rates. Thus, although there are certainly other factors shaping these patterns, the mechanism proposed here cannot be ignored.

The focus of this paper is on the medium-long run. However, our model would lend itself naturally to the study of how income shares exacerbate or mitigate fluctuations. The setup is also suitable for analyzing questions linking inequality and asset pricing. In particular, ours is a two-factor asset pricing model in which the capital share and the relative wealth of financial intermediaries are factors pricing the "cross-section" of assets. Both business cycles and asset prices are topics for further research.

Finally, we have abstracted from many frictions that can either directly affect financial markets, such as inflation, liquidity concerns, default risk, etc., or that can indirectly affect the financial sector through links to the real economy, such as wage and price rigidities. The interactions of these frictions with our mechanism are interesting paths for future research.

References

Auclert, A., F. Martenet, H. Malmberg, and M. Rognlie, "Wealth and Demographics in the 21st Century," Technical Report, Working Paper 2019.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J Van Reenen, "The fall of the labor share and the rise of superstar firms," Technical Report, NBER Working Paper 2017.

Bansal, Ravi and Amir Yaron, "Risks for the long run: A potential resolution of asset pricing puzzles," *The Journal of Finance*, 2004, *59* (4), 1481–1509.

Barkai, Simcha, "Declining labor and capital shares," *Journal of Finance*, 2019, *Forthcoming.*

Barro, Robert, "Rare disasters, asset prices, and welfare costs," *American Economic Review*, 2009, *99* (1), 243–64.

Bates, T. W., K. M. Kahle, and R. M. Stulz, "Why do US firms hold so much more cash than they used to?," *The Journal of Finance*, 2009, *64* (5), 1985–2021.

Benigno, Gianluca and Luca Fornaro, "Stagnation traps," *The Review of Economic Studies*, 2018, *85* (3), 1425–1470.

Bentolila, Samuel and Gilles Saint-Paul, "Explaining movements in the labor share," *Contributions in Macroeconomics*, 2003, *3* (1).

Bergstresser, Daniel and James Poterba, "Asset allocation and asset location: Household evidence from the Survey of Consumer Finances," *Journal of Public Economics*, 2004, *88* (9-10), 1893–1915.

Bernanke, Ben and Mark Gertler, "Agency costs, net worth, and business fluctuations," *The American Economic Review*, 1989, pp. 14–31.

Bertaut, Carol C. and Martha Starr, "Household portfolios in the United States," Technical Report, FEDS Working Paper 2000.

Blanchard, Oivier, William D. Nordhaus, and Edmund S. Phelps, "The medium run," *Brookings Papers on Economic Activity*, 1997, (2), 89–158.

Blanchard, Olivier and Francesco Giavazzi, "Macroeconomic effects of regulation and deregulation in goods and labor markets," *The Quarterly Journal of Economics*, 2003, *118* (3), 879–907.

Bocola, Luigi and Guido Lorenzoni, "Risk Sharing Externalities," Working Paper 26985, National Bureau of Economic Research April 2020.

Brunnermeier, Markus K. and Yuliy Sannikov, "A macroeconomic model with a financial sector," *The American Economic Review*, 2014, *104* (2), 379–421.

Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas, "Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares," *American Economic Review*, 2017, *107* (5), 614–20.

Campbell, John Y., "Household finance," *The Journal of Finance*, 2006, *61* (4), 1553–1604.

Cantore, Cristiano, Filippo Ferroni, and Miguel A. León-Ledesma, "The missing link: monetary policy and the labor share," Technical Report, CEPR Working Paper 2019.

Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio, "Demographics and real interest rates: inspecting the mechanism," *European Economic Review*, 2016, *88*, 208–226.

Chen, Peter, Loukas Karabarbounis, and Brent Neiman, "The global rise of corporate saving," *Journal of Monetary Economics*, 2017, *89*, 1–19.

Crump, Richard K., Stefano Eusepi, Andrea Tambalotti, and Giorgio Topa, "Subjective intertemporal substitution," Staff Reports 734, Federal Reserve Bank of New York July 2015.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger, "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 2020.

Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, "Safety, liquidity, and the natural rate of interest," *Brookings Papers on Economic Activity*, 2017, *2017* (1), 235–316.

_ , Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The great escape? A quantitative evaluation of the Fed's liquidity facilities," *American Economic Review*, 2017, *107* (3), 824–57.

DeMarzo, Peter M. and Michael J. Fishman, "Optimal Long-Term Financial Contracting," *Review of Financial Studies*, November 2007, *20* (6), 2079–2128.

_ and Yuliy Sannikov, "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model," *Journal of Finance*, December 2006, *61* (6), 2681–2724.

Di Tella, Sebastian, "Uncertainty Shocks and Balance Sheet Recessions," *Journal of Political Economy*, 2017, *125* (6), 2038–2081.

_ , "Optimal regulation of financial intermediaries," *American Economic Review*, 2019, *109* (1), 271–313.

Eggertsson, Gauti B., Jacob A Robbins, and Ella Getz Wold, "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States," Technical Report, National Bureau of Economic Research 2018.

_ **, Neil R. Mehrotra, and Jacob A. Robbins**, "A model of secular stagnation: Theory and quantitative evaluation," *American Economic Journal: Macroeconomics*, 2019, *11* (1), 1–48.

Farhi, Emmanuel and Francois Gourio, "Accounting for macro-finance trends: Market power, intangibles, and risk premia," Technical Report, NBER Working Paper 2018.

Foley, C. F., J. C. Hartzell, S. Titman, and G. Twite, "Why do firms hold so much cash? A tax-based explanation," *Journal of Financial Economics*, 2007, *86* (3), 579–607.

Geanakoplos, John D and Heraklis M Polemarchakis, "Existence, regularity, and constrained suboptimality of competitive allocations," *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication*, 1986, *3*, 65.

Gourio, Francois, "Disaster risk and business cycles," *American Economic Review*, 2012, *102* (6), 2734–66.

Grossman, G. M., E. Helpman, E. Oberfield, and T. Sampson, "The productivity slowdown and the declining labor share: A neoclassical exploration," Technical Report, NBER Working Paper 2017.

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi, "Insurance within the firm," *Journal of Political Economy*, 2005, *113* (5), 1054–1087.

Gutierrez-Gallardo, Gérman and Sophie Piton, "Revisiting the global decline of the (non-housing) labor share," Technical Report, Bank of England Working Paper 2019.

Hansen, Alvin H., "Economic progress and declining population growth," *The American Economic Review*, 1939, *29* (1), 1–15.

Hansen, Lars Peter, John C. Heaton, and Nan Li, "Consumption strikes back? Measuring long-run risk," *Journal of Political Economy*, 2008, *116* (2), 260–302.

Harrison, Ann, "Has globalization eroded labor's share? Some cross-country evidence," Technical Report, Working Paper 2005.

He, Zhiguo and Arvind Krishnamurthy, "A model of capital and crises," *The Review of Economic Studies*, 2012, *79* (2), 735–777.

Heaton, John and Deborah Lucas, "Portfolio choice and asset prices: The importance of entrepreneurial risk," *The Journal of Finance*, 2000, *55* (3), 1163–1198.

Homer, Sidney and Richard Sylla, *A history of interest rates*, Rutgers University Press, 1996.

Hopenhayn, Hugo, Julian Neira, and Rish Singhania, "From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share," Technical Report, NBER Working Paper December 2018.

Inklaar, Robert and Marcel P. Timmer, "Capital, Labor, and TFP in PWT8.0," Technical Report, Working Paper, Groningen Growth and Development Centre 2013.

Jones, Charles, "Growth, capital shares, and a new perspective on production functions," Technical Report, NBER Working Paper 2003.

Karabarbounis, Loukas and Brent Neiman, "The global decline of the labor share," *The Quarterly Journal of Economics*, 2014, *129* (1), 61–103.

Kiyotaki, Nobuhiro and John Moore, "Credit chains," *Journal of Political Economy*, 1997, *105* (21), 211–248.

Koh, Dongya, Raul Santaeulàlia-Llopis, and Yu Zheng, "Labor share decline and intellectual property products capital," Technical Report, Working Paper 2018.

Krishnamurthy, Arvind, "Collateral constraints and the amplification mechanism," *Journal of Economic Theory*, 2003, *111* (2), 277–292.

León-Ledesma, Miguel A. and Mathan Satchi, "Appropriate technology and balanced growth," *The Review of Economic Studies*, 2018, *86* (2), 807–835.

Marx, Magali, Benoit Mojon, and Francois R. Velde, "Why Have Interest Rates Fallen Far Below the Return on Capital," Technical Report, BIS Working Paper 2019.

Merton, Robert C., *Continuous-time finance, revised edition*, Blackwell Publishing, 1992.

Oberfield, Ezra and Devesh Raval, "Micro data and macro technology," Technical Report, NBER Working Paper 2014.

Opler, T., L. Pinkowitz, R. Stulz, and R. Williamson, "The determinants and implications of corporate cash holdings," *Journal of Financial Economics*, 1999, *52* (1), 3–46.

Piketty, Thomas and Emmanuel Saez, "The evolution of top incomes: a historical and international perspective," *American Economic Review*, 2006, *96* (2), 200–205.

Poterba, James and Andrew Samwick, "Household portfolio allocation over the life cycle," in "Aging Issues in the United States and Japan," University of Chicago Press, 2001, pp. 65–104.

Ríos-Rull, Jose Victor and Raul Santaeulàlia-Llopis, "Redistributive shocks and productivity shocks," *Journal of Monetary Economics*, November 2010, *57* (8), 931–948.

Rodriguez, Francisco and Arjun Jayadev, "The declining labor share of income," *Journal of Globalization and Development*, 2010, *3* (2), 1–18.

Rognlie, Matthew, "Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?," *Brookings Papers on Economic Activity*, 2015, *46* (1), 1–69.

Schmitt-Grohé, Stephanie and Martin Uribe, "The making of a great contraction with a liquidity trap and a jobless recovery," Technical Report, NBER Working Paper 2012.

Shiller, Robert J., *Market volatility*, MIT Press, 1992.

_ , *Irrational exuberance: Revised and expanded third edition*, Princeton University Press, 2015.

Stiglitz, Joseph E, "The inefficiency of the stock market equilibrium," *The Review of Economic Studies*, 1982, *49* (2), 241–261.

Summers, Larry, "Reflections on the 'new secular stagnation hypothesis'," *Secular Stagnation: Facts, Causes and Cures*, 2014, pp. 27–40.

A Additional tables

Table 4: Correlation of simulated variables - Table for the appendix of the paper

Capital Share	Risk Free Position	Risky Position	Risk Free Rate	Consumer Share	Investment	Risk Premium
1.00	-0.97	0.85	-0.99	-0.01	-0.79	0.81
-0.97	1.00	-0.91	0.98	-0.12	0.78	-0.86
0.85	-0.91	1.00	-0.90	0.51	-0.68	0.99
-0.99	0.98	-0.90	1.00	-0.11	0.79	-0.88
-0.01	-0.12	0.51	-0.11	1.00	-0.01	0.58
-0.79	0.78	-0.68	0.79	-0.01	1.00	-0.65
0.81	-0.86	0.99	-0.88	0.58	-0.65	1.00

B Stylized facts

B.1 Post 1980's

Figure 4 plots four statistics which we consider important for empirically validating our work. Panel A shows that the share of risk-free assets in U.S. firms' portfolios has been increasing in the last 30 years. We define risk-free assets as the sum of private foreign deposits, checkable deposit and currency, total time and savings deposits, money market fund shares, security repurchase agreements, commercial paper, treasury securities, agency and GSE backed securities, municipal securities and mutual fund shares. The amount of risk-free assets is then normalized by nominal GDP. Our definition of risk-free assets is similar to the one of broad liquid assets for nonfinancial corporations given by the Board of Governors of the Federal Reserve; ours differs as we consider both financial and nonfinancial corporations and exclude corporate equity, as we will treat equity differently from other liquid assets. The indicator on corporate risk-free debt has received a lot of attention in the recent years because of what is now known as the "Corporate savings glut." This pattern is not peculiar to the U.S. but it is evident in many countries. [22]

Panel B shows an increase in the share of risky assets in households' portfolios as a proportion of GDP. Indeed, households' equity holdings have almost tripled from 1980

[22] This kind of measure is used for example in the empirical literature on the determinants of corporate cash holdings (see Opler et al. (1999)) and in the literature on intangible capital. See Chen et al. (2017) for a characterization of patterns of sectoral saving and investment for a large set of countries over the past three decades. Among other possibilities, increased net lending can be associated with accumulation of cash, repayment of debt, or increasing equity buybacks net of issuance, as highlighted by Bates et al. (2009) and Foley et al. (2007).

to 2014, with a clear upward trend. To compute the series, we normalize the amount of households' directly and indirectly held corporate equities over nominal GDP. This kind of indicator is generally considered for cross sectional analyses in the household finance literature.[23]

Panel C depicts the 10-year U.S. real interest rate, which has been falling continuously since the '80s. This feature has been widely documented; explanations for this falling rate range from demographics, to passing from a secular stagnation, to a sudden increase in uncertainty.[24] Long term nominal interest rates and the consumer price index are taken from the FRED database.

Panel D plots the time series of the U.S. labor share, which hovers around 0.7 until the 2000's, after which experiences a decline. This highlighting that, although one could say that the Kaldor facts hold in the long run, the labor share is moving in the medium run. We chose to compute the labor and capital share similarly to Piketty and Saez (2006)[25]; capital is defined as the sum of consumption of fixed capital and net operating surplus less business current transfer payments, and labor is defined as compensation of employees. We obtain capital and labor shares by normalizing the two quantities over nominal GDP (less taxes). The magnitude and the consequences of the drop in the labor share are controversial issues which are being analyzed in current studies.[26]

B.2 Post WWII and Data Sources

Long-term interest rates. Long-term interest rates are taken from two sources. First, we use FRED data from 1962 to 2017, series DGS10, source Board of Governors of the Federal Reserve System (US), release: H.15 Selected Interest Rates. Units are in percent, not seasonally adjusted, at a monthly frequency. The rates are averages of business days. Inflation is computed from the Consumer Price Index for all urban consumers series CPIAUCSL also taken from FRED. The index series is sampled at monthly frequency, with base year 1982-1984, seasonally adjusted. Data before 1962 are taken from Robert Shiller's update of data shown in Chapter 26 of Shiller (1992), and Shiller (2015). For the long term

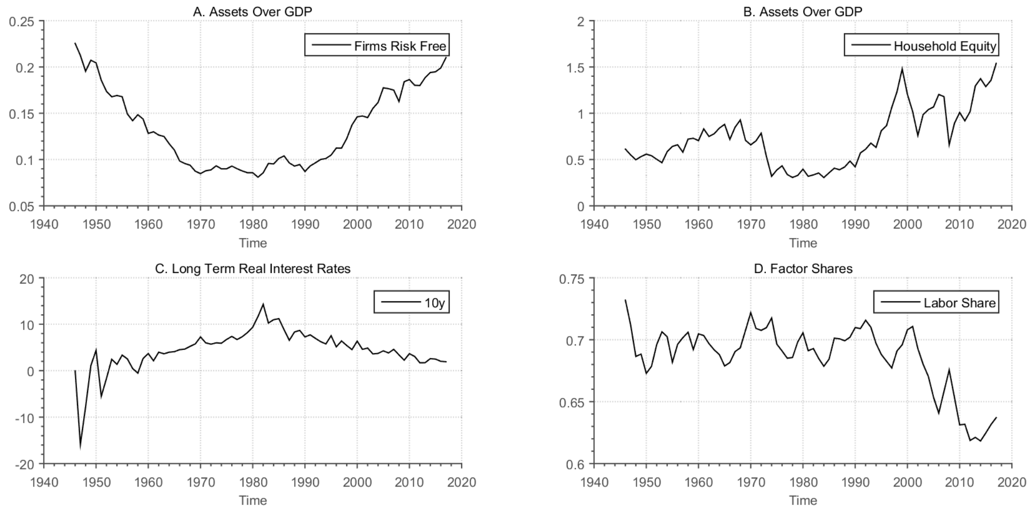
[23]See Campbell (2006). Most papers use SCF data to look at these statistics (Bergstresser and Poterba (2004), Bertaut and Starr (2000), Heaton and Lucas (2000), Poterba and Samwick (2001), while we use the Flow of Funds data to construct a longer series that we show Appendix B.2.

[24]See for example Karabarbounis and Neiman (2014), Caballero et al. (2017), Carvalho et al. (2016) and Summers (2014).

[25]There are a number or ways to compute the labor share, see Karabarbounis and Neiman (2014), Blanchard et al. (1997), Blanchard and Giavazzi (2003), Jones (2003), Bentolila and Saint-Paul (2003), Harrison (2005) and Rodriguez and Jayadev (2010)). FRED has similar calculations from 1950 on, but we chose to use our own measure to construct a longer serie that we show Appendix B.2.

[26]See Rognlie (2015) and Autor et al. (2017).

Figure 6: Saving and equity gluts



Note: U.S. Data. Data are from 1946 on.

interest rate, the author uses the 10-year Treasury after 1953; before 1953, it is government bond yields from Homer and Sylla (1996). Shiller uses the CPI (Consumer Price Index-All Urban Consumers) published by the U.S. Bureau of Labor Statistics. We compute the monthly inflation as the percentage change of the consumer price index and obtain the real 10-year rate by subtracting the inflation, times 100, from the nominal interest rate.

Labor Share. Data from NIPA Table 1.14; Similarly to Piketty and Saez (2006), capital is defined as the sum of consumption of fixed capital and net operating surplus less business current transfer payments, and labor is defined as compensation of employees. We obtain capital and labor shares by normalizing the two quantities over nominal GDP (less taxes).

Assets. Data are from the Board of Governors of the Federal Reserve system. Firms' risk-free debt is defined as the sum of private foreign deposits, checkable deposit and currency, total time and savings deposits, money market fund shares, security repurchase agreements, commercial paper, treasury securities, agency and gse backed securities, municipal securities and mutual fund shares. All the series are in table L.102 Nonfinancial Business, Billions of dollars; amounts outstanding end of period, not seasonally adjusted. To compute household equity we use the B.101.e Balance Sheet of Households and Nonprofit Organizations with Equity Detail, Billions of dollars; amounts outstanding end of period, not seasonally adjusted, precisely the entry Directly and indirectly held corporate equities (serie LM153064475). Both quantities are normalized over nominal GDP.

C Proofs

C.1 Profits are linear in the effective capital stock.

This result is valid for both the two period model and the infinite horizon economy. Recall that the technology is:

$$y(s,i,k,l) = F(g_i g_s k, l) + (1-\delta) g_i g_s k_i$$

where F is homogeneous of degree one. The profits are given by:

$$\pi(s,i,k,l) = \max_l \{F(g_i g_s k, l) + (1-\delta) g_i g_s k_i - \omega(s) l\}$$

Because the technology is CRS, we can write it as:

$$\begin{aligned}
\pi(s,i,k,l) &= \left[\max_{\frac{l}{g_i g_s k_i}} \left\{ F(1, \frac{l}{g_i g_s k_i}) + (1-\delta) - \frac{\omega(s) l}{g_i g_s k_i} \right\}\right] g_i g_s k \\
&= \left[\max_{\tilde{l}} \left\{ F(1,\tilde{l}) + (1-\delta) - \omega(s)\tilde{l} \right\}\right] g_i g_s k \\
&= r(\omega(s)) g_s g_i k \\
&= r(s) g_i k_i
\end{aligned}$$

where we have defined:

$$r(\omega(s)) \equiv \left[\max_{\tilde{l}} \left\{ F(1,\tilde{l}) + (1-\delta) - \omega(s)\tilde{l} \right\}\right]$$

Thus, the shock to capital, including the effect in depreciation, renders the problem linear in individual capital and the idiosyncratic shock. As mentioned before the gross return $r(s)$ includes the depreciation rate. The net return on capital is $r^n(s) \equiv r(\omega(s)) - (1-\delta)$.

C.2 Proof of Propositions 1 and 2

Here we characterize the solution to the two period model. Recall equation (9),

$$\frac{u'(e_1)}{u'(c_1)} = \frac{\mathbb{E}_i[(-\phi(s)Y_2(s) + \alpha(s)Y_2(s)g_i)^{-\sigma}]}{(\phi(s)Y_2(s) + (1-\alpha(s))Y_2(s))^{-\sigma}}$$

Replacing the budget constraint in the period 1 by the individuals consumptions, dividing all the components in period 1 by Y_1, and cancelling $Y_2(s)$ in the right hand side of (9), the last equation can be written as:

$$\left(\frac{y_1 + \sum_s p(s)g_s\phi(s)}{1 - y_1 - \sum_s p(s)g_s\phi(s)}\right)^{-\sigma} = \frac{\mathbb{E}_i[(-\phi(s) + \alpha(s)(s)g_i)^{-\sigma}]}{(\phi(s) + (1 - \alpha(s)))^{-\sigma}}$$

where $y_1 = \alpha_1 + E_1/Y_1$ is the share of resources in hands of entrepreneurs in period 1.

Define the grow adjusted Arrow-Debreu prices as: $\hat{p}(s) = p(s)g_s$. Using the definition of consumption equivalent in (16), and taking into account that the Arrow-Debreu prices satisfy $p(s) = \Pi(s)\frac{\mathbb{E}_i u'(e_2(s,i))}{u'(e_1)}$, we can characterize the equilibrium as the functions $\{\phi(s), \hat{p}(s)\}$ satisfying:

$$\frac{y_1 + \sum_s \hat{p}(s)\phi(s)}{1 - y_1 - \sum_s \hat{p}(s)\phi(s)} = \frac{-\phi(s) + \alpha(s)(s)g^{ce}(s)}{\phi(s) + (1 - \alpha(s))} \tag{45}$$

$$\hat{p}(s) = \Pi(s)\left(\frac{y_1 + \sum_s \hat{p}(s)\phi(s)}{-\phi(s) + \alpha(s)g^{ce}(s)}\right)^{\sigma} g_s^{1-\sigma} \tag{46}$$

Notice that $\sum_s \hat{p}(s)\phi(s)$ are the total entrepreneur's savings in units of time 1 output. Thus $y_1 + \sum_s \hat{p}(s)\phi(s)$ is the normalized consumption of the capitalist. With these definitions the total economy's wealth in units of period output is $1 + \sum_s \hat{p}(s)$. Similarly, the worker's present value of resources, normalized by Y_1, is: $1 - y_1 + \sum_s \hat{p}(s)(1 - \alpha(s))$. Therefore, the equilibrium initial wealth ratio is:

$$x_1 = \frac{1 - y_1 + \sum_s \hat{p}(s)(1 - \alpha(s))}{1 + \sum_s \hat{p}(s)} \tag{47}$$

To simplify notation define:

$$P = \sum_s \hat{p}(s) \in \mathbb{R}_+; \quad \text{and} \quad P_\phi = \sum_s \hat{p}(s)\phi(s) \in \mathbb{R}$$

Operating with equation (45) we can write:

$$\phi(s) = \alpha(s)g^{ce}(s)[1 - y_1 - P_\phi] - (1 - \alpha(s))[y_1 + P_\phi]; \;\; \forall s \tag{48}$$

Multiplying the last by $\hat{p}(s)$ and adding up we obtain:

$$P_\phi = \sum_s \hat{p}(s)\alpha(s)g^{ce}(s)[1 - y_1 - P_\phi] - \sum_s \hat{p}(s)(1 - \alpha(s))[y_1 + P_\phi]$$

$$P_{\phi} = \sum_{s} \hat{p}(s)\alpha(s) - P[y_1 + P_{\phi}] + \sum_{s} \hat{p}(s)\alpha(s)[g^{ce}(s) - 1][1 - y_1 - P_{\phi}]$$

Adding y_1 in both sides of the last equation and reorganizing generates:

$$y_1 + P_{\phi} = \frac{y_1 + \sum_s \hat{p}(s)\alpha(s)}{1+P} + \frac{\sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1][1 - y_1 - P_{\phi}]}{1+P}$$

$$y_1 + P_{\phi} = 1 - x_1 + \hat{c} \sum_{s} \hat{p}(s)\alpha(s)[g^{ce}(s) - 1] \tag{49}$$

where $\hat{c} = \frac{1-y_1-P_{\phi}}{1+P} \geq 0$. Hence, equations (48) and (49) generate the solution for the quantities of transacted assets.

To solve for the prices note that equation (48) can be rewritten as:

$$[y_1 + P_{\phi}][(1 - \alpha(s)) + \alpha(s)g^{ce}(s)] = -\phi(s) + \alpha(s)g^{ce}(s)$$

Using the last relationship in the price equation (46) we obtain:

$$\hat{p}(s) = \Pi(s) \left(\frac{1}{1 + \alpha(s)[g^{ce}(s) - 1]} \right)^{\sigma} g_s^{1-\sigma} \tag{50}$$

Proof of Proposition 1. When there is no idiosyncratic risk $g^{ce}(s) = 1$ for all s. Thus, combining equations (48) and (49) with $g^{ce}(s) = 1$ we obtain:

$$\phi(s) = \alpha(s) - (1 - x_1); \quad \forall s$$

While equation (50) generates:

$$\hat{p}(s) = \Pi(s) g_s^{1-\sigma}; \quad \forall s$$

Using the fact that $\hat{p}(s) = p(s)g_s$, the last two equations deliver the results shown in equation (12), part b) of Proposition 1. Moreover, because the second period is the last one, the wealth ratio is equal to the income ratio: $1 - x_2 = -\phi(s) + \alpha(s)$. Finally, since $-\phi(s) + \alpha(s) = 1 - x_1$ by the last results, it follows that $x_1 = x_2$. The wealth ratio is constant, delivering part a) of Proposition 1.

Proof of Proposition 2. The results of Proposition 2 are conditional on equal wealth ratios. Since prices differ with complete and incomplete markets, one can think there are different distributions of initial assets that equalize them. To be precise, using equation (47), for any x_1^{CM}, given initial asset holdings E_1, there exists an alternative $\hat{E}_1$ such that $x_1 = x_1^{CM}$. This allows as to focus on the risk effect postponing the discussion of wealth effects to the full dynamic model of Section 3. Thus, from now on we work under the assumption that $x_1 = x_1^{CM}$ and later we addressed the issue on how wealth shares differ.

Precautionary savings. Recall that $y_1 + P_\phi$ is the capitalist's normalized consumption and also its consumption share out of output. In the complete markets economy $y_1 + P_\phi = 1 - x_1$, so that the consumption share is equal to the wealth ratio. However, by equation (49), when markets are incomplete $y_1 + P_\phi < 1 - x_1$ because $g^{ce}(s) < 1$, $\forall s$. Due to the presence of uninsured idiosyncratic risk, capitalists consume a smaller proportion of their wealth. If they are consuming less, it is because they are saving more. Using equation (49) the difference in total value of the worker's savings satisfies:

$$P_\phi^{IM} - P_\phi^{CM} = \hat{c} \sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1] < 0$$

which proves part a) of Proposition 2. By market clearing the capitalists are saving more. This does not depend on P_ϕ being positive or negative. If P_ϕ is positive with complete markets, given the same wealth ratio, it is less positive (or even negative) with incomplete markets. If P_ϕ is already negative, it is more negative when there is idiosyncratic risk.

Using $\hat{p}(s) = p(s)g_s$ in equation (50) generates the first equation in (18). To obtain the second relationship, reorganize equation (48):

$$\phi(s) = \alpha(s)g^{ce}(s) - (y_1 + P_\phi) + \alpha(s)[1 - g^{ce}(s)][y_1 + P_\phi]$$

Replacing (49) in the last:

$$\phi(s) = \alpha(s)g^{ce}(s) + [\alpha(s)(1 - g^{ce}(s)) - 1][1 - x_1 + \hat{c} \sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1]]$$

Define:

$$\Gamma := -\hat{c} \sum_s \hat{p}(s)\alpha(s)[g^{ce}(s) - 1]] > 0$$

Because $g^{ce}(s) < 0$ for all s, then:

$$\phi(s) = \alpha(s)g^{ce}(s) + [\alpha(s)(1 - g^{ce}(s)) - 1][1 - x_1 - \Gamma]$$

$$\phi(s) = \alpha(s) - (1 - x_1) + \Gamma + \alpha(s)(1 - g^{ce}(s))(-x_1 - \Gamma)$$

The last result is the second equation in (18) of Proposition 2.

To prove part c) notice that equation (45) can also be written as:

$$\frac{e_1}{1-e_1} = \frac{-\phi(s) + \alpha(s)(s)g^{ce}(s)}{\phi(s) + (1-\alpha(s))}$$

where e_1 is the normalized consumption of the capitalist, and thus $\phi(s) = \alpha(s)g^{ce}(s) + \alpha(s)(1-g^{ce}(s))e_1 - e_1$, which generates:

$$\frac{\partial\phi(s)}{\partial\alpha} = \frac{\partial[\alpha(s)g^{ce}(s)]}{\partial\alpha}(1-e_1) + e_1$$

If markets are complete, then $g^{ce}(s) = 1$ and $\frac{\partial\phi(s)}{\partial\alpha} = 1$, while in Appendix C.3 we show that $\frac{\partial[\alpha(s)g^{ce}(s)]}{\partial\alpha} < 1$ as long as $g^{ce}(s) < 1$.

Constant shares. When the income shares are constant the expression for ϕ can be further simplified. Since $\alpha(s) = \alpha$ and $g^{ce}(s) = g^{ce}$, for all s, equation (49) can be written as:

$$y_1 + P_\phi = 1 - x_1 + \frac{1 - y_1 - P_\phi}{1+P} P\alpha[g^{ce} - 1]$$

Which yields:

$$y_1 + P_\phi = \frac{y_1 + \alpha g^{ce} P}{1 + (1-\alpha+\alpha g^{ce})P}$$

Using the last in (48) we obtain:

$$\phi = \frac{\alpha g^{ce} - y_1(1-\alpha+\alpha g^{ce})}{1 + (1-\alpha+\alpha g^{ce})P}$$

Denoting by ϕ_1 the worker's initial financial position, then $y_1 = -\phi_1 + \alpha$, which replaced in the above generates:

$$\phi = \frac{\phi_1[g^{ce}(\phi)\alpha + 1 - \alpha] + \alpha(1-\alpha)[g^{ce}(\phi) - 1]}{1 + P[g^{ce}(\phi)\alpha + 1 - \alpha]} \tag{51}$$

The workers ratio of wealth at period 1 is $x_1 = [\phi_1 + (1-\alpha)(1+P)]/(1+P)$ while in the second period the equivalent ratio is $x_2 = \phi + 1 - \alpha$. Thus, the worker's proportion of wealth decreases whenever:

$$\phi \leq \frac{\phi_1}{1+P}$$

Using the last with (51) it follows that the wealth ratio decreases always that $g^{ce}(\phi) < 1$ and $\phi_1 > -(1-\alpha)(1+P)$. But the last condition must always be satisfied because otherwise the worker would have negative consumption.

C.3 Proof that $\frac{\partial[\alpha(s)g^{ce}(s)]}{\partial\alpha} < 1$

Replace the solution for $\phi(s)$ in equation (16) and use the second order approximation of the RHS as in equation (15). Then we have:

$$(-\alpha(s)e_1 + \alpha(s)g^{ce}(s)e_1 + e_1)^{-\sigma} = (-\alpha(s)g^{ce}(s)(1-e_1) - \alpha(s)e_1 + e_1 + \alpha(s))^{-\sigma} + \frac{\sigma(1+\sigma)\alpha(s)^2\mathbb{V}ar(g_i)/2}{(-\alpha(s)g^{ce}(s)(1-e_1) - \alpha(s)e_1 + e_1 + \alpha(s))^{\sigma+2}}; \quad \forall s.$$

Define $y(s) \equiv \alpha(s)g^{ce}(s) - \alpha(s)$. The previous equation becomes:

$$(1+y(s))e_1 = [e_1 - y(s)(1-e_1)]\left(1 + \frac{\sigma(1+\sigma)\alpha(s)^2}{(e_1 - y(s)(1-e_1))^2}\frac{\mathbb{V}ar(g_i)}{2}\right)^{-1/\sigma}; \quad \forall s.$$

Define:

$$M \equiv (1+y(s))e_1 - [e_1 - y(s)(1-e_1)]\left(\underbrace{1 + \frac{\sigma(1+\sigma)\alpha(s)^2}{(e_1 - y(s)(1-e_1))^2}\frac{\mathbb{V}ar(g_i)}{2}}_{\aleph}\right)^{-1/\sigma} = 0$$

Hence,

$$\frac{\partial M}{\partial y}dy + \frac{\partial M}{\partial \alpha}d\alpha = 0 \;\Rightarrow\; \frac{dy}{d\alpha} = -\frac{\frac{\partial M}{\partial \alpha}}{\frac{\partial M}{\partial y}}.$$

Differentiating $M(\cdot)$ we obtain:

$$\begin{aligned}
\frac{\partial M}{\partial \alpha} &= [e_1 - y(s)(1-e_1)]^{-1}\left((1+\sigma)\mathbb{V}ar(g_i)\right)\aleph^{-\frac{1}{\sigma}-1}\alpha \\
\frac{\partial M}{\partial y} &= e_1 + (1-e_1)\aleph^{-\frac{1}{\sigma}} + [e_1 - y(s)(1-e_1)]^{-2}\left((1+\sigma)\mathbb{V}ar(g_i)\right)\aleph^{-\frac{1}{\sigma}-1}\alpha^2(1-e_1)
\end{aligned}$$

Combining the two:

$$\frac{\partial M}{\partial y} = e_1 + (1-e_1)\aleph^{-\frac{1}{\sigma}} + \frac{\partial M}{\partial \alpha}[e_1 - y(s)(1-e_1)]^{-1}\alpha(1-e_1)$$

Then,

$$\frac{dy}{d\alpha} = -\left[\underbrace{\frac{e_1 + (1-e_1)\aleph^{-\frac{1}{\sigma}}}{\frac{\partial M}{\partial \alpha}}}_{>0} + \underbrace{\frac{\alpha(1-e_1)}{e_1 - y(s)(1-e_1)}}_{>0}\right]^{-1}$$

As a result:

$$\frac{dy}{d\alpha} < 0 \quad \Rightarrow \frac{d(\alpha(s)g^{ce}(s) - \alpha)}{d\alpha} < 0 \quad \Rightarrow \frac{d\alpha(s)g^{ce}(s)}{d\alpha} < 1$$

C.4 Derivation Equations (13) and (14)

In equilibrium $A^c + A^e = 0$ and $B^c + B^e = 0$. Consider the problem of the consumer. Note that there is a one-to-one mapping between the second period's payoffs of the AD securities $\phi(s)$ and of the portfolio with two assets:

$$\phi(L)Y(L) = R_L B^c + A^c \alpha(L)Y(L)$$
$$\phi(H)Y(H) = R_L B^c + A^c \alpha(H)Y(H)$$

The latter implies positions and prices given by

$$A^c = \frac{\phi(H)Y(H) - \phi(L)Y(L)}{\alpha(H)Y(H) - \alpha(L)Y(L)} \tag{52}$$

$$R_L B^c = \frac{Y(L)Y(H)(\phi(L)\alpha(H) - \alpha(L)\phi(H))}{\alpha(H)Y(H) - \alpha(L)Y(L)} \tag{53}$$

$$R_L = \frac{1}{\sum_s p(s)} \tag{54}$$

$$P_A = \sum_s p(s)\alpha(s)Y(s) \tag{55}$$

Where $p(s)$ is the price of the AD securities, then both the consumer and the entrepreneur are optimizing. Using the results of Proposition 1 we have equations (13) and (14).

C.5 Capital share in the CES production function

The firms maximizes $\pi(s,i) = \left[\alpha\left(g_i g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - \omega L$, which implies $L^d(s,i) = \alpha^{\frac{\rho}{\rho-1}}\left[\left(\frac{\omega}{1-\alpha}\right)^{\rho-1} - (1-\alpha)\right]^{\frac{\rho}{1-\rho}} g_i g_s k$. From the labor market clearing condition $1 = L^s = L^d(s) = \mathbb{E}(L^d(s,i))$ we obtain the wage:

$$\omega(s) = (1-\alpha)\left[\alpha\left(g_s k\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)\right]^{\frac{1}{\rho-1}}$$

Moreover recall that:

$$\alpha(s,i) = \frac{\partial y(s,i)}{\partial k}\frac{k}{y(s,i)} = \frac{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_i g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)(L)^{\frac{\rho-1}{\rho}}}$$

so $\alpha(s) = \mathbb{E}_i(\alpha(s,i))$ is given by:

$$\alpha(s) = \frac{\alpha(g_s k)^{\frac{\rho-1}{\rho}}}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

and, given that $Y(s) = \mathbb{E}(y(s,i))$, in the same way, the labor share is:

$$(1-\alpha(s)) = \frac{\partial Y}{\partial L}\frac{L}{Y} = \frac{(1-\alpha)}{\alpha(g_s k)^{\frac{\rho-1}{\rho}} + (1-\alpha)}$$

Then $\omega(s) = (1-\alpha(s))Y(s)$. Given the wage, $L^d(s,i) = g_i$ and therefore $\pi(s,i) = \alpha(s)Y(s)g_i$.

C.6 Proof of Proposition 3

First notice that when capitalists can fully insure their idiosyncratic risk $m(s) = 1$, for all s. Then from equations (24) and (35) follows that $\zeta(s) = \vartheta(s)$. In addition, in Appendix F we show that the law of motion of the wealth ratio can be written as:

$$x(s'|s) = \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s',i,s)\vartheta(s)(1-x) + \phi^c(s'|s)\zeta(s)x},$$

where $\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}$ satisfies:

$$\left(\mathbb{E}_i o(s',i;\phi^e)^{-\sigma}\right)^{-\frac{1}{\sigma}} = \tilde{\beta}(s',s)\frac{(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))};\ \ \forall s,s'$$

When all idiosyncratic risk is insured $\mathbb{E}_i o(s',i;\phi^e)^{-\sigma} = [\mathbb{E}_i o(s',i;\phi^e)]^{-\sigma}$, thus using this and $\zeta(s) = \vartheta(s)$, from the last equation and (23) we obtain:

$$\mathbb{E}_i o(s',i;\phi^e) = \frac{\tilde{\beta}(s',s)(1-\vartheta(s))}{\vartheta(s)(1-\vartheta(s'))} = \frac{\tilde{\beta}(s',s)(1-\zeta(s))}{\zeta(s)(1-\zeta(s'))} = \phi^c(s'|s);\ \ \forall s,s'$$

Using the last equation in the first delivers the result.

C.7 Proof of Proposition 4

Assume that $\delta = 1$ and guess an equilibrium with constant x. To do this, assume that entrepreneurs have a different discount rate β^e. We'll pick its value to make sure that x is constant. We use a guess and verify strategy, guessing that prices and the risk adjustment factor satisfy:

$$p(s'|s) = A_0 \beta \Pi(s'|s) \tilde{g}(s'|s)^{-\sigma} \qquad \textit{and} \qquad m(s) = m; \qquad \forall s, s' \tag{56}$$

for some constant $A_0 > 0$ and $m \geq 1$; $\forall x$. Later we verify this guess. We prove this proposition in a series of steps showing that: 1) the savings rates are independent of the state, 2) holdings of contingent assets are proportional to growth, 3) the investment rate and portfolio allocations are constant, 4) the wealth growth rates are independent of the state. In the final steps we verify the guesses in (56).

Savings rates are independent of aggregate shock. Now we have two $\tilde{\beta}(s'|s)$, using the guessed prices in $\tilde{\beta}(s'|s)$ we obtain:

$$\tilde{\beta}(s'|s) = \left[\frac{\beta \Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} = \left[\frac{\beta \Pi(s'|s)}{A_0 \beta \Pi(s'|s) \tilde{g}(s'|s)^{-\sigma}}\right]^{1/\sigma} = \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}}$$

but also:

$$\tilde{\beta}^e(s'|s) = \left[\frac{\beta^e \Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} = \left[\frac{\beta^e \Pi(s'|s)}{A_0 \beta \Pi(s'|s) \tilde{g}(s'|s)^{-\sigma}}\right]^{1/\sigma} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}}$$

Guess that the savings rates are constant. The last in equation (24) together with the guessed price implies that the solution for the worker's saving rate is:

$$\zeta(s') = \zeta(s) = \zeta = \beta \frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}; \qquad \forall s, s'$$

Doing similar calculations with equation (35) we obtain:

$$\vartheta(s) = \vartheta = \beta \frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{m A_0^{\frac{1-\sigma}{\sigma}}} \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}; \qquad \forall s, s' \tag{57}$$

AD securities proportional to $\tilde{g}(s'|s)$. From equation (23) and the computed value of $\tilde{\beta}(s'|s)$ we obtain:

$$\phi^c(s'|s) = \frac{\tilde{g}(s'|s)}{\zeta A_0^{1/\sigma}} = \frac{\tilde{g}(s'|s)}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}.$$

The equivalent condition for the entrepreneur (see equation (74) in the online appendix) generates:

$$\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \frac{\tilde{g}(s'|s)}{\vartheta A_0^{1/\sigma}} = \frac{\tilde{g}(s'|s)m}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}} \qquad \forall s, s' \tag{58}$$

Note that this implies that $\phi^c(s'|s) = \frac{\tilde{\beta}(s'|s)}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}$ and $\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = \frac{\tilde{\beta}^e(s'|s)m}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}$

The capitalist's growth rate of wealth is $o(s', i; \phi^e) \equiv [(1-\nu(s))\phi^e(s'|s) + \nu(s)r(s')g_i]$. Define the portfolio allocation as:

$$\frac{(1-\nu(s))\phi^e(s'|s)}{\nu(s)r(s')} = \frac{1}{D}$$

Here we are guessing that D is constant. With this equation it is immediate (recall $\sum_{s'} p(s'|s)\phi^e(s'|s) = 1$) that

$$m \equiv \sum_{s'|s} p(s'|s) \left(\mathbb{E}_i o(s', i; \phi^e)^{-\sigma}\right)^{-1/\sigma} = (1-\nu(s)) \left[\mathbb{E}_i (1 + D g_i)^{-\sigma}\right]^{-1/\sigma} \tag{59}$$

As a result:

$$\left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = (1-\nu(s))\phi^e(s'|s) \left[\mathbb{E}_i (1 + D g_i)^{-\sigma}\right]^{-1/\sigma} = \frac{\tilde{\beta}(s'|s)m}{\sum_{s'} p(s'|s)\tilde{\beta}(s'|s)}$$

Because of (59) the last implies $\phi^e(s'|s) = \phi^c(s'|s), \quad \forall s, s'$

Investment rates are constant. Using portfolio choice D in equation (33) :

$$\mathbb{E}_{s',i|s}\left[\left[(1-\nu(s))\phi^e(s'|s)(1+Dg_i)\right]^{-\sigma}\left(\frac{(1-\nu(s))\phi^e(s'|s)}{\nu(s)}Dg - \frac{1}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{-\sigma}}\right)\right] = 0$$

Since i is independent of s we can write:

$$\mathbb{E}_{s'|s}\left[\frac{\left[(1-\nu(s))\phi^e(s'|s)\right]^{1-\sigma}}{\nu(s)}\right]\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right) = \frac{\mathbb{E}_{s'|s}\left[(1-\nu(s))\phi^e(s'|s)\right]^{-\sigma}\mathbb{E}_i(1+Dg_i)^{-\sigma}}{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{-\sigma}}$$

$$\frac{(1-\nu(s))}{\nu(s)}\mathbb{E}_{s'|s}[\phi^e(s'|s)]^{1-\sigma}\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right) = \frac{\mathbb{E}_{s'|s}\left[\phi^e(s'|s)\right]^{-\sigma}\mathbb{E}_i(1+Dg_i)^{-\sigma}}{\beta A_0 \mathbb{E}_{s'}\tilde{g}(s'|s)^{-\sigma}}$$

Doing similar calculations as before to replace $\phi^e(s'|s)$ we obtain:

$$\frac{\mathbb{E}_i\left((1+Dg_i)^{-\sigma}Dg_i\right)}{\mathbb{E}_i(1+Dg_i)^{-\sigma}} = \frac{\nu}{(1-\nu)} \tag{60}$$

So ν is constant whenever D is constant. Also, with D and ν we can compute the value of $m(s)$ given by equation (59), which confirms that m is constant. To solve for D use portfolio equation to get:

$$\frac{(1-\nu)}{\nu}D = \frac{r(s')}{\phi^e(s'|s)} = \frac{r(s')}{\phi^c(s'|s)} = \frac{r(s')}{\tilde{g}(s'|s)}\beta A_0 \mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}$$

With an AK model $r(s')$ is exogenous, so the above equation pins down D,which it won't depend on x. In a more general model $r(s')$ would depend on aggregate capital. With constant shares $r(s') = \alpha\frac{y(s')}{K'}$, which can be written as $r(s') = \alpha\tilde{g}(s'|s)\frac{y(s)}{K'}$. But since the capital law of motion is $K' = \vartheta\nu(1-x)W^T(s)$, we can write the previous equation as:

$$r(s') = \frac{\tilde{g}(s'|s)}{\beta A_0 \mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}\frac{(1-\nu)}{\nu}D$$

$$\alpha\tilde{g}(s'|s)\frac{y(s)}{\vartheta\nu(1-x)W^T(s)} = \frac{\tilde{g}(s'|s)}{\beta A_0 \mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}\frac{(1-\nu)}{\nu}D$$

$$\frac{\alpha}{\vartheta(1-x)}\frac{y(s)}{W^T(s)} = \frac{(1-\nu)D}{\beta A_0 \mathbb{E}_{s'|s}\tilde{g}(s'|s)^{1-\sigma}}$$

Now replacing ϑ from the previously found value:

$$\frac{\alpha m A_0^{\frac{1}{\sigma}}}{(1-x)}\frac{y(s)}{W^T(s)} = (1-\nu)D\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}$$

$$\frac{\alpha\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{-1/\sigma}A_0^{\frac{1}{\sigma}}}{(1-x)}\frac{y(s)}{W^T(s)} = D\left(\frac{\beta^e}{\beta}\right)^{1/\sigma} \tag{61}$$

Where in the last step we have replaced m from equation (59). This equation solves for D, then (60) delivers ν, and then with (59) we obtain m. All this variables are constant if 1) α is constant and 2) the ratio $\frac{y(s)}{W^T(s)}$ is constant. What we need in general is that this solution is independent of the aggregate shock, it could depend on x or k as long as it does it in deterministic way. Here we are considering the case of x constant to simplify the calculations, but it is not needed for the argument about the amplification effects. We consider these cases in the following extensions of this proposition.

GDP-wealth ratio is constant. Suppose $\delta = 1$ Note that (56) implies that $\sum_{s'|s} p(s'|s) = \beta A_0 \mathbb{E}\tilde{g}(s'|s)^{-\sigma} = \tilde{p}$. Using equation (44) with constant shares we obtain:

$$\frac{w(s') + h(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{[1 - \alpha(s') + \tilde{h}(s')]}{1 + \tilde{h}(s)} = \tilde{g}(s'|s)\frac{(1-\alpha)[1 + G(s')]}{1 + (1-\alpha)G(s)}$$

where $G(s) = \sum_{s'|s} p(s'|s)\left[(1 + G(s'))\tilde{g}(s'|s)\right]$ is the present value of a constant dividend unit with grow factor $\tilde{g}(s'|s)$ Similarly:

$$\frac{W^T(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{1 + (1-\alpha)G(s')}{1 + (1-\alpha)G(s)}$$

Because the distributions of growth rates are independent of the state and using the guessed pricing function, it is straightforward to show that $G(s') = G(s) = G, \ \forall s, s'$; then it follows that:

$$\frac{W^T(s')}{W^T(s)} = \tilde{g}(s'|s); \qquad \frac{w(s') + h(s')}{W^T(s)} = \tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1 + (1-\alpha)G}.$$

Also,

$$\frac{y(s)}{W^T(s)} = \frac{y(s)}{y(s) + h(s')} = \frac{1}{1 + \tilde{h}(s)} = \frac{1}{1 + (1-\alpha)G}$$

Thus the ratio $\frac{y(s)}{W^T(s)}$ is constant and therefore also ν, D and m are.

Use feasibility to find A_0. We can use feasibility to dig further into the solutions. Replacing (59) in (57) generates:

$$\vartheta(1-\nu) = \beta\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\left[\mathbb{E}_i(1 + Dg_i)^{-\sigma}\right]^{1/\sigma}$$

We can use the last equation in the feasibility constraint:

$$\zeta(x)x + \vartheta(s)(1 - \nu(s))(1 - x) = 1 - \frac{y(s)}{W^T(s)}; \ \forall s$$

$$\beta\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1-\sigma}{\sigma}}}\left[x + (1-x)\left[\mathbb{E}_i(1 + Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right] = \frac{(1-\alpha)G}{1 + (1-\alpha)G}; \ \forall s \qquad (62)$$

Computing the present values with the guessed function for $p(s'|s)$ we obtain:

$$G = \frac{\beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{1 - \beta A_0 \mathbb{E}\tilde{g}(s'|s)^{1-\sigma}} \tag{63}$$

Hence, equation (62) generates the value of A_0.

Verify guessed prices. For the AD prices we compute a price equation akin to that in the online Appendix F (see equation (86)). To make the proof self-contained we replicate some calculations adapted to this environment. Recall that the AD securities must satisfy:

$$\phi^c(s'|s) = \left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\zeta(s))}{(1-\zeta(s'))\zeta(s)}; \quad \left[\mathbb{E}_i o(s', i, \phi^e)^{-\sigma}\right]^{-1/\sigma} = \left[\frac{\beta^e\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}, \forall s, s'$$

Using the definition of D, the second equality can be written as:

$$(1-\nu)\phi^e(s'|s)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{-1/\sigma} = \left[\frac{\beta^e\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \frac{(1-\vartheta(s))}{(1-\vartheta(s'))\vartheta(s)}, \forall s, s'$$

Replacing the last two in the assets' market clearing we obtain:

$$\left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \left[x + (1-x)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right] = \tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1+(1-\alpha)G}; \quad \forall s, s'$$

Using equation (62) the above becomes:

$$\left[\frac{\beta\Pi(s'|s)}{p(s'|s)}\right]^{1/\sigma} \left[\frac{(1-\alpha)G}{1+(1-\alpha)G}A_0^{\frac{1-\sigma}{\sigma}}\right] = \beta\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\tilde{g}(s'|s)\frac{(1-\alpha)(1+G)}{1+(1-\alpha)G}; \quad \forall s, s'$$

Therefore,

$$p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}\left(\beta\frac{(1+G)}{A_0^{\frac{1-\sigma}{\sigma}}G}\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\right)^{-\sigma}$$

Using the solution for G from (63), we obtain the initially guessed price function $p(s'|s) = \beta\Pi(s'|s)\tilde{g}(s'|s)^{-\sigma}A_0$.

Verify that x is constant with the appropriate choice of β^e. To solve for the evolution of x, recall that

$$x(s'|s) = \frac{\phi^c(s'|s)\zeta(s)x}{\mathbb{E}_i o(s', i, s)\vartheta(s)(1-x) + \phi^c(s'|s)\zeta(s)x}$$

Using definition of $o(s', i, s)$ and because we already showed that $\phi^e(s', s) = \phi^c(s', s)$ we can write the last as:

$$x' = \frac{\zeta x}{[(1-\nu)(1+D)]\vartheta(1-x)+\zeta x}$$

Now, note that savings rates satisfy

$$\frac{\vartheta}{\zeta} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\frac{1}{m} = \left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\frac{[\mathbb{E}_i(1+Dg_i)^{-\sigma}]^{1/\sigma}}{1-\nu}$$

where in the last step we have used (59). Thus, the last in the law of motion of x generates:

$$x' = \frac{x}{\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}(1+D)\,[\mathbb{E}_i(1+Dg_i)^{-\sigma}]^{1/\sigma}(1-x)+x}$$

Which implies that for $x' = x$ to be true β^e must satisfy:

$$\beta^e = \beta\frac{(1+D)^{-\sigma}}{\mathbb{E}_i(1+Dg_i)^{-\sigma}} \tag{64}$$

Because of he Jensen's inequality and the convexity of the marginal utility, so that $\mathbb{E}_i(1+Dg_i)^{-\sigma} \geq (1+D)^{-\sigma}$, it is clear that $\beta^e < \beta$. The entrepreneurs must have a smaller discount factor, otherwise x would converge to zero. The correction in the discount factor corrects the upwards drift in the capitalist's savings needs. When there is not exposure to idiosyncratic risk, there is no need for the correction.

Check solution is correct. Alternatively, the x's law of motion is characterized by:

$$x(s'|s) = \phi^c(s'|s)\zeta(s)\frac{W^T(s)}{W^T(s')}x.$$

Replacing the relationships for $\phi^c(s'|s)$, $\zeta(s)$ and the grow rate of wealth implies:

$$x(s'|s) = \frac{\tilde{g}(s'|s)}{A_0^{1/\sigma}}\frac{1}{\tilde{g}(s'|s)}x$$

Hence, if $x' = x$, it must be that $A_0 = 1$. Is this true? The value of A_0 is determined by equation (62), which can be written as:

$$\beta A_0\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}}\left[x+(1-x)\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right] = 1-\frac{y}{W^T}$$

To show that indeed $A_0 = 1$ first notice that equation (64) implies

$$\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}=1-D\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}$$

Therefore, replacing the latter in the former:

$$\beta A_0\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}}\left[1-(1-x)D\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}\right]=1-\frac{y}{W^T}$$

Now, collecting the term $(1-x)D\left[\mathbb{E}_i(1+Dg_i)^{-\sigma}\right]^{1/\sigma}\left(\frac{\beta^e}{\beta}\right)^{1/\sigma}$ in equation (61) and replacing in the above we obtain:

$$\beta A_0\frac{\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{A_0^{\frac{1}{\sigma}}}\left[1-\alpha A_0^{\frac{1}{\sigma}}\frac{y}{W^T}\right]=1-\frac{y}{W^T}$$

Therefore we have:

$$\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\left[\frac{1}{A_0^{\frac{1}{\sigma}}}+\frac{(1-\alpha\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma})}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}\frac{y}{W^T}\right]=1$$

We showed before that $\frac{y}{W^T}=\frac{1}{1+(1-\alpha)G}$, which using equation (63) generates

$$\frac{y}{W^T}=\frac{1-\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{1-\alpha\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}$$

Replacing the latter in the former:

$$\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}\left[\frac{1}{A_0^{\frac{1}{\sigma}}}+\frac{1-\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}{\beta A_0\mathbb{E}\tilde{g}(s'|s)^{1-\sigma}}\right]=1$$

Which can only be true if $A_0=1$.

D Numerical Appendix

a. Using $g_s k$, compute once and for all $Y(s)$, $r(s)$ and $w(s)$

b. Step A: Solve given a guess.

   (a) Guess

      i. $\tilde{\beta}(s'|s)$ and $p_0(s'|s)$ and compute $W^T(s)$ and $h(s)$.

      ii. "ratio" $Ra(s',s)$. Start with $Ra(s',s)=1, \forall s,s'$.

(b) Find $\zeta(s)$. Use $\tilde{\beta}(s'|s)$ and $p_0(s'|s)$ in (24) to solve for $\zeta(s)$. Then use (23) to get: $\phi^c(s'|s)$.

(c) Find $\vartheta(s)$. Use $\tilde{\beta}(s'|s)$, $p_0(s'|s)$ and $Ra(s', s)$ to solve for $\vartheta(s)$. To this end use (82).

(d) Use $\zeta(s)$, $\vartheta(s)$ and $\phi(s'|s)$ in the market clearing for assets (75) to solve for $\phi^e(s'|s)$. The explicit formula for $\phi^e(s'|s)$ is given by equation (87).

(e) Use $\zeta(s)$ and $\vartheta(s)$ in (76) to solve for $\nu(s)$. The explicit formula for $\nu(s)$ is given by equation (88).

(f) End of Step A. At this point, given the initial guess all the solutions have been computed.

c. Step B: Update

(a) Use $\phi^e(s'|s)$ and $\nu(s)$ in (81) to update the guess for $Ra(s'|s)$.

(b) Using (83) and (84) compute a new law of motion of s: $\Pi_1(s'|s)$. Use linear interpolation.

(c) Use (85) to compute a new $\tilde{\beta}_1(s', s)$ and $p_1(s', s) = \beta\Pi_1(s'|s)\tilde{\beta}(s', s)^{-\sigma}$

d. Step C: check convergence. If $\Pi_1 = \Pi_0$ and $p_1 = p_0$, stop otherwise update and start again in step 2 with:

$$\Pi_0 = 0.5\Pi_0 + 0.5\Pi_1; \quad p_0 = 0.5p_0 + 0.5p_1; \quad \tilde{\beta}_0 = 0.5\tilde{\beta}_0 + 0.5\tilde{\beta}_1.$$